

Distributed Compressed Sampling Architecture for Maximum Likelihood Signal Detection

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Abstract

Cognitive radios are a new technology introduced to resolve the spectrum scarcity problem by superimposing new services in the already allocated bands under a non-interference constraint. It has been recently demonstrated that the challenging implementation of the signal detectors can be facilitated by using the theory of compressive sampling. In this paper, we consider a distributed network of secondary nodes that cooperate to detect the primary signals. Each secondary node samples the signal periodically at a rate much smaller than the Nyquist rate. The delays inherent to the propagation channel are used to implement a periodic non-uniform sampling detector when the secondary nodes combine their observations. We demonstrate that the proposed detector can efficiently detect the primary user signal, even under fading channels.

I. INTRODUCTION

Many studies have shown that static frequency allocation for wireless communication systems is responsible for the inefficient use of the spectrum. Cognitive radios (CR) resolve the problem by first detecting the still available frequency resources and then using them for their own transmission. Efforts are being made to develop efficient signal detectors capable of scanning a wide bandwidth with a large dynamic range [1]. Recently the theory of compressed sampling (CS) has received an increasing attention as it may help in relaxing the constraints on the design of the CR system [2]. It is based on the fact that a frequency sparse signal may be sampled at a rate significantly lower than the Nyquist rate without losing information. This may potentially facilitate the implementation of the analog-to-digital (A/D) converters and digital processors.

In the CS framework, signal recovery is classically achieved through expensive algorithms. The signal reconstruction may be expressed as a problem of ℓ_1 -norm minimization and typically solved with linear programming algorithms [3]. Other methods such as the orthogonal matching pursuit algorithm have been proposed [4]. However signal reconstruction is not necessary in many signal processing applications, as the designer is only interested in solving an inference problem. Davenport et al. demonstrate that it is possible to tackle the problem of the detection of a known signal in noise or the classification of signals directly in the compressive domain without first resorting to full signal reconstruction [5], [6]. We have recently extended [6] to the optimal maximum-likelihood (ML) detection of linearly modulated signals of unknown parameters in a set of predetermined subchannels [7].

On the other hand, the use of distributed spectrum sensing algorithms is recommended to cope with the fading phenomenon present in all wireless communications systems. Distributed algorithms rely on the exchange of the observations made at the spatially distributed nodes on a control channel of capacity unfortunately limited in practice. When the observed

signal at each node is sparse, it is advantageous to first compress the information using the CS theory before communicating it to the coordinator node. The existing contributions in the literature aim at reconstructing the wideband signal spectrum at the coordinator node by defining a sparsity model common to all sensing nodes [8], [9].

In this paper, we focus on the detection of primary signals directly in the compressive domain (as in [7]). Therefore the proposed scheme does not necessitate the signal reconstruction as in [8], [9]. We demonstrate that the CS architecture can advantageously be implemented by a distributed network of sensing nodes, each sampling the signal periodically at a low rate. Because the propagation delays to reach the sensing nodes are inherently different, the distributed network can be viewed as a periodic non-uniform sampling architecture when the signals are combined at the coordinator node.

