INTERNATIONAL MIGRATION AND UNCERTAINTY: A NON-FACTOR PRICE EQUALIZATION OVERLAPPING GENERATIONS MODEL

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ABSTRACT:
This paper proposes a framework to analyze international migrations under uncertainty in the Diamond (1965) two period overlapping generations model with two countries. Considering a random country-shock on both labor and capital, we show that each autarkic Cobb-Douglas economy converges to a unique expected steady-state equilibrium which is a function of country-specific shocks and parameters. Opening the borders of the two countries in steady-state equilibrium allows for individual migrations. Since risk is country-specific, incentives for migration exist and there is no factor price equalization. Since individuals are heterogeneous with respect to their risk aversion, some migrate and some do not. Countries are always populated. Both temporary and return migrations appear.

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INTRODUCTION

Differences in factor prices across countries appear to be a persistent feature of many open economies. The presence of wide variations in factor prices raises the question of why these differences in wages and interest rates do not fall to clear international labor and capital markets. It is surprising that to our knowledge, there is no overlapping generations (OLG) models capable of reproducing these empirical differences. For that reason we develop such a OLG model with uncertainty where international migrations cease when expected wages and expected interest rates equalize across the two countries (and not necessarily their respective levels, as in the current OLG literature) explaining why post-migration equilibrium is characterized by non-factor prices equalization. In addition to these new results, our framework is also capable of reproducing international return migration in OLG models.

The literature on international economics can broadly be classified into two main categories. The literature under certainty studies factor price equalization (FPE) models and the literature under uncertainty studies the non-factor price equalization (NFPE) models.

The first category with perfect information, initiated by the Heckscher-Ohlin-Samuelson (HOS) model, shows how differences in scarcity of substitutable factors of production explain exchanges between countries. In the OLG literature on international economic migration under certainty, post-migration competitive equilibrium is based on factor price equalization models across countries. OLG models have often been used to explain international migration. Following Galor (1986), a wide class of models studies the aggregate implications on migration of life-cycle saving by heterogeneous agents. Different time preferences across countries (Crettez, Michel and Vidal, 1996, 1998) as well as different degree of altruism (Gaumont and Mesnard, 2000, 2001) lead to various intensities of capital accumulation as well as a trade-off between the wage differential and the interest rate differential generates migration flows. This class of models offers a framework in general equilibrium to compare the welfare implications of labor versus capital mobility (see Galor, 1992). Under perfect information and competitive markets, if there is some incentives for migration, individuals migrate. International migration ceases when prices equalize.

The literature on international migration under uncertainty mainly developed partial equilibrium modeling, rather than the dynamic general equilibrium of OLG models. This literature can be divided into a number of categories. For our present purpose, we shall concentrate on just three of them; for more complete surveys see Drinkwater, Levine, Lotti and Pearlman (2003) and Zimmermann and Constant (2004), as well as Chaabane (2011). In partial equilibrium, migration decisions are affected by the presence of uncertainty on the capital market (1), on the labor

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2 Labor migrates from high (low) to the low (high) time preference country if both countries under (over) invest relative to the Golden Rule. Unilateral migration worsens (improves) the welfare of non-migrants in the immigration (emigration) country. Bilateral migration improves (worsens) the welfare of non-migrants that are characterized by identical (different) preferences to those of the migrants from their country.
market (2) and on the production side (3).

1) On the capital market with uncertainty, Dustmann (1997) presents a life-cycle model where migrants determine their return migration and consumption simultaneously. Depending on the relative levels of risk abroad and at home and their correlation, migrant’s precautionary savings can be proved to be above or below those of native inhabitants. Consequently, return migration is ambiguous and depends on the size of the wage differential and the relative risk the migrant experienced on the two labor markets. Berninghaus and Seifert-Vogt (1993) study the possibility of return migration at each period. A migrant may decide to return migrate in order to invest his capital in a small business or may stay abroad to work and earn money. The properties of the optimal saving and return migration strategies can be derived by formulating the migrant’s decision problem as a Stopped Markovian Decision Process. This framework provides a lot of interesting testable results concerning return migration behavior of a targeted saver. Finally, since uncertainty affects both labor and capital markets, it also affects income. Considering migration as an investment under uncertainty on future returns, Burda (1995) shows that postponing migration one period has a positive value for a range of individuals who otherwise would have migrated on a net present value basis. This option is related to the future differentials between home and abroad of uncertain interest rates and incomes. In inter-temporal equilibrium, low migration rates may coexist with current wage differentials.

2) On the labor market with uncertainty, Todarro (1969) considers migrants having a probability of finding a better job abroad. Before migrating, risk-averse individuals compare expected future flows of domestic and foreign income. Hatton and Williamson (1998) provide empirical evidence of the above approach for European countries. In partial equilibrium, Harris and Todarro (1970) consider a random job selection process, so that both employment and unemployment are random. Individuals take into account their expected utility when deciding whether to return migrate or not. Using a large sample of farmers Kanwar (1998) shows that both labor market and production risks are not causally related in the Granger sense.

3) On the production side with uncertainty, El-Gamal (1994) studies a dynamic version of the Harris-Todaro’s migration model where a finite population of infinitely lived Bayesian agents choose consumption and migration as a function of their histories. Only the government knows the production functions. The government maximizes its welfare function using wage subsidies in the two sectors, and a migration tax. The effects of government policies on the population distribution are analyzed. Traca (2005) studies the effects of trade liberalization on factor returns with industry-specific uncertainty. Under certain conditions, the return to capital increases and real wages and the welfare of workers decrease along with an expansion of wage dispersion and volatility.

Our contribution incorporates the above partial equilibrium approach into an OLG model. Taking uncertainty into account in a general dynamic competitive equilibrium is an important issue for international considerations. In our model in
autarky, heterogeneous risk-averse individuals maximize their expected utility in a Levy-Markowitz (1979) sense. Expected payoff and variance play a role in migration decision, as they do in the saving decision. The traditional methods used in the theory of decision under uncertainty apply (see for instance Levy and Markowitz (1979)). In our model, as long as the shocks on labor and capital differ across countries, the two autarkic steady states differ. In the open economy, individuals make the decision to migrate (or not) with respect to their expected post-migration utility. Equalizing the expected wages and the expected return on saving implies that incentives for migration cease for a particular distribution of individuals in the integrated economy. Since differences in capital per worker only result from differences in random shocks, there is no factor price equalization across countries. This directly involves that NFPE may occur in post-migration equilibrium. Since prices do not equalize over time across countries, individuals may return migrate. This result is particularly new. Moreover, since low risk-averse individuals migrate while high risk-averse ones do not, the two countries are always populated.

The first Section is devoted to the study of the autarkic equilibrium, the second is devoted to international migration and the third to welfare implications of the non-factor price equalization. The last section concludes.

1. AUTARKY

In autarky, the country risks arise on both labor and capital markets, each market being affected by a specific random shock. The probability distribution of each shock is common knowledge to firm, individuals and government. The firm maximizes its profit after the shocks have occurred. Individuals differ in their risk aversion $\theta$ and maximize the sum of their first-period consumption utility and second-period expected consumption utility, in a sense that will be defined below. Individuals know their own risk aversion. Government maintains the equilibrium of the unemployment insurance.

