Software for Time Series Analysis

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Summary : This paper is devoted to the presentation of (a) a general approach in univariate time series analysis, and (b) the corresponding computer software called ANSECH. The methodology is similar to that of Box and Jenkins(1970) but the class of models is wider. The main algorithms for estimation and forecasting are briefly described.

Keywords : time series, Box and Jenkins method, statistical algorithms.

1. Introduction

Given a time series $w = \{w_t ; t=1, \dots, n\}$, a parametric model belonging to the class of stationary and invertible autoregressive-moving average (ARMA) models is searched for. The model is defined by

$$w_t = \phi_1 w_{t-1} + \dots + \phi_p w_{t-p} + a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q}, \quad (1)$$

$t \in \mathbb{Z}$, $a = \{a_t ; t=1, \dots, n\}$ being a white-noise process. Since the book of Box and Jenkins(1970), the tools of the time-series analyst have been improved, see the review by Newbold(1981). Many authors, including Box and Jenkins themselves, have admitted that the lack of stationarity of many time series implies that extensions of ARMA models should be used. Most computer programs on the market have few of these extensions. In this paper, we will describe a comprehensive class of models supported by a software called ANSECH (ANalyse des SEries CHronologiques), various estimation procedures and some aspects of the computer program.

Hipel and McLeod(1980) and Khobie-Zeitoune(1982) have had similar concerns.

2. The class of models

We will use the following notations :

- (a) $y_t^I, y_t^D, y_t^W, y_t^M$ and y_t^S are arbitrary functions of t ;
- (b) f_t and g_t are positive functions of t ;
- (c) μ_t and μ_t' are polynomial functions of t ;
- (d) m_t and m_t' are periodic functions of t ;
- (e) ϕ_{it} and θ_{jt} are either constants (ϕ_i, θ_j) , or functions of t ;
- (f) $C(.)$ defines a transformation which is invariant with respect to t ;
- (g) φ_i and ϑ_j are constants.

We denote by d, D, p, q, P, Q, s , non-negative integers, $s \neq 0$. Let $v = 1 - B$ and $v_s = 1 - B^s$, where B is the backshift operator ($B \cdot t = \cdot_{t-1}$). Finally, $z = \{z_t ; 1 \leq t \leq n\}$ is a time series and $a = \{a_t ; 1 \leq t \leq n\}$ is seen as a realization of a white-noise process with mean 0 and variance σ^2 .

The following equations, called respectively the variable sub-model, the exten-

ded ARMA sub-model and the innovation sub-model, yield a_t from the z_t :

$$w_t = \nabla^D \nabla_S^D [C(z_t - y_t^I)/f_t] - (y_t^D + \mu_t + m_t) \quad (2)$$

$$b_t = \frac{w_t - \sum_{i=1}^P \phi_i w_{t-i} + \sum_{j=1}^Q \theta_j b_{t-j} + y_t^W}{1 + \sum_{i=1}^P \phi_i w_{t-i} - \sum_{j=1}^Q \theta_j b_{t-j}} \quad (3)$$

$$a_t = \exp(-y_t^S) [b_t/g_t - (y_t^M + \mu_t' + m_t')] \quad (4)$$

The subscript t will be deleted when the context is clear. The parameters of the model are σ^2 and those included in (a-g). The analytical form of the functions (a-f) must be specified. The following remarks are intended to justify (2-4) :

- 1° Functions (a) can describe the effects of interventions. In particular, y^I has been used by Box and Tiao(1975) and y^S is concerned with the innovation variance. Instead of relying on transfer function models, a parametrization via piece-wise linear functions is used (Mélard,1981). This is much more efficient in all respects, e.g. when there are several outliers.
- 2° Several options exist for (b) in order to model a trend on the scale parameter of the time series (Mélard,1977).
- 3° Functions (c-d) can be used when deterministic trend or seasonal are better than the corresponding stochastic components provided by differencing. The seasonal component can be parcimoniously parametrized in terms of seasonal coefficients, Fourier coefficients (Abraham and Box, 1978) or a mixture of both.
- 4° The introduction of μ' and m' in (c-d) has been examined by Mélard(1982a).
- 5° In the extended ARMA sub-model, the coefficients ϕ_t and θ_t can vary linearly or exponentially with t (Mélard and Kiehm, 1981). It is also possible to handle the natural parameters of various short-term forecasting methods (Mélard and Rouland, 1979).
- 6° The transformation (f) is based on the power family as in Box and Cox(1964) and Ansley et al(1977) :

$$C(x_t) = \lambda^{-1} (x_t^\lambda - 1) \text{ for } \lambda \neq 0, C(x_t) = \log x_t \text{ for } \lambda = 0.$$

- 7° The non-linear model including (g) has been used by Lentz and Mélard(1981). The most useful elements among these extensions appear to be : (a) y^I , y^D and y^S ; (b) g and (f).