II. SYSTEM MODEL

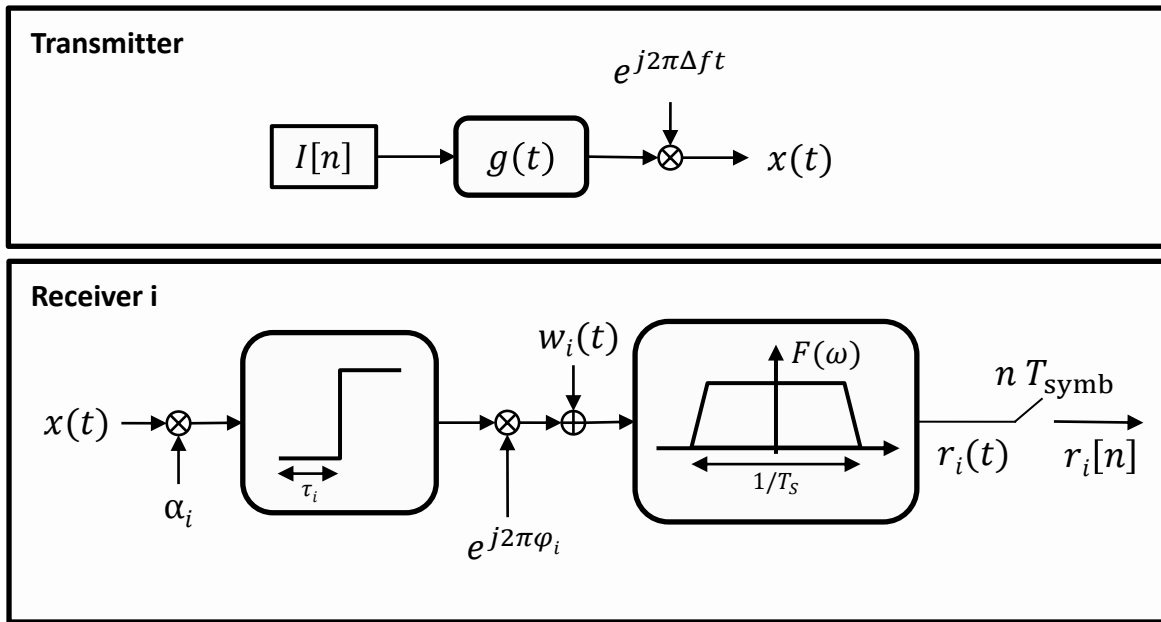


Fig. 1. Baseband system model.

Fig. 1 describes the baseband system model. It is assumed that the overall bandwidth is divided in a set of M uniformly spaced frequency bands. The primary system is composed of one base station transmitting a signal on one of the subbands. The secondary system is composed of a set of Q cooperative sensing nodes that send their observations to a coordinator node to detect which band is occupied. The transmitter transmits a finite sequence of symbols $I[n]$ of length N ($n = 1 \dots N$). The complex symbols are supposed independent and identically distributed (i.i.d.) of variance σ_I^2 . The symbol duration is denoted T_{symb} . This sequence of symbols is lowpass filtered by the pulse-shaping filter $g(t)$ (eg. halfroot Nyquist). The signal is then shifted in the frequency domain to the frequency Δf . The transmitted signal is therefore:

$$x(t) = \sum_{n=1}^N I[n] g(t - nT_{\text{symb}}) e^{j2\pi\Delta ft}. \quad (1)$$

Each secondary received signal is affected by a different propagation delay τ_i . The signal at each receiver is multiplied by a coefficient α_i modeling the fading channel. A phase

shift, modeled by the parameter φ_i , affects the signal at each receiver. Before sampling at the receiver, the signal is lowpass-filtered with an ideal filter $f(t)$ of bandwidth $1/T_s$ where $T_s = T_{\text{symp}}/M$ is the sampling period required to satisfy the Nyquist criterion. The signal is corrupted by the addition of an additive white Gaussian noise (AWGN) $w_i(t)$ of variance σ_w^2 . The signal received at node i is:

$$r_i(t) = \alpha_i e^{j\varphi_i} x(t - \tau_i) * f(t) + v_i(t), \quad (2)$$

where $v_i(t) = w_i(t) * f(t)$ is the noise $w_i(t)$ filtered by the ideal lowpass filter. We assume that it is periodically sampled at the symbol rate $1/T_{\text{symp}}$ (and not at the Nyquist rate $1/T_s$):

$$r_i[m] := r_i(t = mT_{\text{symp}}) \quad (3)$$

$$\begin{aligned} &= \alpha_i e^{j(2\pi\Delta f(mT_{\text{symp}} - \tau_i) + \varphi_i)} \sum_{n=1}^N I[n] g_i[m - n] \\ &\quad + v_i(mT_{\text{symp}}) \end{aligned} \quad (4)$$

where:

$$g_i[n] := g(nT_{\text{symp}} - \tau_i). \quad (5)$$

Because a finite number of symbols is transmitted and assuming that the shaping pulse is of finite length L , an equivalent matrix model can be built :

$$\underline{r}_i = \underline{H}_i \underline{I} + \underline{v}_i. \quad (6)$$

where:

- Vector \underline{I} is composed of the N transmitted symbols.
- Vectors \underline{r}_i and \underline{v}_i are composed of the $N + L - 1$ received and noise elements.
- Matrix \underline{H}_i is defined as the product $\underline{H}_i = \alpha_i \underline{\Phi}_i \underline{G}_i$ where \underline{G}_i is the matrix (of dimension $N + L - 1 \times N$) representing the convolution with the pulse shaping filter, including the time shift τ_i . The element (j, k) of \underline{G}_i is given by:

$$\underline{G}_i(j, k) = g_i[j - k + 1]. \quad (7)$$

Matrix $\underline{\Phi}_i$ performs the shift to frequency Δf . The element (j, k) is given by:

$$\underline{\Phi}_i(j, k) = \delta_{jk} e^{j(2\pi\Delta f(kT_{\text{symp}} - \tau_i) + \varphi_i)}. \quad (8)$$

The received vectors are communicated to the coordinator node where the combined received vector \underline{r} is composed:

$$\underline{r} = [\underline{r}_1^T \quad \underline{r}_2^T \quad \cdots \quad \underline{r}_Q^T]^T. \quad (9)$$

III. LIKELIHOOD FUNCTION

The occupied frequency band is estimated by using a Maximum Likelihood Frequency Estimator (MLFE):

$$\hat{\Delta f} = \arg \max_{\Delta f} p(\underline{r} | \Delta f). \quad (10)$$

The distributed detection relies on the pre-estimation of the signal delays, the phase shifts and fading coefficients that are therefore assumed to be known in this study. The relation (10) can be rewritten as:

$$\hat{\Delta f} = \arg \max_{\Delta f} \mathbb{E} [p(\underline{r} | \Delta f, \underline{I})] \quad (11)$$

where $\mathbb{E}[x]$ stands for the mathematical expectation of x . In the expression (6) of the received signal, the first term is deterministic; only the second term is random and has a Gaussian probability density function (PDF). Thus, \underline{r} is a Gaussian random variable of mean $\left[\underline{H}_1 \underline{I} \quad \underline{H}_2 \underline{I} \quad \cdots \quad \underline{H}_Q \underline{I} \right]$ and variance $\sigma_w^2 \mathbb{I}_P$ where \mathbb{I}_P denotes the size P identity matrix. We obtain:

$$\begin{aligned} p(\underline{r}|\Delta f, \underline{I}) &= C \exp \left(\frac{-1}{2\sigma_w^2} \sum_{i=1}^Q \left(r_i - \underline{H}_i \underline{I} \right)^H \left(r_i - \underline{H}_i \underline{I} \right) \right) \\ &= C \exp \left(\frac{-1}{2\sigma_w^2} \sum_{i=1}^Q \left[r_i^H r_i - \underline{I}^H \underline{y}_i - \underline{y}_i^H \underline{I} + \lambda_i \right] \right) \end{aligned} \quad (12)$$

where:

- C is a constant independent of Δf and \underline{I} ;
- $r_i^H r_i$ is the correlation of the observation independent of Δf and \underline{I} ;
- $\underline{y}_i = \underline{H}_i^H r_i$ is the vector that results from the application of a matched filter to r_i .
- $\lambda_i = \underline{I}^H \underline{H}_i^H \underline{H}_i \underline{I}$ is approximately equal to the constant $N\sigma_f^2 |\alpha_i|^2$ when N is sufficiently large and if the pulse shaping filters are normalized.