1.1. THE MODEL

We consider a two-country world, where the countries are denoted by $i = 1, 2$, with an infinite horizon, discrete time and uncertainty. During each period $N_{t+1}$ individuals are born in country $i$. There is a constant population growth $N_{t+1} = (1 + n)N_t$. The rate of population growth is the same in both countries. We assume for the moment that there are $N_{t}^\theta(\theta)$ low risk-averse individuals, $\theta \in [0, \bar{\theta}]$, where $\theta$ is an individual parameter that captures the risk aversion. There are $N_{t}^{\bar{\theta}}(\bar{\theta})$ high risk-averse individuals, $\bar{\theta} \in [\bar{\theta}, \infty]$. The true number $\hat{\theta}$ will be calculated precisely latter on. Firms act competitively.

Uncertainty occurs within the economy as follows. There are two shocks. The first affects the quantity of labor (by job destruction) and the second affects the quantity
The two different states of nature are as follows:

\[ \Omega^i = \{ \omega^i_L, \omega^i_K \} \]

The state of the nature \( \omega^i_L \) occurs with the exogenous probability \( 1 > q^i > 0 \), which is common knowledge. In this state, there are \( q^i N^i_t \) jobs destroyed, and with the complementary probability \( 1 - q^i \) there is no job destroyed. The state of the nature \( \omega^i_K \) occurs with the exogenous probability \( 1 > p^i > 0 \), which is common knowledge. In that state of the nature, the capital \( p^i K^i_t \) units of capital are destroyed before the end of the current period, in which case they do not provide any return. The origins of these shocks are for instance earthquakes, meteorological events, capital default, unexpected obsolescence, war etc. Consequently, \( p^i \neq q^i \) is possible, and we assume this for the rest of the paper. With the complementary probability \( 1 - p^i \) the capital is preserved until the end of the period and provides a return \( R^i_t \). If it is not destroyed during the current period, capital fully depreciates after one period.

1.2. The Firm

The technology is the same in each country. There are two factors of production, capital in quantity \( K^i_t \) and labor in quantity \( L^i_t \), producing a single good. In period \( t \), the prices of capital and labor in country \( i \) are \( r^i_t \) and \( w^i_t \). Output is the numeraire and is allocated to current consumption \( c^i_t \) and investment \( I^i_t \) which constitutes the next period’s capital \( K^i_{t+1} \). Thus, a price system is a positive vector \( \psi^i_t = (w^i_t, R^i_t) \), where \( R^i_t = 1 + r^i_t \) is the interest factor. For each country, labor demand in each period is endogenous, whereas the labor supply is exogenous. The capital is the output produced but not consumed in the preceding period. The type of uncertainty that we study in this section is characterized by the fact that individuals cannot influence the probability of an event.

Production in each country occurs within a period according to a Cobb-Douglas production function: \( F(K^i_t, L^i_t) = A(Ke^i)^{\alpha} (Le^i)^{1-\alpha} \). The production function is independent of time and invariant across countries. Prices are state of the nature dependent. For a given price system \( \psi^i_t(\omega^i_L, \omega^i_K) \), the demand for factors of production in country \( i \) is chosen so as to maximize the profit of the expected variables, which are denoted \( \widehat{K}^i_t = (1 - p^i)K^i_t \) and \( \widehat{L}^i_t = (1 - q^i)N^i_t \). Considering the expected capital per worker, \( \widehat{k}^i_t = \widehat{K}^i_t / \widehat{L}^i_t \) the profit per expected
capita is $\pi^i_t = A\left(k^i_t\right)^\alpha - w^i_t - R^i_t k^i_t$. The maximization of this profit combined with the free entry condition (i.e. the zero-profit condition) gives:

$$R^i_t = \alpha A\left(k^i_t\right)^{\alpha-1},$$  \hspace{1cm} (1)

$$w^i_t = A\left(k^i_t\right)^\alpha - R^i_t k^i_t = (1-\alpha) A\left(k^i_t\right)^\alpha.$$  \hspace{1cm} (2)

### 1.3. The individual

All through the life-cycle, the shock probabilities are common knowledge, but no individual knows which state Nature will choose. In autarky, it is important to specify that during the first period, individuals make a saving choice between different random alternatives, see Section 1.6.

### 1.4. Life-cycle decisions

The life-cycle decision process of an individual who lives for two periods in country $i$ is the following. During period $t$, the length of which is normalized to 1, a young individual supplies one unit of labor, which is paid $w^i_t$, consumes $c^i_t(\theta)$ units of goods and saves $s^i_t(\theta)$. During the second period he retires and consumes the return on his first period saving. The level of the first period saving depends on whether he saved on the basis of net income or unemployment benefits.

#### 1.4.1. The equilibrium of government unemployment insurance

Due to job destruction, an individual keeps his job with probability $1-q^i$, in which case he earns the net labor income $(1-\tau^i)w^i_t$, where $\tau^i$ is the level of unemployment insurance contributions. If the job is destroyed, with the publicly-known and exogenous probability $q^i$, he receives unemployment benefits $b^i_t$. The expression of the expected first period income is $(1-q^i)(1-\tau^i)w^i_t + q^i b^i_t$.

Government unemployment insurance make no profit, so that $N^i_t \tau^i(1-q^i)w^i_t = q^i N^i_t b^i_t$. Simplify by $N^i_t$ and put the expression of $q^i b^i_t$ into the expected first period income. Finally, the individual expected income simplifies as $E(w^i_t) = (1-q^i)w^i_t$, where $E$ is the linear expectation operator.
1.4.2. The equilibrium of capital insurance

The exogenous probability of capital destruction is $p^i$. During the first period, the $N_i(\theta)$ low risk-averse individuals (as defined in Section 1.5) choose to save on the basis of their net income, i.e., $(1-\tau^i)w^i$. The $\bar{N}_i(\bar{\theta})$ high risk-averse individuals choose to invest on the basis of the unemployment benefit $b^i$.

Individuals can buy a capital insurance, denoted $z^i_{t+1}(\theta)$, at price $\lambda^i R_{t+1}^i s^i(\theta)$. The second period income is thus: $(1-p^i)(1-\lambda^i)R_{t+1}^i s^i(\theta) + p^i z^i_{t+1}(\theta)$. Individuals are insured in proportion to their contribution to capital. In the case of a bad event, low risk-averse individuals, who save more and own more capital, get reimbursed in proportion of $s^i(\theta)$, their true contribution to the aggregate capital. The same applies for high risk-averse individuals.

Since all the old buy this insurance, the zero profit condition of the competitive insurance companies implies that:

$$N_i p^i \sum_{\theta} z^i_{t+1}(\theta) = \lambda^i (1-p^i) R_{t+1}^i \left[ N_i(\theta)s^i(\theta) + \bar{N}_i(\bar{\theta})s^i(\bar{\theta}) \right],$$

so that $p^i z^i_{t+1}(\theta) = \lambda^i (1-p^i) R_{t+1}^i s^i(\theta)$.

