

Scaling Analysis and Evolution Equation of the North Atlantic Oscillation Index Fluctuations

C. Collette(*) and M. Ausloos

SUPRATECS, B5, Sart Tilman, B-4000 Liège, Belgium

(*) present address: Active Structures Laboratory, Université Libre de Bruxelles,

ULB - CP165/42 av. F.D. Roosevelt, 50 B-1050 Brussels, Belgium

(February 8, 2008)

Abstract

The North Atlantic Oscillation (NAO) monthly index is studied from 1825 till 2002 in order to identify the scaling ranges of its fluctuations upon different delay times and to find out whether or not it can be regarded as a Markov process. A Hurst rescaled range analysis and a detrended fluctuation analysis both indicate the existence of weakly persistent long range time correlations for the whole scaling range and time span hereby studied. Such correlations are similar to Brownian fluctuations. The Fokker-Planck equation is derived and Kramers-Moyal coefficients estimated from the data. They are interpreted in terms of a drift and a diffusion coefficient as in fluid mechanics. All partial distribution functions of the NAO monthly index fluctuations have a form close to a Gaussian, for all time lags, in agreement with the findings of the scaling analyses. This indicates the lack of predictive power of the present NAO monthly index. Yet

there are some deviations for large (and thus rare) events. Whence suggestions for other measurements are made if some improved predictability of the weather/climate in the North Atlantic is of interest. The subsequent Langevin equation of the NAO signal fluctuations is explicitly written in terms of the diffusion and drift parameters, and a characteristic time scale for these is given in appendix.

PACS numbers: 05.45.Tp, 05.45.Gg, 93.30.Fd, 89.69.+x; 02.50.Le, 05.40.-a, 47.27.Ak, 87.23.Ge

I. INTRODUCTION

In order to establish a sound understanding for any scientific phenomenon, one has to record numerical data and from the latter to obtain laws which can be next derived theoretically from so called first principle models. The so called inverse model method, starting from raw data and using statistical analysis as a first step, is of great interest since it is model free. Some difficulty arises in particular in nonlinear dynamical systems because of the need to sort out noise from both chaos and deterministic components^{1,2}. Whence to extract meaningful model-free dynamical equations from chaotic-like data is an enormous challenge³. Practically one is often led to empirical relationships. This is often the case in the meteorology/climatology field where there is a widely mixed set of various (sometimes) unknown influences, over different time and space scales. Often the fast variations are taken as noise terms in some sort of Langevin equation(s)^{4,5} as for the *el Niño* Southern Oscillation Index (SOI).

In order to quantify weather and climate events in Europe and report large-scale variability an index has been imagined⁶ the so called North Atlantic Oscillation (NAO) index ([http : //www.ldeo.columbia.edu/NAO/](http://www.ldeo.columbia.edu/NAO/); [http : //www.met.rdg.ac.uk/cag/NAO/](http://www.met.rdg.ac.uk/cag/NAO/); [http : //www.cru.uea.ac.uk/cru/info/nao/](http://www.cru.uea.ac.uk/cru/info/nao/);

[http : //www - bprc.mps.ohio - state.edu/gpl/NAO/Naobibliography.htm](http://www-bprc.mps.ohio-state.edu/gpl/NAO/Naobibliography.htm)). It is the normalized sea level pressure (SLP) difference between a station at Ponta Delgada, Azores and one at Akureyri, Iceland.

Since about the mid-50's the NAO index was trending from negative to positive values, but is mostly positive since 1980, a variation attributed to global warming. It is thought⁷ that the influence of slow changes in the ocean and in the greenhouse gases maybe picked

up as the fundamental causes of a prolonged (upward) trend.

Until a few years ago, the NAO was not receiving intense attention⁷, because it was thought that its phase and amplitude were rather unpredictable, because both involve many (time and space) scales which are often intrinsic to chaotic behavior; see also reviews⁸⁻¹⁰. Yet, evidence has been presented that NAO exhibits 'long-range' dependence having winter values residually correlated over many years, with short-term 2-5 year variations and decadal trends. Note that Wallace¹¹ has argued that the NAO is a local expression for a Northern Annular Mode (NAM), also called Arctic Oscillation^{12,13}.