3. Estimation of the parameters of the extended model

Let α be the vector of all the parameters except σ^2 , and assume that the a_t are n.i.d. $(0, \sigma^2)$. The likelihood function, conditional on a_t and z_t , $t \leq 0$, is given by

$$L(\alpha, \sigma^2; z) = (2\pi)^{-n/2} J(\alpha) \sigma^{-n} \exp\left\{-\frac{1}{2\sigma^2} \sum_{t=1}^n a_t^2(\alpha)\right\}, \quad (5)$$

where $a_t(\alpha)$ is derived from (2-4) and the Jacobian $J(\alpha) = |\partial z_t / \partial a_s|$ is

$$J(\alpha) = \prod_{t=1}^n \{f_t g_t \exp(y_t^S) | 1 + \sum_i \tilde{\phi}_i w_{t-i} - \sum_j \tilde{\theta}_j b_{t-j} | (z_t - y_t^I)^{\lambda-1}\}. \quad (6)$$

By letting $\sigma_* = \sigma(J(\alpha))^{1/n}$, the concentrated likelihood is

$$(2\pi)^{-n/2} \sigma_*^{-n} \exp\{-S(\alpha)/(2\sigma_*^2)\}, \text{ where } S(\alpha) = \sum_t \{a_t(\alpha)/(J(\alpha))^{1/n}\}^2. \quad (7)$$

Maximizing (5) is equivalent to finding $\hat{\alpha}$ such that $S(\hat{\alpha})$ is minimum. Then an estimate $\hat{\sigma}^2$ of σ^2 is deduced from $\hat{\sigma}_*^2 = S(\hat{\alpha})/n$. In practice, the constraints $\prod f_t = \prod g_t = \prod \exp(y_t^S) = 1$ are imposed and the Box-Cox transformation is normalized.

Brubacher(1977) has noted that if λ is included in the model, the information matrix may not be estimated by

$$\hat{\sigma}^2 \Sigma \frac{\partial a_t}{\partial \alpha} \frac{\partial a_t}{\partial \alpha} \quad \text{but by} \quad \hat{\sigma}^2 \Sigma \left\{ \frac{\partial a_t}{\partial \alpha} \frac{\partial a_t}{\partial \alpha} + a_t \frac{\partial^2 a_t}{\partial \alpha^2} \right\}. \quad (8)$$

The same is true when f , g or y contains parameters.

The algorithm for computing a by the conditional method is divided in 14 steps. The steps 1 to $(r-1)$ are executed only once, where the r -th step is the first in which parameters are involved. The optimization routine calls steps r to 14 with the current parameters values, as many times as necessary. Only three auxiliary arrays $X1$, $X2$ and $X3$, with length not greater than $(n+d+Ds)$, are needed. Let $X4$ contain a . The algorithm is summarized in Table 1.

Table 1.- Algorithm for computing a by the conditional method

Exit : Y(es) if parameters first encountered at the next step, N(o) otherwise

Entry : Y if parameters first encountered at the current step, N otherwise

Parameters : indicates where they are included

$X1, X2, X3, X4$: operations performed (by default : on the same array)

List of operators : (op) $\in \{+, -, ., /, \nabla^d, \nabla_s^D, ARMA\}$

An operation in column X_i should be understood as an assignment $X_i =: X_i(\text{op})$ operand, except for "=" which is : $X_i =: \text{operand}$.

Step	Exit	Entry	Parameters	X1	X2	X3	X4
0	Y	Y		$= z$			
1	Y	Y		$- y^I$			
2	Y	Y	$C(.)$	$= C(.)$			
3	N	Y	f	$/ f$		$= f$	
4	N	N		∇^d			
5	Y	N		∇_s^D			
6	Y	Y	y^D, y^M	$- y^D$	$= X1$		
7	Y	Y	μ, μ'	$- \mu$			
8	Y	Y	m, m'	$- m$			
9	Y	Y	$\phi, \theta, \tilde{\phi}, \tilde{\theta}$				
10	N	Y	g			$= g$	
11	Y	N					
12	N	Y	y^W, y^S		$= -y^W$		
13	N	N			$ARMA(X1)$		
14	Y	N					$= X2/X3 - (y^M + \mu' + m') / \exp(y^S)$

Forecasting can be performed in two stages :

- 1° recovery of intermediary arrays, see Table 2 ;
- 2° computation of the forecasts, see Table 3. Four other arrays are necessary, denoted by X5, X6, X7 and X8.

Table 2.- Algorithm for forecasting : stage 1.