Depending on the two possible levels of individual risk aversion, $\theta$ and $\bar{\theta}$, the second period expected utility can be written as:

$$E\left[d^i_{t+1}(\theta)\right] = (1-p^i) R_{t+1}^i s^i(\theta), \quad (3)$$

and

$$E\left[d^i_{t+1}(\bar{\theta})\right] = (1-p^i) R_{t+1}^i s^i(\bar{\theta}). \quad (4)$$

We now turn to study the individual’s preferences.

1.5. Preferences

When the risk is exogenous, i.e. no individual decision can change its occurrence (like the type of risks presented above on labor and capital markets), individuals take into consideration the expected income. If they have to choose between two (or more) risky alternatives, as in the saving decision, they have to consider not only the expected income but also the dispersion of risk. We have therefore chosen a utility function, drawn directly from Levy and Markowitz (1979), that allows us to
eliminate the dispersion of risk. For that reason, we choose the following function, directly extracted. Since this is not so well known in the overlapping generations literature, we present a summary of the main results obtained by Levy and Markowitz (1979) in Appendix A.

**Proposition 1.** The utility of expected consumptions is variance independent and can be written as follows:

\[
U(c_i'(\theta), E[d_{i+1}'](\theta)) = \log(c_i'(\theta)) + \beta \log\left[E(d_{i+1}'(\theta))\right].
\]

Proof is given in Appendix B.

### 1.6. The Individual’s Solutions

The objective of this subsection is to determine the solutions for the individuals and to characterize the threshold of the risk aversion below which individuals choose a high level of savings, and above which they choose a low level.

In order to accomplish this, we have to consider two cases. The low risk-averse individuals save on the basis of their net wage. The high risk-averse individuals save on the basis of the unemployment benefits. We first solve the two types of program, then we determine the threshold \( \tilde{\theta} \). Time preference is denoted \( \beta \), and a \( \theta \)-risk-averse individual is characterized by his expected life-cycle utility:

\[
U(c_i'(\theta), d_i'(\theta)) = \log(c_i'(\theta)) + \beta \log\left[E(d_{i+1}'(\theta))\right], \text{ with } 0 < \beta i = 1, 2.
\]  

For a \( \theta \)-individual, the problem to be solved is:

\[
\begin{align*}
\max_{c_i'(\theta), d_{i+1}'(\theta)} & \quad U(c_i'(\theta), E[d_{i+1}'(\theta)]) \\
\text{s.t.} & \quad c_i'(\theta) + s_i'(\theta) = (1 - \tau)w_i', \\
& \quad E[d_{i+1}'(\theta)] = (1 - p^\prime)R_{i+1}'s_i'(\theta), \\
& \quad c_i'(\theta) > 0, d_{i+1}'(\theta) > 0.
\end{align*}
\]

Replace the expression of the saving from the first period constraint into the second period constraint. Put the new expression of the second period consumption into the objective and solve the following program:

\[
\begin{align*}
\max_{c_i'} & \quad \log(c_i'(\theta)) + \beta \log\left[(1 - p^\prime)R_{i+1}'(1 - \tau)w_i' - c_i'(\theta)\right] \\
& \quad c_i'(\theta) > 0.
\end{align*}
\]
The first-order conditions are:

\[ \begin{cases} 
\frac{1}{c_i'(Q)} = \frac{(1 - p')\beta R_{i+1}'}{d_{i+1}'(Q)}, \\
(c_i'(Q)) = \frac{d_{i+1}'(Q)}{(1 - p')\beta R_{i+1}'}, \\
d_{i+1}'(Q) = (1 - p')R_{i+1}'((1 - \tau')w'_i - c_i'(Q)). 
\end{cases} \tag{7} \]

Solve the above system to obtain the individual’s optimal choices. The solutions are:

\[ c_i'(Q) = \frac{(1 - \tau')}{1 + \beta} w'_i > 0, \tag{9} \]

\[ s'_i(Q) = \beta \frac{(1 - \tau')}{1 + \beta} w'_i > 0. \tag{10} \]

Replace (7) into the second period budget constraint

\[ d_{i+1}'(Q) = \frac{\beta}{1 + \beta} (1 - p')(1 - \tau')R_{i+1}'w'_i > 0. \tag{11} \]

The utility of a \( \bar{\theta} \) type individual is

\[ U \left[ c_i'(Q), E \left[ d_{i+1}'(Q) \right] \right] = \log \left[ c_i'(Q) \right] + \beta \log \left[ E \left[ d_{i+1}'(Q) \right] \right], \] with \( 0 < \beta i = 1, 2. \tag{12} \]

For a \( \bar{\theta} \) Individual, the problem to be solved is:

\[ \max_{c_i'(\bar{\theta}), E[d_{i+1}'(\bar{\theta})]} U \left( c_i'(\bar{\theta}), E[d_{i+1}'(\bar{\theta})] \right) \]

s.t.

\[ \begin{cases} 
(c_i'(Q)) + s'_i(Q) = b'_i, \\
E \left[ d_{i+1}'(Q) \right] = (1 - p')R_{i+1}'s'_i(\bar{\theta}), \\
c_i'(\bar{\theta}) \geq 0, E \left[ d_{i+1}'(\bar{\theta}) \right] \geq 0. 
\end{cases} \]

Using the same methodology, we obtain:

\[ c_i'(\bar{\theta}) = \frac{b'_i}{1 + \beta}. \]
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\[ s'_i(\theta) = \frac{\beta b_i}{1+\beta}, \]

\[ d'_{i+1}(\theta) = \frac{\beta(1-p')R_{i+1}b'_i}{1+\beta}. \]

From the equilibrium of the government unemployment insurance, we have

\[ c'_i(\theta) = \frac{\tau' (1-q')w'_i}{q'(1+\beta)}, \quad (13) \]

\[ s'_i(\theta) = \frac{\beta \tau'(1-q')w'_i}{q'(1+\beta)}, \quad (14) \]

\[ d'_{i+1}(\theta) = \frac{\beta(1-p')R_{i+1} \left[ \tau' (1-q')w'_i \right]}{q'(1+\beta)}. \quad (15) \]

1.7. Determination of the threshold \( \hat{\Theta}^i \)

The threshold \( \hat{\Theta}^i \) is determined by comparing the two possible saving options. A rational individual compares \( f_\theta(E'_i(s'_i(\theta)), V'_i(s'_i(\theta)), U) \) in country \( i \) with \( f_\theta(E'_i(s'_i(\theta)), V'_i(s'_i(\theta)), U) \) in country \( i \) too. He chooses the saving for which the expected utility of the second period consumption, approximated as Levy and Markowitz (1979) suggested by the quadratic (23) is the highest.

**Proposition 2.** When low risk-averse individuals save on the basis of their first period net income and high risk aversion individuals on the basis of their first period unemployment benefits, the condition on the individual’s risk aversion is

\[ \frac{1-2\tau^i}{1-q^i(1-\tau^i)-\frac{1}{q^i}} \geq \hat{\Theta}^i, \quad \frac{\tilde{b}^2}{\tilde{a}^2} < 0. \quad (16) \]

The number of low risk aversion individuals is decreasing with the probability of job destruction.