The PDF defined in (12) is thus approximately given by:

$$p(\underline{r}|\Delta f, \underline{I}) = C' \Omega(\Delta f, \underline{I}) \quad (13)$$

where C' is a constant and:

$$\Omega(\Delta f, \underline{I}) := \exp \left(\frac{1}{2\sigma_w^2} \sum_{i=1}^Q (\underline{y}_i^H \underline{I} + \underline{I}^H \underline{y}_i) \right). \quad (14)$$

The exponential in (14) is expanded as a Maclaurin serie in order to be able to average the function over the symbols:

$$\exp(\lambda) = \sum_{\ell=0}^{\infty} \frac{\lambda^\ell}{\ell!}. \quad (15)$$

IV. FREQUENCY ESTIMATOR

The ML criterion reduces to the simplified expression hereafter:

$$\begin{aligned} \hat{\Delta f} &\approx \arg \max_{\Delta f} \mathbb{E}_{\underline{I}} [\Omega(\Delta f, \underline{I})] \\ &\approx \arg \max_{\Delta f} \mathbb{E}_{\underline{I}} \left[\exp \left(\frac{1}{2\sigma_w^2} \sum_{i=1}^Q (\underline{y}_i^H \underline{I} + \underline{I}^H \underline{y}_i) \right) \right] \\ &\approx \arg \max_{\Delta f} \mathbb{E}_{\underline{I}} \left[\sum_{\ell=0}^{\infty} \frac{1}{\ell!} \left(\frac{1}{2\sigma_w^2} \sum_{i=1}^Q (\underline{y}_i^H \underline{I} + \underline{I}^H \underline{y}_i) \right)^\ell \right] \\ &\approx \arg \max_{\Delta f} \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \left(\frac{1}{2\sigma_w^2} \right)^\ell \mathbb{E}_{\underline{I}} \left[\left(\sum_{i=1}^Q \underline{y}_i^H \underline{I} + \underline{I}^H \underline{y}_i \right)^\ell \right]. \end{aligned} \quad (16)$$

To compute the expectation, we use the binomial theorem that states:

$$(x + y)^n = \sum_{\xi=0}^n \binom{n}{\xi} x^{n-\xi} y^\xi. \quad (17)$$

Thus:

$$\Delta f \approx \arg \max_{\Delta f} \sum_{\ell=0}^{\infty} \sum_{\xi=0}^{\ell} \frac{1}{\ell!} \left(\frac{1}{2\sigma_w^2} \right)^{\ell} \binom{\ell}{\xi} \cdot \mathbb{E}_{\underline{I}} \left[\left(\sum_{i=1}^Q y_i^H \underline{I} \right)^{\ell-\xi} \left(\sum_{i=1}^Q \underline{I}^H y_i \right)^{\xi} \right]. \quad (18)$$

Only the terms where $\left(\sum_{i=1}^Q y_i^H \underline{I} \right)$ and $\left(\underline{I}^H \sum_{i=1}^Q y_i \right)$ are elevated to the same power are different from zero because the symbols are i.i.d. by hypothesis and:

$$\mathbb{E}_{\underline{I}} [(I[n])^\gamma] = 0 \quad \forall \gamma \in \mathbb{N}. \quad (19)$$

This happens only for even orders ℓ and for $\xi = \frac{\ell}{2}$. We have:

$$\Delta f \approx \arg \max_{\Delta f} \sum_{\ell=1}^{\infty} \frac{1}{(\ell! 2^\ell \sigma_w^{2\ell})^2} \mathbb{E}_{\underline{I}} \left[\left(\sum_{i=1}^Q y_i^H \underline{I} \right)^{\ell} \left(\sum_{i=1}^Q \underline{I}^H y_i \right)^{\ell} \right] \quad (20)$$

$$\approx \arg \max_{\Delta f} \left[1 + \frac{\sigma_I^2}{4\sigma_w^4} \left| \sum_{i=1}^Q y_i \right|^2 + \dots \right]. \quad (21)$$

The dominant terms in (21) may be much larger than the second order. Fortunately, the terms in (21) are monotonically increasing with the terms that compose the second order as ℓ grows, since each term is composed of a sum of powers of those terms and only terms that are redundant with others are subtracted from the sum. Thus, it is equivalent to keep only the second-order term in the objective function. Thus, the function to maximize reduces to:

$$F(\Delta f) := \left| \sum_{i=1}^Q y_i \right|^2 = \sum_{n=1}^N \left| \sum_{i=1}^Q y_i[n] \right|^2. \quad (22)$$