In view of the above it seems of pertinent interest to consider again the NAO and adopt specific data analysis techniques when searching for scaling ranges and stochasticity features. We start with the Hurst (R/S) method^{14,15} followed by a detrended fluctuation analysis (DFA)^{16,17} of the monthly averaged NAO signal. Such tests supplement classical analyses based on frequency spectra¹⁸⁻²⁰ which are debatable due to the non stationarity of the data. Interestingly the data histogram have so called fat tails, resembling the Lévy flights, signatures of *self-organizing* systems, today emerging in many areas of physics as those mentioned here above. Again these facts seem to exclude low dimensional chaos but support the conjecture of Markov dynamics for atmospheric evolution, as already suggested in fact many years ago^{8,21-23}.

In the following, we adopt a Markov assumption in order to derive the FPE and to write down the Chapman-Kolmogorov equation for the conditional probability of the increments Δx (of the NAO index) over different time intervals Δt . This leads to a numerical derivation of the Kramers-Moyal coefficients which are the moments of such probability distributions. Up to the second moments, this leads to the *diffusion* and *drift* coefficients appearing in the Fokker-Planck equation (FPE) and are basic to the

Langevin equation²⁴. It will be noticed that the analytical form of both drift $D^{(1)}$ and diffusion $D^{(2)}$ coefficients are simple. It will be found that the *experimental* probability density functions (pdf) have all a Gaussian form when excluding the (rare) large (so called extreme) events.

The methods, applied in this paper, are briefly explained in Sect. 2.2 and 2.3 : (1) the rescaled range analysis and (2) the detrended fluctuation analysis. In all cases results are tested against surrogate or shortened data for error bar evaluation. Both methods lead to an exponent characteristic of the classical random walker position fluctuation correlations. Next, in Sect. 3, we examine how to describe the statistical evolution of increments for different time scales i.e. establishing a Fokker-Planck equation within a Markov process assumption. A few comments pertain to considerations on NA weather causes and predictions in a discussion and conclusion sections.

II. DATA AND THEORETICAL ANALYSIS

A. Data

The monthly averaged NAO index (available on the web sites [http : //www.cru.uea.ac.uk/ftpdata/nao.dat](http://www.cru.uea.ac.uk/ftpdata/nao.dat) or [http : //www.cru.uea.ac.uk/cru/data/nao.htm](http://www.cru.uea.ac.uk/cru/data/nao.htm) and updated at [http : //www.cru.uea.ac.uk/timo/projpages/nao_update.htm](http://www.cru.uea.ac.uk/timo/projpages/nao_update.htm)), i.e. as the normalized sea level pressure (SLP) difference between a station at Ponta Delgada, Azores (26°W,38°N) and one at Akureyri, Iceland (18°W,66°N) is represented on Fig.1 from

January 1825 to November 2002 (2135 points).¹

It is a standard procedure that in order to reduce spurious noise effects, the study is performed on the *integrated* series (Fig.2). Such values can be interpreted as mimicking the successive positions of a random walker¹⁵. The amplitude correlations should allow us below to understand the drift and diffusion process (as that of a walker).

B. The rescaled range analysis

Introduced by Hurst^{14,29}, the rescaled range analysis method computes a ratio R/S defined as follows. The time series $X = \{x_t, t = 1, \dots, N\}$ is divided into l intervals of equal length n . In the k^{th} box, ($k = 1, \dots, l$), there are n elements, $X_j^{(k)}(n) = \{x_j, j = (k-1)n + 1, \dots, (k-1)n + n (\equiv kn)\}$. The local fluctuation at point j in the k^{th} -box, i.e. $(x_j^{(k)} - \langle x \rangle_n^{(k)})$ is calculated as the deviation from the mean $\langle x \rangle_n^{(k)} = \frac{1}{n} \sum_{j=1}^n x_j^{(k)}$, in that k^{th} -box. The *cumulative* departure $Y_m^{(k)}(n)$ up to the m^{th} point in the k^{th} -box (of size n) is next calculated

$$Y_m^{(k)}(n) = \sum_{j=1}^m (x_j^{(k)} - \langle x \rangle_n^{(k)}) = \left(\sum_{j=1}^m x_j^{(k)} \right) - m \langle x \rangle_n^{(k)} \quad (1)$$

for $m = 1, \dots, n$ and in all k boxes and where $\langle x \rangle_n^{(k)} = \frac{1}{n} \sum_{j=1}^n x_j^{(k)}$. The rescaled range function is defined by

$$\frac{R^{(k)}}{S^{(k)}}(n) = \frac{\max_{1 \leq m \leq n} (Y_m^{(k)}(n)) - \min_{1 \leq m \leq n} (Y_m^{(k)}(n))}{\sqrt{\left(\frac{1}{n}\right) \sum_{j=1}^n (x_j^{(k)} - \langle x \rangle_n^{(k)})^2}} \quad k = 1, \dots, l. \quad (2)$$

¹For completeness let us point out that early instrumental or paleoclimatic data can be used to extend the North Atlantic Oscillation index back to 1823 or even 1675²⁵⁻²⁸.