The proof is given in Appendix C. We now turn to the study of the autarkic equilibrium.
1.8. THE DYNAMICS OF THE ECONOMY IN AUTARKY

We study the dynamics of the autarkic economy.

**Proposition 3.** The economy converges to an expected autarkic steady-state equilibrium

\[
(k^i)^* = \left[ \frac{1 - \beta^i}{1 - q^i} \right]^{\frac{\sigma^i}{1 - \alpha^i}} \left[ \beta^i(\hat{\beta}^i)(1 - \tau^i) \right] \left( 1 - \delta^i(\hat{\beta}^i) \right) \left( \frac{1 - q^i}{q^i} \right) \left[ \frac{\beta(1 - \alpha^i)A}{(1 + n)(1 + \beta)} \right]^{\frac{1}{1 - \alpha^i}}
\]

and we have \( \partial(k^i)^*/\partial q^i < 0 \), \( \partial(k^i)^*/\partial p^i < 0 \), \( \partial(k^i)^*/\partial \beta > 0 \).

Proof: A dynamic equilibrium satisfies the following equalities:

\[
\begin{align*}
N_{i+1}^t &= (1 + n)N_i^t, & \text{Active population} \quad (17) \\
E_i^t &= (1 - q^i)N_i^t, & \text{employment} \quad (18) \\
U_i^t &= q^iN_i^t, & \text{unemployment} \quad (19) \\
K_i^t &= E_i^t + U_i^t, & \quad (20) \\
K_i^{t+1} &= N_i^{(i)}N_i^t + N_i^{(e)}N_i^t + N_i^{(l)}N_i^t, & \quad (21) \\
N_i^{(e)} + N_i^{(l)} + N_i^{(o)} &= Y_i^t. & \quad (22)
\end{align*}
\]

Remark that \( q^i = U_i^t / N_i^t \) is the unemployment rate, and we will use this interpretation for the rest of the paper. Let \( \delta^i \) denote the ratio of low risk-averse young over to the total number of old, \( \delta^i(\hat{\beta}^i) = N_i^{(l)}(\hat{\beta}^i) / N_i^{(o)}(\hat{\beta}^i) \). The equation of the capital accumulation (21) can be rewritten by using the solution of the individual (10), (14) and (2) of the firm

\[
(1 + n)K_{i+1}^{t+1} = \left[ \delta^i(\hat{\beta}^i)(1 - \tau^i) + \left( 1 - \delta^i(\hat{\beta}^i) \right) \frac{\tau^i(1 - q^i)}{q^i} \right] \frac{\beta}{1 + \beta} w_i^t,
\]

\[
(1 + n)K_{i+1}^t = \left[ \delta^i(\hat{\beta}^i)(1 - \tau^i) + \left( 1 - \delta^i(\hat{\beta}^i) \right) \frac{\tau^i(1 - q^i)}{q^i} \right] \frac{\beta}{1 + \beta} (1 - \alpha)A(k_i)^\alpha.
\]

The dynamics of the economy is
The economy converges to the following autarkic steady-state equilibrium

\( K_{t+1}^i = \left[ \delta^i(\beta^i)(1 - r^i) + \left(1 - \delta^i(\beta^i)\right) \frac{\tau^i(1 - q^i)}{q^i} \right] \frac{\beta(1 - \alpha)A}{(1 + \alpha)(1 + \beta)} \left[1 - p^i\right]^{\tau^i} k_{t+1}^i. \)

The economy converges to the following autarkic steady-state equilibrium

\[ (k^i)^* = \left[ \frac{1 - p^i}{1 - q^i} \right]^{1 - \alpha} \left[ \delta^i(\beta^i)(1 - r^i) \right. \]

\[ + \left. \left(1 - \delta^i(\beta^i)\right) \frac{\tau^i(1 - q^i)}{q^i} \right] \frac{\beta(1 - \alpha)A}{(1 + \alpha)(1 + \beta)} \left[1 - p^i\right]^{\tau^i} k_{t+1}^i. \]

Note that as long as the two countries differ in \( q^i \) and \( p^i \), there is no way for the two steady-state equilibria to be equal. As the two \( \beta^i \) are different, so is the capital per worker. Depending on parameters, the steady-state capital per worker ratio may be increasing or decreasing with the proportion of low risk aversion young over the total number of old individuals.

2. INTERNATIONAL MIGRATION

This section is devoted to the international labor migration. In autarky, the two countries converge to an expected steady-state equilibrium. Like Galor (1986), we open the borders in expected steady-state equilibrium and let individuals migrate if they want to. However, the logic of the migration decision in our model is slightly different from that of Galor. Although the borders are open, risk cannot be considered as constant across countries. Individuals face different country risks. Individuals take into consideration both variations of risk on both labor and capital (i.e., variance \( \sigma_w^2 \) and \( \sigma_k^2 \) in (23)) before deciding whether or not to migrate. In the case of an international migration, an individual migrates with his own capital.

The autarkic steady-state indirect utility is obtained by substituting (9) and (11) into the individual’s utility. Since in autarkic steady-state equilibrium time index is irrelevant, we can write \( E(w^i) = (1 - q^i) w^i \) and \( E(R') = (1 - p^i) R' \).

\[ V\left[ c'(\theta), d'(\theta) \right] = \log \left[ c'(\theta) \right] + \beta \log \left[ E\left[ d'(\theta) \right] \right] \]

Recall the individual’s solutions. From (9), (11), (13) and (15) we have
Knowing that the utility is defined to any increasing monotonic transformation, we can always write
\[ V(c(\theta), d(\theta)) = (1 + \beta) \log \left( E(w') \right) + \beta \log \left( E(R') \right) \]

Note that the indirect utility is the sum of the first period indirect utility (only depending on the wage) and the second period indirect utility (only depending on the interest rate). Such a structure implies that individuals are in a position to choose at each period in which country they want to live.

Let us now adapt the Levy and Markowitz quadratic expression to the individuals’ logarithmic preferences. We first study the incentives for migration and show that there is no factor price equalization. Second, we study the pattern of international migration (return migration can be observed) and show that neither country completely empties into the other.

2.1. INCENTIVES FOR MIGRATION

Since individuals are distributed according to their risk aversion, they take into consideration the risk of migrating. They perfectly know their own risk-aversion type. Rational individuals compare \( f_o(E_1^1, V_1^1, U) \) in country 1 with \( f_o(E_2^2, V_2^2, U) \) in country 2 and choose the country for which the expected utility, approximated as Levy and Markowitz (1979) suggested by the quadratic (23) is the highest. The condition for living in country 1 is thus

\[ f_o(E_1^1, V_1^1, U) \geq f_o(E_2^2, V_2^2, U), \]

\[ \log \left( E(W') \right) + \frac{\log \left( (1 - \theta^2 V(W') / E(W')^2) \right)}{2\theta^2} \geq \log \left( E(W^2) \right) + \frac{\log \left( (1 - \theta^2 V(W^2) / E(W^2)^2) \right)}{2\theta^2}, \]
\[
\log \left[ \frac{E(w_1^3)}{E(w_2^3)} \right] \geq \log \left[ \frac{(1-\theta^2 V(w_1^2) / E(w_2^3))}{(1-\theta^2 V(w_1^1) / E(w_1^3))} \right],
\]

\[
\frac{E(w_1^3) - \theta^2 V(w_1^2)}{E(w_1^2)} \geq \frac{E(w_2^3) - \theta^2 V(w_2)^3}{E(w_2^2)},
\]

That is also to say that all individuals who have a low risk aversion will choose country 1, and the others will choose country 2 according to the following rule:

\[
\frac{E(w_1^3) - E(w_2^3)}{V(w_1^3) / E(w_1^3) - V(w_2^3) / E(w_2^3)} \geq \theta^2.
\]

**Proposition 4.** In this model we have the two following results.

1.- Generically, it is never the case that both wages equalize across countries \(w_1 = w_2\) and simultaneously that both interest rates equalize across countries, \(R_1 = R_2\). If wage equalize, then interest rate differ, and reciprocally. Since these two conditions cannot be satisfied at the same time, there is no factor price equalization.