The sequence $y_i[n]$ is obtained by applying a matched filter on each of the candidate bands:

$$y_i[n] = \alpha_i^* e^{-j(2\pi\Delta f(nT_{\text{symb}} - \tau_i) + \varphi_i)} \sum_m r_i[m] g_i^*[m - n]. \quad (23)$$

The signal detector reduces to a matched filter applied independently at each sensing node at the symbol rate. The results are transmitted to the coordinator node where the sum is taken to obtain the final metric based on which the occupied band is estimated.

V. PERFORMANCE ANALYSIS

The subchannel estimation error probability is assessed numerically. A 100 MHz bandwidth is uniformly divided in $M = 20$ subchannels. A sequence of 50 symbols is transmitted at the rate 5 Msps on each subchannel. The symbols are shaped with a halfroot Nyquist filter of roll-off factor equal to 0.2. In our simulations, 5000 realizations of the Rayleigh fading channels have been generated.

Fig. 2 illustrates the probability of error as a function of the signal-to-noise ratio (SNR) in three scenarios: (i) the ideal channel, (ii) a common Rayleigh channel obtained by assuming the nodes are co-located, (iii) the distributed Rayleigh channel considered in this paper. Rayleigh fading incurs a performance loss compared to the ideal channel. In the case of a distributed sensing network, the system benefits from spatial diversity explaining the performance gain. Fig. 3 illustrates that the performance improves significantly when the number of sensing nodes is increased thanks to the increasing average sampling frequency and the additional source of diversity.

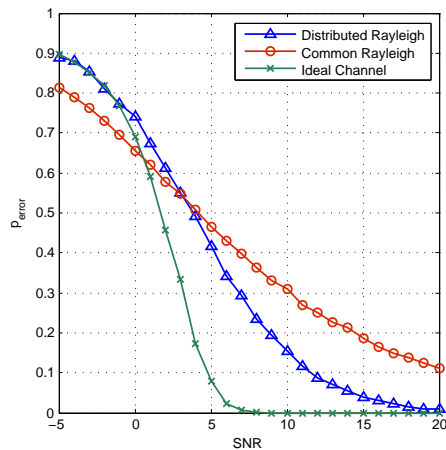


Fig. 2. Error probability as a function of the SNR. Comparison of the performance obtained under the ideal, Rayleigh, distributed Rayleigh channels. $Q = 5$.

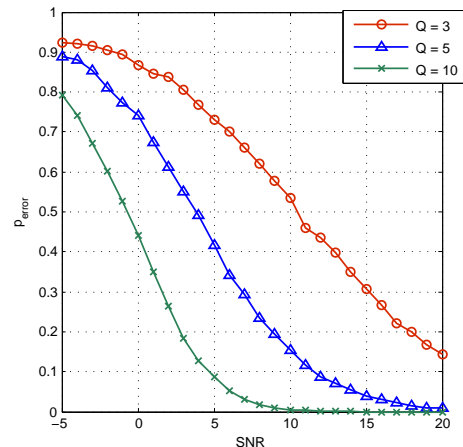


Fig. 3. Error probability as a function of the SNR. Impact of the number of sensing nodes.

VI. CONCLUSION

This paper investigates the signal detection in a distributed network. The network is composed of a set of sensing nodes that sample the signal at a low rate and send their observations to a coordinator node. The propagation delays inherent to the distributed network of sensing nodes are used to form a global compressive sampling architecture. Contrary to state-of-the-art solutions, the proposed signal detector works directly in the compressive domain and therefore does not require the reconstruction of the received signal. Performance results show that the system benefits from the spatial diversity present in the distributed network. Future work will investigate the implementation of the detector in a real-life testbed (estimation of the Rayleigh fading coefficients and delays...).

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