The macro-instruction [1,m] (X1,X2,X3,X4) means that steps 1 to m of the algorithm of Table 1 are executed.

Instruction	Result
[1,11] (X1,X2,X3,X4)	$X1 = w$
[1, 6] (X4,X2,X3,X4)	$X2 = w + y^D + \mu + m$ and $X3 = f$
$X2 =: X2 - X1$	$X2 = y^D + \mu + m$
[1, 1] (X1,X4,X4,X4)	$X1 = z - y^I$
$X4 =: z - X1$	$X4 = y^I$
[1, 3] (X1,X3,X3,X3)	$X1 = C(z - y^I)/f$

Table 3.- Algorithm for forecasting : stage 2.

The distinction between the observation and the forecast intervals is made by the concatenation operator " ; "

Instruction	Result
[1,14] (X5,X6,X7,X8)	$X5 = w$
$X8 =: w + X2$	$X8 = w + y^D + \mu + m$
$X8 =: X1 ; X8$	$X8 = C(z - y^I)/f ; w + y^D + \mu + m$
$X8 =: X8 ; (v_{v_S}^D)^{-1} X8$	$X8 = C(z - y^I)/f$
$X8 =: C^{-1}(X8.X3) + X4$	$X8 = z$

Three other estimation methods can be used in the program :

- 1° The unconditional method, with the backforecasting procedure (Box and Jenkins, 1970), iterated on request. The algorithm is more complex than the algorithm for the conditional method because of the time-dependency of the coefficients ϕ and θ (Kiehm et al, 1980) and presence of μ' and m' (Mélard, 1982a).
- 2° The exact likelihood method for ARMA models with constant coefficients, without y^S , g , μ' , m' , $\tilde{\phi}$, $\tilde{\theta}$. The algorithm (Mélard, 1982c) is faster than the approximate unconditional method.
- 3° The exact likelihood method for ARMA models with time-dependent models, without μ' , m' , $\tilde{\phi}$, $\tilde{\theta}$ (Mélard, 1982b).

We will not give here the corresponding modifications to Table 1. In all cases, a sum of squares $S(\alpha)$ is minimized with respect to α by the Marquardt algorithm.

4. General description of the software

In the sequel of this paper we will mention a number of features provided by the computer program ANSECH (Mélard, 1975), written in Extended Fortran C.D.C. for a Cyber 170/750 system. Each problem is decomposed into sections, as shown in Table 4. The problem is described by a command sentence using separated commands of the form "identifier" or "identifier=option". Each identifier is associated with two default option values, one to be supplied when the identifier is absent from the command sentence, the other, when the first form of the command is used. Examples of identifiers are shown in Table 4.

Table 4.- Decomposition of a problem into sections, with some examples of commands

Section	Purpose	Examples of commands
A	input validation	ESTIM must be present if estimation is requested
B	access to data	KEY = name of data set (in case of random access)
C	model definition	DIFF = degree of regular difference (value of d) BOXCOX for normalized logarithmic transformation
D	parameter meaning	HOLT = 1 or 2 Holt or Winters model (instead of ARMA)
E	estimation	ITERATIONS = maximum number of iterations (Marquardt) LIKELIHOOD = estimation method
F	plots, diagnostics	CHECKING = a string of code letters defining the requests (A for autocorrelation, L for Ljung-Box,...)
G	forecasting	LEADTIME = number of steps ahead for which forecasts are asked for

Further lines of directives are required for

- preliminary transformations,
- selection of parameters : type, order, initial value,
- description of interventions (Mélard, 1981).

There are fourteen types of parameters, plus the various parameters in the intervention directives. For example, the letters M and N identify respectively the regular and seasonal moving average polynomials. The following two lines can be used to fit the airline passengers data (Box and Jenkins, 1970, Ch 9) by the model

$$\nabla \nabla \log z = \frac{a}{t} - \frac{\theta}{1} \frac{a}{t-1} - \frac{\phi}{1} \frac{a}{t-12} + \frac{\theta}{1} \frac{\phi}{1} \frac{a}{t-13}$$

ESTIM,READ,AUXIN=1,KEY=BJ-SERG,BOXCOX,DIFF,DIFFS

M 1 0.5 N 1 0.6

("READ,AUXIN=1" specifies reading from input file # 1, DIFFS corresponds to ∇_s).

The command language is easy to learn, as observed with about a hundred students and researchers, many of them without any experience in data processing. However, a program has been written recently that questions the user who wishes to enter new

data or solve simple problems. Command sentences are composed, shown to the user, who can supplement them.

It is not possible to detail other features of ANSECH such as : data management, dating facility, extended residual tests, calculation of the likelihood on grids of points, transformations, Monte Carlo processor (which can yield a ready-to-use SPSS input file).

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