2.- Since incentives for migration cease when the first period expected wage are equal across both countries, and when the expected return on saving are equal across countries, the proportion of low risk aversion migrants of each country is different from zero, so that no country completely empties into the other one.

3.- The risk aversion threshold of the first period is wage dependent, the risk aversion threshold of the second period is interest rate dependent. Consequently, they are different, so that return migration is possible.

**Proof.** Incentives for migration during the first period cease when \(E(w_1^1) = E(w_2^3)\), i.e., when

\[
(1-q_1^1)w_1^1 = (1-q_2^3)w_2^3,
\]

\[
(1-q_1^1)(1-\alpha)A(k^1)^\alpha = (1-q_2^3)(1-\alpha)A(k^2)^\alpha,
\]

\[
(1-q_1^1)A(k^1)^\alpha = (1-q_2^3)(k^2)^\alpha,
\]

\[
\frac{k_1^1}{k_2^2} = \left[ \frac{1-q_2^3}{1-q_1^1} \right]^{\frac{\alpha}{2}},
\]

Incentives for migration during the second period cease when \(E(R_1^1) = E(R_2^3)\), i.e., when

\[
(1-p_1^1)R_1^1 = (1-p_2^3)R_2^3,
\]

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\[(1 - q^1)\alpha A(k^1)^{\alpha - 1} = (1 - q^2)\alpha A(k^2)^{\alpha - 1},\]

\[(1 - q^1)(k^1)^{\alpha - 1} = (1 - q^2)(k^2)^{\alpha - 1},\]

\[
\frac{k^1}{k^2} = \left[\frac{1 - p^1}{1 - p^2}\right]^{\frac{\alpha}{\gamma}},
\]

Which completes the proof of the first part of the proposition.

Let us rewrite the expression of each capital per worker ratio using the expected steady-state equilibrium of each country after simplifications as follows:

\[
\begin{align*}
\frac{k^1}{k^2} &= \left[\frac{1 - p^1}{1 - q^1}\right]^{\frac{\alpha}{\gamma}} \left[1 - \tau^1\right] \delta^1 + \left(1 - \delta^1\right) \frac{\tau^1(1 - q^1)^{\frac{\alpha}{\gamma}}}{q^1} = \left[1 - q^2\right]^{\frac{1}{1 - \alpha}}, \\
\frac{k^1}{k^2} &= \left[\frac{1 - p^2}{1 - q^2}\right]^{\frac{\alpha}{\gamma}} \left[1 - \tau^2\right] \delta^2 + \left(1 - \delta^2\right) \frac{\tau^2(1 - q^2)^{\frac{\alpha}{\gamma}}}{q^2} = \left[1 - p^1\right]^{\frac{1}{1 - \alpha}}.
\end{align*}
\]

Let us define the following coefficients:

\[
A^1 = \left[\frac{1 - p^1}{1 - q^1}\right] \left(1 - \tau^1\right)B^1 = \left[\frac{1 - p^1}{1 - q^1}\right] \frac{\tau^1(1 - q^1)^{\frac{\alpha}{\gamma}}}{q^1},
\]

\[
A^2 = \left[\frac{1 - p^2}{1 - q^2}\right] \left(1 - \tau^2\right)B^2 = \left[\frac{1 - p^2}{1 - q^2}\right] \frac{\tau^2(1 - q^2)^{\frac{\alpha}{\gamma}}}{q^2}.
\]

Using these coefficients, the system of equations to be solved is simple:
\[
\begin{align*}
\frac{A^1\delta^1 + B^1(1-\delta^1)}{A^2\delta^2 + B^2(1-\delta^2)} &= C1,
\frac{A^1\delta^2 + B^1(1-\delta^2)}{A^2\delta^2 + B^2(1-\delta^2)} &= C2,
\end{align*}
\]

which gives the solutions:

\[
\begin{align*}
\delta^1 &= \frac{B^1}{B^1 - A^1} \neq 0, \\
\delta^2 &= \frac{B^2}{B^2 - A^2} \neq 0.
\end{align*}
\]

Note that \( \forall i = 1, 2, B^i - A^i = \frac{\theta}{\tau} - 1 > 0 \iff \tau^i > q^i \), which is a reasonable case.

We now deal with the third part of the Proposition. We calculate the thresholds of risk aversion that affect migration. Let us recall the Markovitz’s criterion. When it is not possible to conclude that the alternative \( a \) having the highest expected return \( E(a_i) \geq E(a_2) \) is also the less risky one \( \sigma_{a1} < \sigma_{a2} \), Markovitz proposes to solve the following problem

\[
\max_a E(a) - \theta \sigma_a 
\]

where \( \theta \) is the decider’s risk aversion.

The Markovitz’s criterion is

\[
\alpha_1 \geq \alpha_2 \iff \frac{E(a_1) - E(a_2)}{\sigma_{a1} - \sigma_{a2}} > \theta.
\]

According to Proposition 1, it is as if individuals make decisions conditional on their expected utility. But this does not mean that they do not take into account the variance of risk. It only means that their optimal decision is unchanged compared with VNM paradigm. Consequently, the Proposition 1 only simplifies the computing of individuals’ solutions; it does not mean that there is any change in individuals’ behaviors under uncertainty compared with Levy-Markovitz paradigm.

In order to prove that the thresholds of risk aversion are different from each other, we define \( V(w) = E(w^2) - E(w)^2 \), and apply it to our problem.

\[
\sigma^2[w'] = (1 - q')(1 - \tau')w'^2 + q(b')^2 - [(1 - q')w']^2.
\]

Using that \( qb' = \tau'(1 - q')w' \), we have
Develop to finally obtain:

\[ V(w') = \frac{1}{q'} \left[ q' - \tau' \right]^2 (w')^2. \]

Replace this expression into the Markowitz criterion of decision and define:

\[ \tilde{\theta}_{w'} = \frac{(1 - q')w^i - (1 - q')w^j}{(q^i - \tau^i)^2 w^i - (q^j - \tau^j)^2 w^j}. \]

Replace the expressions of the first-period expected income and its variance into the Markowitz criterion of decision and define:

\[ \tilde{\theta}_{R^i} = \frac{(1 - p^i)R^i s^i - (1 - p^j)R^j s^j}{(p^i - \lambda^i)^2 R^i s^i - (p^j - \lambda^j)^2 R^j s^j}. \]

Note that since \( R^i s^i \neq w^i \) and as long as \( \tau^i \neq \lambda^i \) and \( q^i \neq p^i \) the two thresholds are different. Consequently, return migration is possible.