The average of the rescaled range in all boxes with an equal size n is next obtained and denoted by $\langle R/S \rangle$. The above computation is then repeated for different values of n to provide a relationship between $\langle R/S \rangle$ and n , - which is expected to be a power law $\langle R/S \rangle \simeq n^H$ if some scaling range and law exist; H is called the Hurst exponent. If $H = 0.5$, the signal is uncorrelated (white noise); the "walk" is like a Brownian motion. If $H < 0.5$, the signal is anticorrelated (blue noise); if $H > 0.5$, there are positive correlations in the signal (red noise). From Fig. 3 it is found that $H = 0.55 \pm 0.02$ for the NAO index variations from 10 to 300 months (about 25 years). The departure from strict linearity on a log-log plot is usually attributed to too small box sizes, or to some periodic mode not finally taken into account through the assumed constant base line in Eq. (1); see discussions in references quoted here above and in the introduction. In order to test the robustness of the result we have checked the scaling properties of NAO index data series that are shorter than the original one by 5% (107 data points). In both cases when we delete the first 107 data points or the last 107 data points, the R/S analysis exponent has roughly the same value. The same goes true (not shown) for surrogate data series, i.e. when amplitudes are randomly displaced or multiplied by a random sign. It is certain that the H value near 0.55 indicates a weak deviation of the signal from Brownian motion.

C. The detrended fluctuation analysis

The detrended fluctuation analysis method has been recently much used in the meteorological field^{19,30-32}. The method has the advantages of avoiding (seasonal-like) trends and non stationarity effects, intrinsic to the finite size of the data. The method consists

in dividing the time series $X = \{x_i, i = 1, \dots, N\}$ into l boxes of equal size n . In the k^{th} box, the *cumulative* sum $Y_m^{(k)}$ can be calculated as above, in the so called the first order DFA. Let $Y_{fit,i}^{(k)}(n)$, be the best linear fit to the data in the k^{th} box. The detrended fluctuation function is next calculated, i.e. dropping the (n) , $\phi_i^{(k)} = Y_i^{(k)} - Y_{fit,i}^{(k)}$. The root mean square fluctuation is then given by

$$F(n) = \sqrt{\frac{1}{N} \sum_{i=1}^N [\phi_i]^2}. \quad (3)$$

If the values of the time series are correlated, there is a power-law relationship between $F(n)$ and n : $F(n) \simeq n^\alpha$. A departure from linearity on a log-log plot, and the existence of crossovers (hereby one occurs near 240 months, see Fig.4) has been discussed¹⁷. Fig. 4 shows that $\alpha = 0.54 \pm 0.02$ below 240 months, for NAO index variations. Again the robustness of the result is confirmed by analysing shorter or surrogate data series. Various considerations¹⁵ indicate that α should be equal to H . Thus we can conclude that the value of the scaling exponent α is roughly the same as the one obtained within the rescaled range analysis. Notice that the DFA method, is clearly leading to extend the scaling properties of the NAO index toward smaller scales, less than 10 months as is found for the R/S analysis.

The above findings confirm the existence of non trivial correlations (since $\alpha \neq 0.5$, even though it is close to 0.50) within precise interval time ranges. They point out to the existence of physical phenomena described as fractional Brownian motions (Mandelbrot 1982) thus with a fractal-like hierarchy of *time* scales. The result of α values larger than 0.5 can be interpreted again through a persistence effect in the fluctuations^{15,29}.

D. The Fokker-Planck equation

In view of the above it is of interest to search whether these weak and persistent correlations can be found through the solution of a phenomenological *evolution equation*, like the Fokker-Planck equation³⁴. Thus we focus on the variations Δx of the elements of the NAO series and the more so on their distribution in time. In order to do so we follow the method of Friedrich et al.³⁵ and reproduce it *almost in extenso* here below, for the technique is not necessarily familiar to most readers.

In order to characterize the statistics of NAO changes, increments $\Delta x_1, \Delta x_2, \dots$ for delay times $\Delta t_1, \Delta t_2, \dots$ at the same time t are considered. This leads to a set of $p(\Delta x_i, \Delta t_i)$. Next the joint probability density functions are evaluated for various time delays $\Delta t_1 > \Delta t_2 > \Delta t_3 > \dots$ directly from the given data set, e.g. $p(\Delta x_1, \Delta t_1; \Delta x_2, \Delta t_2)$. Of course if two increments i.e. Δx_1 and Δx_2 are statistically independent, the joint pdf should factorize into a product of two probability density functions:

$$p(\Delta x_1, \Delta t_1; \Delta x_2, \Delta t_2) = p(\Delta x_1, \Delta t_1)p(\Delta x_2, \Delta t_2). \quad (4)$$

leading to an isotropic single hill landscape in the $\Delta x_1, \Delta x_2$ plane.