We now turn to the study of migration.

2.2. THE PATTERN OF INTERNATIONAL MIGRATION

In this model, whatever their type, individuals may decide to migrate during both periods or to stay in their country for both periods, to work abroad and to return migrate for retirement, to work home and migrate for retirement, depending on the relative position of the two bounds \( \tilde{\theta}_{w^i R^i} \) and \( \tilde{\theta}_{R^i}. \)

Again, as long as the probabilities differ the incentives for migration never cease and the Markowitz-individuals have an incentive for migration. Remark that incentives for migration are not the same for all the different type of risk-averse individuals. Some may want to migrate while others not, depending on the individual value of the risk aversion \( \tilde{\theta}. \) We now study the pattern of international migration depending on the values of the two thresholds \( \tilde{\theta}_{w^i} \) and \( \tilde{\theta}_{R^i}. \)
2.2.1. Migrate for both periods

The conditions for a $\theta$-type individual born in country $i$ to migrate in country $j$ for the two periods are

\[ \begin{align*}
\theta^i_{w, t} &\geq \theta^j, \\
\theta^i_{R, t} &\geq \theta^j.
\end{align*} \]

Individuals with a low risk aversion, i.e. $\theta^i < \min\{\theta^i_{w, t}, \theta^i_{R, t}\}$, migrate for the two periods, while others adopt different forms of behavior, depending on the respective values of the thresholds $\theta^i_{w, t}$ and $\theta^i_{R, t}$. These forms of behavior are depicted in the next subsections. Low type individuals born in country $j$ never migrate, they work and retire in their home country.

2.2.2. Work in the home country, migrate for retirement

The conditions for a $\theta$-type individual born in country $i$ to work home and migrate for retirement are

\[ \begin{align*}
\theta^i_{w, t} &< \theta^j, \\
\theta^i_{R, t} &\geq \theta^j.
\end{align*} \]

In an economy characterized by $\theta^i_{w, t} < \theta^i_{R, t}$, such a case only exists for $\theta$-type individuals if $\theta^i_{w, t} < \theta^j < \theta^i_{R, t}$, i.e. for intermediate values of the coefficient of risk aversion. Individuals born in country $j$ work abroad and return migrate home for retirement.

2.2.3. Work abroad, return migrate for retirement

The conditions for a $\theta$-type individual to work abroad and return migrate for retirement are

\[ \begin{align*}
\theta^i_{w, t} &\geq \theta^j, \\
\theta^i_{R, t} &< \theta^j.
\end{align*} \]

Such a case only exists for $\theta$-type individuals if $\theta^i_{w, t} < \theta^i_{R, t} < \theta^j$, i.e. for intermediate values of the coefficient of risk aversion and the right ordering of the two thresholds. Individuals born in country $j$ want to work home and migrate abroad for retirement.
2.2.4. Stay home for both periods

The conditions for a \( \theta \) -type-individual to stay home for the two periods are

\[
\begin{align*}
\bar{\theta}_w^i &< \theta^j, \\
\bar{\theta}_r^i &< \theta^j.
\end{align*}
\]

This is only possible for individuals who have a very high risk-aversion, i.e. which satisfies the following condition: \( \theta^j > \max\{\bar{\theta}_w^i, \bar{\theta}_r^i\} \). Obviously, individuals born in country \( j \) want to migrate for both periods.

The next two tables summarize all the possible configurations of migration for an individual born in country \( i \) who may or may not migrate to country \( j \). Obviously, the same type of table could be drawn up for an inhabitant of country \( i \).

<table>
<thead>
<tr>
<th>Period 2</th>
<th>Migrate</th>
<th>Migrate</th>
<th>Stay home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>Migrate</td>
<td>Stay home</td>
<td>Stay home</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period 2</th>
<th>Migrate</th>
<th>Return Migrate</th>
<th>Stay home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>Migrate</td>
<td>Migrate</td>
<td>Stay home</td>
</tr>
</tbody>
</table>

3. Welfare Analysis

The Golden Rule is defined as the particular value of the expected steady-state capital per worker that maximizes the consumer’s utility. The problem to solve is:

\[
\max_{\bar{\theta}_w^i, \bar{\theta}_r^i} \log \left( \bar{\theta}_w^i \right) + \beta \log \left( \bar{\theta}_r^i \right)
\]

\[
\bar{\theta}_w^i + \frac{\bar{\theta}_r^i}{1 + n} + (1 + n)\bar{k}^i = A(\bar{k}^i)\gamma.
\]

The first order condition is

\[
\alpha A \left( \frac{1 - \beta}{1 - \gamma} \right)^\gamma \bar{k}^i = 1 + n.
\]
Isolating the capital per worker,

\[ \bar{k}_{GR} = \left( \frac{1 - p^i}{1 - q^i} \right)^{1-\alpha} \left( \frac{\alpha A}{1 - \mu} \right)^{\frac{1}{1-\alpha}}. \]

Rewriting the steady-state capital per worker, we obtain:

\[ \left( \bar{k}^i \right)^* = \left( \frac{1 - p^i}{1 - q^i} \right)^{1-\alpha} \left( \frac{\alpha A}{1 - \mu} \right)^{\frac{1}{1-\alpha}} \left[ \delta^i(\beta^i)(1 - \tau^i) + \frac{\tau^i(1 - q^i)}{q^i} \frac{\beta(1 - \alpha)}{\alpha(1 + \beta)} \right]^{\frac{1}{1-\alpha}} \]

and finally

\[ \left( \bar{k}^i \right)^* = \left[ \delta^i(\beta^i)(1 - \tau^i) + \frac{\tau^i(1 - q^i)}{q^i} \frac{\beta(1 - \alpha)}{\alpha(1 + \beta)} \right]^{\frac{1}{1-\alpha}} \bar{k}_{GR}. \]

The steady-state capital per worker is characterized by a steady-state equilibrium with under-capitalization if \( \left( \bar{k}^i \right)^* < \bar{k}_{GR} \), or by a steady-state equilibrium with over accumulation of capital if \( \left( \bar{k}^i \right)^* > \bar{k}_{GR} \). The tax system on the first period wage can be chosen in order to reach the Golden Rule:

\[ \tau_{CR} = \frac{q^i \alpha(1 + \beta)(1 - \delta^i)}{(1 - q^i)\beta(1 - \delta^i) - (1 + \beta)q^i\alpha\delta^i}. \]

The borders can be open in the Golden Rule and migration is welfare improving.

The question of post-migration welfare is quite complex from a mathematical point of view, because there is no clear influence of \( \delta^i(\beta^i) \) on the steady-state equilibrium and because wages and interest rates across the two countries cannot be equal at the same time, except for some special values of parameters, a condition that has no chance of being reached. However, a simple line of reasoning may help explain what the post-migration welfare could be. In post-migration equilibrium, if a country has more low risk-averse individuals than in autarky, its savings should be increased, thereby improving by the same way the welfare in this country, but not in the other country, where individual welfare will worsen. Reciprocally, in post-migration equilibrium, if a country has more high risk-averse individuals than in the autarkic equilibrium, its savings should be reduced, so that its post-migration welfare is worsened, while it is improved in the other country.
CONCLUSION

This paper has studied international migration under uncertainty in a two period overlapping generations model. It has produced some new results. First of all, under uncertainty, when individuals make decisions within the mean - variance paradigm, it is shown that with a log linear utility function, the variance term can be eliminated, so that everything is as if individuals were likely making decision according to expected utility, the so-called VNM paradigm. This property of the model dramatically simplifies calculations. When the two countries only differ in their shocks on labor and / or on capital, the capital per worker differs across country. Consequently, incentives for migration exist because one of the two country exhibits a high level of capital per worker than the other. Since individuals are allowed to make international migration decisions according to their expected utility in steady-state equilibrium, migration cease when expected price equalize, and not prices. Since the solely causes of country differences are random, there is no reason for international migration should cause the post-migration equilibrium to converge to some particular post-migration price.

Why does one country not completely empty into the other? The reason is because individuals are heterogeneous with respect to their risk aversion. They maximize the ratio of their marginal expected payoff of migration over the marginal risk taken when migrating, which is exactly the type of behavior Levy and Markowitz (1979) suggested. Due to their heterogeneity, for a given expected payoff, some accept to migrate, some refuse. Consequently, both countries are always populated.

This result is very useful when we consider that individual may return migrate, since their country is still populated when they return home for retirement. Our model is capable of reproducing return migrations in the overlapping-generations models. This is new. The advantage of this approach is that it avoids the anticipation by all foresighted migrants of the effect of their return migration choice on prices for all subsequent generations (which really involves complexity). This is because in our paper, their migration has no impact on prices, since countries only differ by the economic shocks that affect labor and capital. We avoid the so-called generations conflict that always appear in the traditional literature with certainty: parents want to live in one country, while children want to live in another.

We have chosen such a framework to highlight the fact that the convergence principle that is alleged to be at work in open economies may not be as rapid as expected, even if we keep a dynamic competitive setting, and may even never happen.

REFERENCES


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A. APPENDIX

Levy and Markowitz (1979) consider an investor who maximizes the expected value of some utility function $U(R)$, where $R$ is the rate of return of the portfolio. It has been shown that it is more efficient to determine the set of mean-variance efficient portfolio than to find the portfolio that maximizes the expected utility. The authors provide various simulations showing that their methods better fit the optimum than simply the Von Neumann and Morgenstern (1941) model does. Usually, there are two ways to approximate the expected utility by a function $f_\theta(E,V)$ of the mean $E$ and the variance $V$, where index $\theta$ captures the risk aversion of the individual. The first is based on a Taylor series around zero, the second one is a Taylor-series around $E$. Both approximations involve fitting a quadratic $Q_\theta$ to the utility $U(R)$, based on the properties of $R$ (i.e., on the shape of the utility function, the first and second derivatives) at only one point ($0$ or $E$). The authors conjectured that a better approximation could be found by fitting the quadratic $Q_\theta$ to three judiciously chosen points on $U(R)$, which are $(E-\theta\sigma)$, $(E)$ and $(E+\theta\sigma)$ where $\theta$ captures the individual’s risk aversion and $\sigma$ the standard deviation.

B. APPENDIX

The quadratic $Q_\theta$ passing through these three points can be written as:

$$Q_\theta = a_\theta + b_\theta (R - E) + c_\theta (R - E)^2.$$ 

Proof of lemma 1: To simplify notation we will adopt the following notations $EQ$ the expected quadratic, and $a,b,c$, the subscript $\theta$ being omitted.

$$EQ = a + cV,$$

$$U(E-\theta\sigma) = a + b((E-\theta\sigma) - E) + c((E-\theta\sigma) - E)^2 = a - b\theta^2\sigma + c\theta^2\sigma^2,$$

$$U(E) = a + b\theta + c\theta^2,$$

$$U(E+\theta\sigma) = a + b\theta^2\sigma + c\theta^2\sigma^2.$$ 

Solve this system for $a,b$ and $c$ to have:

$$a = U(E)$$

$$b = \frac{U(E+\theta\sigma) - U(E-\theta\sigma)}{2\theta\sigma}$$

$$c = \frac{U(E+\theta\sigma) + U(E-\theta\sigma) - 2U(E)}{2\theta^2\sigma^2}$$

We define the approximation of a utility function $U$ by $f_\theta(E,V,U)$. Hence
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\[ f_0(E, V, U) = EQ = U(E) + \frac{U(E + \theta \sigma) + U(E - \theta \sigma) - 2U(E)}{2\theta^2 \sigma^2}. \]

Simplify by \(\sigma^2\):

\[ f_0(E, V, U) = U(E) + \frac{U(E + \theta \sigma) + U(E - \theta \sigma) - 2U(E)}{2\theta^2}. \] (23)

We choose in this paper a log-linear function to represent the individual’s preferences, as Levy and Markowitz (1979) do, among other utility functions, but the following development is not in their paper.

Using (23) and the properties of the logarithmic function, we have for the second period consumption \(E(d'_{i,t+1})\):

\[ f_0(E, V, U) = \log \left[ E(d'_{i,t+1}) \right] + \frac{\log \left[ E(d'_{i,t+1}) + \theta \sigma \right] + \log \left[ E(d'_{i,t+1}) - \theta \sigma \right] - 2 \log \left[ E(d'_{i,t+1}) \right]}{2\theta^2}. \]

\[ f_0(E, V, U) = \log \left[ E(d'_{i,t+1}) \right] + \frac{\log \left[ E(d'_{i,t+1}) + \theta \sigma \right] - \log \left[ E(d'_{i,t+1}) \right]}{2\theta^2} - \frac{\log \left[ E(d'_{i,t+1})^2 - \theta^2 \sigma^2 \right]}{2\theta^2}. \]

\[ f_0(E, V, U) = \frac{\theta^2 - 1}{\theta^2} \log \left[ E(d'_{i,t+1}) \right] + \frac{\log \left[ E(d'_{i,t+1})^2 (1 - \theta^2 V(d'_{i,t+1}) / E(d'_{i,t+1})^2) \right]}{2\theta^2}. \]

\[ f_0(E, V, U) = \frac{\theta^2 - 1}{\theta^2} \log \left[ E(d'_{i,t+1}) \right] + \frac{\log \left[ E(d'_{i,t+1}) \right]}{2\theta^2} + \frac{\log \left[ (1 - \theta^2 V(d'_{i,t+1}) / E(d'_{i,t+1})^2) \right]}{2\theta^2}. \]

\[ Like\ Jovanovic\ and\ Nyarko\ (1994),\ we\ argue\ that\ the\ choice\ of\ a\ functional\ form\ for\ the\ utility\ function\ and\ for\ the\ production\ function\ is\ important.\ Here,\ the\ Cobb-Douglas\ production\ function\ allows\ us\ to\ easily\ calculate\ the\ wage\ and\ the\ rate\ of\ return\ of\ the\ capital.\ We\ also\ choose\ a\ logarithmic\ utility\ function\ which\ allows\ us\ to\ explicitly\ model\ savings\ as\ a\ simple\ function\ of\ the\ current\ wage,\ eliminating\ the\ interest\ rate.\ Consequently\ this\ drastically\ simplifies\ the\ expressions\ and\ the\ calculation\ of\ the\ steady-state.\]
We already know the expression of the expected income at each period of time. Let us now evaluate the dispersion, i.e., the Variance, of the second period consumption. By definition

\[ \sigma^2 [d_{i+1}^t(\theta)] = (1 - p') \left[ (1 - \lambda^i) R_{t+1}^s(\theta) \right]^2 + p' (z_{i+1}^t(\theta))^2 - \left[ (1 - p') R_{t+1}^s(\theta) \right]^2, \]