A complete characterization of the statistical properties of the data set in general requires the evaluation of joint pdf's $p^N(\Delta x_1, \Delta t_1; \dots; \Delta x_N, \Delta t_N)$ depending on N variables (for arbitrarily large N). In the case of a Markov process (a process without memory but governed by probabilistic conditions), an important simplification arises: The N -point pdf p^N is generated by the mere product of conditional probabilities $p(\Delta x_{i+l}, \Delta t_{i+l} | \Delta x_i, \Delta t_i)$ itself equal to $p(\Delta x_{i+l}, \Delta t_{i+l}; \Delta x_i, \Delta t_i) / p(\Delta x_i, \Delta t_i)$ for $i = 1, \dots, N - 1$. The conditional probability is given by the probability of finding Δx_{i+1} values for fixed Δx_i . As a necessary

condition of Markov processes, the Chapman-Kolmogorov equation in its integral form reads

$$p(\Delta x_2, \Delta t_2 | \Delta x_1, \Delta t_1) = \int d(\Delta x_i) p(\Delta x_2, \Delta t_2 | \Delta x_i, \Delta t_i) p(\Delta x_i, \Delta t_i | \Delta x_1, \Delta t_1) \quad (5)$$

and should hold for any value of Δt_i , with $\Delta t_2 < \Delta t_i < \Delta t_1$; see Appendix A for a discussion in particular concerning large (and thus rare) events. As is well known, such a Chapman-Kolmogorov equation yields an evolution equation for the change of the conditional distribution functions $p(\Delta x, \Delta t | \Delta x_1, \Delta t_1)$ and $p(\Delta x, \Delta t)$ across the scales Δt . The Chapman-Kolmogorov equation when formulated in *differential* form yields a master equation, which can take the form of a Fokker-Planck equation^{35–37,24}. It is useful to use reduced time units, like $\tau = \log_2(16/\Delta t)$,²

$$\frac{d}{d\tau} p(\Delta x, \tau) = \left[-\frac{\partial}{\partial \Delta x} D^{(1)}(\Delta x, \tau) + \frac{\partial}{\partial^2 \Delta x^2} D^{(2)}(\Delta x, \tau) \right] p(\Delta x, \tau) \quad (6)$$

in terms of a drift $D^{(1)}(\Delta x, \tau)$ and a diffusion coefficient $D^{(2)}(\Delta x, \tau)$ (thus values of τ represent Δt_i , $i = 1, \dots$) Their functional dependence can be estimated directly from the moments $M^{(k)}$ (known as Kramers-Moyal coefficients) of the conditional probability distributions:

$$M^{(k)} = \frac{1}{\Delta \tau} \int d\Delta x' (\Delta x' - \Delta x)^k p(\Delta x', \tau + \Delta \tau | \Delta x, \tau) \quad (7)$$

for different small $\Delta \tau$'s, such that for $\Delta \tau \rightarrow 0$,

$$D^{(k)}(\Delta x, \tau) \simeq \frac{1}{k!} \lim_{\Delta \tau \rightarrow 0} M^{(k)}. \quad (8)$$

²Why "16" is chosen to be the normalizing value will be made clear below, but it has obviously not much effect at this stage.

After calculating such moments from the conditional probabilities, we find (Fig. 5) that the coefficient $M^{(1)}$ shows a linear dependence for small Δx , while $M^{(2)}$ can be approximated by a polynomial of degree two in Δx . Therefore the type of fluctuation probability drift term $D^{(1)}$ is well approximated by a linear function of Δx , whereas the diffusion term $D^{(2)}$ follows a function quadratic in Δx . For very large values of Δx the statistics becomes poorer and the uncertainty increases.