Using that \( p' z_{i+1}^t(\theta) = \lambda^i (1 - p') R_{t+1}^s(\theta) \) so that we have

\[ (z_{i+1}^t(\theta))^2 = \left[ \frac{\lambda^i (1 - p') R_{t+1}^s(\theta)}{p'} \right]^2, \]

\[ \sigma^2 [d_{i+1}^t(\theta)] = (1 - p') \left[ (1 - \lambda^i) R_{t+1}^s(\theta) \right]^2 + p' \left[ \frac{\lambda^i (1 - p') R_{t+1}^s(\theta)}{p'} \right]^2 - \left[ (1 - p') R_{t+1}^s(\theta) \right]^2, \]

Develop to finally obtain:

\[ V(d_{i+1}^t(\theta)) = \frac{1 - p'}{p'} \left[ q' - \lambda^i \right]^2 (R_{t+1}^s)^2. \]

Calculate the ratio \( V(d_{i+1}^t(\theta)) / E(d_{i+1}^t(\theta))^2 \) we have:

\[ V(d_{i+1}^t(\theta)) / E(d_{i+1}^t(\theta))^2 = \left[ \frac{p' - \tau}{p' (1 - p')} \right]^2. \]

Replace this expression into the utility function to have:

\[ f_\theta(E, V, U) = \log \left[ E(d_{i+1}^t) \right] + \frac{\log \left[ (1 - \theta^2 \frac{(p' - \tau)^2}{p' (1 - p')}) \right]}{2 \theta^2}, \]

Since utility functions are defined to any monotonic and increasing transformation, the constant term depending on the probabilities and risk aversion can be eliminated, so that we have:

\[ U(c_i^t(\theta), E[d_{i+1}^t(\theta)]) = \log \left[ c_i^t(\theta) \right] + \beta \log \left[ E[d_{i+1}^t(\theta)] \right]. \]
C. APPENDIX

Proof. The proof of the Proposition is as follows. The condition for choosing a high level of saving in country \(i\) is

\[f_o(E_i'(s_i'(\varrho)), V_i'(s_i'(\varrho)), U) \geq f_o(E_i'(s_i'(\varrho)), V_i'(s_i'(\varrho)), U),\]

\[\log\left[ E(s_i'(\varrho)) \right] + \frac{\log\left[ (1-(\vartheta)^2) V(s_i'(\varrho)) / E(s_i'(\varrho))^2 \right]}{2(\vartheta)^2} \geq \log\left[ E(s_i'(\varrho))^2 \right] + \frac{\log\left[ (1-(\vartheta)^2) V(s_i'(\varrho)) / E(s_i'(\varrho))^2 \right]}{2(\vartheta)^2},\]

\[\frac{E(s_i'(\varrho))}{E(s_i'(\varrho))^2} \geq \frac{(1-(\vartheta)^2) V(s_i'(\varrho)) / E(s_i'(\varrho))^2)}{(1-(\vartheta)^2) V(s_i'(\varrho)) / E(s_i'(\varrho))^2},\]

\[E(s_i'(\varrho)) - (\vartheta)^2 V(s_i'(\varrho)) / E(s_i'(\varrho)) \geq E(s_i'(\varrho)) - (\vartheta)^2 V(s_i'(\varrho)) / E(s_i'(\varrho)).\]

All individuals who have a low risk aversion \(\vartheta\) will choose to save on the basis of the net income in period 1, and others will save on the basis of the unemployment benefits according to the following rule:

\[\frac{E(s_i'(\varrho)) - E(s_i'(\varrho))}{V(s_i'(\varrho)) / E(s_i'(\varrho)) - V(s_i'(\varrho)) / E(s_i'(\varrho))} \geq (\vartheta)^2.\]

Let us calculate the expected saving and its variance.

\[E\left[ s_i'(\varrho) \right] = (1-q') \frac{\beta(1-\tau')}{1+\beta} w_i',\]

\[V\left[ s_i'(\varrho) \right] = (1-q') \frac{\beta^2}{(1+\beta)^2} (1-\tau')^2 w_i'^2 - (1-q')^2 \frac{\beta^2}{(1+\beta)^2} (1-\tau')^2 w_i'^2 = q'(1-q') \frac{\beta^2}{(1+\beta)^2} (1-\tau')^2 w_i'^2,\]
\[ \frac{V[s'_i(\theta)]}{E[s'_i(\theta)]]} = q' - \frac{\beta}{1 + \beta} (1 - \tau') w'_i, \]

\[ E[s'_i(\bar{\theta})] = q' s'_i(\bar{\theta}) = \frac{\beta}{1 + \beta} q' b'_i = \frac{\beta}{1 + \beta} \tau' (1 - q') w'_i, \]

\[ V[s'_i(\bar{\theta})] = q' \left[ \frac{\beta^2 \tau' (1 - q')^2 w'_i}{q' (1 + \beta)^2} - \frac{\beta^2 (1 - q')^2 \tau' w'_i^2}{(1 + \beta)^2} \right], \]

\[ = \left[ \frac{1 - q'}{q'} \right] \frac{\beta^2 \tau' (1 - q')^2 w'_i}{(1 + \beta)^2}, \]

\[ \frac{V[s'_i(\bar{\theta})]}{E[s'_i(\bar{\theta})]} = \left[ \frac{1 - q'}{q'} \right] \frac{\beta}{1 + \beta} \tau' (1 - q') w'_i. \]

Replace the respective expressions of \( E(s'_i(\theta)) \) and \( V(s'_i(\theta)) \) into the criterion of decision and obtain:

\[ (1 - q') \frac{\beta}{1 + \beta} \tau' w'_i - \frac{\beta}{1 + \beta} \tau' (1 - q') w'_i \geq \tilde{\theta}. \]

Simplify by \( (1 - q') \frac{\beta}{1 + \beta} w'_i \)

\[ \frac{1 - 2 \tau'}{1 - q'} (1 - \tau') - \frac{1}{\tau'} \tilde{\theta} \leq \tilde{\theta}, \frac{\partial \tilde{\theta}}{\partial q'} < 0. \quad (24) \]

As the probability of the labor-market shock increases, the number of risky individuals decreases.