From a careful analysis of the data based on the functional dependences of $M^{(1)}$ and $M^{(2)}$ (Fig. 5 (a-b)), the following approximations hold true:

$$\begin{cases} D^{(1)} = -0.52\Delta x + 0.04 & \text{for } |\Delta x| < 5 \text{ (NAO units)} \\ D^{(2)} = \frac{1}{2} (0.21\Delta x^2 - 0.02\Delta x + 4.2) & \text{for } |\Delta x| < 5 \text{ (NAO units)} \end{cases} \quad (9)$$

Notice the range of validity of the simple analytical forms, thus the limit found for what would be called³⁸ "extreme events" or "outliers". Also observe that except for the independent term, $2D^{(2)} \simeq (D^{(1)})^2$, the strict equal sign being a request for indicating an absolute lack of intermittency in turbulence³⁹⁻⁴².

The FPE for the distribution function is known to be equivalent to a Langevin equation for the variable, i.e. Δx here, within the Ito interpretation^{24,34}

$$\frac{d}{d\tau}\Delta x(\tau) = D^{(1)}(\Delta x(\tau), \tau) + \eta(\tau)\sqrt{D^{(2)}(\Delta x(\tau), \tau)}, \quad (10)$$

where $\eta(\tau)$ is a fluctuating δ -correlated force with Gaussian statistics, i.e. $\langle \eta(\tau) \eta(\tau') \rangle = 2\delta(\tau - \tau')$. An interpretation of the analogy between these *drift* and *diffusion* coefficients and those usually employed in fluid mechanics is given in Appendix. It may be worthwhile to emphasize here that (i) a negative slope value for $D^{(1)}$ indicates a sort of restoring or damping force for the evolution of Δx ; (ii) the observed quadratic dependence of the

diffusion term $D^{(2)}$ is essential for the logarithmic scaling of the intermittence parameter in turbulence^{41,42}.

Using those analytical expressions of the empirically derived Kramers-Moyal coefficients, Eq.(6) can now be integrated, thereby leading to a test of the Markov process assumption. For the above change of variables, $\tau = \log_2(16/\Delta t)$, we take the observed distribution of the time series for $\Delta t = 16$ months thus at "large time", as the initial condition for the integration, - considering that there is no propagation of anomalous correlation to be expected after such a time lag. Indeed the scaling ranges of H and α found here above (Figs. 3 and 4) indicate that the correlations are quasi identical for $\Delta t = 16$ months or $\Delta t = 240 - 400$ months, whence justifying the normalization "16" as the safest lowest boundary range for quasi Brownian fluctuations. Starting that far with a Gaussian as initial condition, the results of the integration (for $\tau = 0,1,2,3,4$, or $\Delta t = 16, 8, 4, 2, 1$) are rather trivial. The variance and the mean are found to vary very weakly. The experimental data pdf's are shown in Fig. 6, - together even with the pdf for $\Delta t = 400$ months, - the latter being indistinguishable from the $\Delta t = 16$ months case. It is readily remarkable that the pdf's are close to the Gaussian form in the interesting NAO (Δx) index range, - with some marginal deviation for the extreme (and rare) events.

E. Discussion

It is therefore confirmed that the NAO is a complex phenomenon which is almost Markovian. This stresses the need to insert appropriate feedback mechanisms into any model evolution equation(s), with an appropriate (red) noise term. This has been recently discussed^{4,5,43,20}. Long-range fractionally integrated noise seems indeed to provide a better

fit of the NAO SLP wintertime index over the period 1864 – 1998 than does either stationary red noise or a non-stationary random walk⁹.

The persistence of the NAO index fluctuations, i.e. SLP fluctuations, is in agreement with the persistence of the sea surface temperature fluctuations at different sites in the North Atlantic as found by Monetti et al.⁴⁴. Some reasons for the above can be found in studies based on circulation-like models⁴⁵.

III. CONCLUSIONS

In summary, the aims of this paper are twofold : (i) to search whether scaling ranges exist in the North Atlantic Oscillation pressure index ; (ii) to derive its FPE and check the validity of the Markovianity assumption. This allows one to examine different time scales on the same footing, - a fundamental need in geophysics⁴⁶.

It is found that the lack of departure from a Gaussian process definitely is a new quality of the NAO index data set, - not perceptible with a rescaled range analysis, or a DFA, as done above, nor with spectral studies.

However, it seems that the actual NAO index is not very useful³. Thus one might have to request other measurements for better predictability of climate and weather in Europe and the Northern Hemisphere, e.g. at other locations. We might also suggest studies on "not-monthly-averaged" indices, - a monthly average being strangely unphysical in our

³This is in agreement with conclusions from a recent paper casting doubt on the NAO-global warming relationship⁴⁷, and indicating a *too strong* influence of the Azores data with respect to the Iceland one. See also Czaja and Frankignoul⁴⁸

opinion.

Yet it is emphasized that the FPE provides the complete knowledge as to how the statistics of correlations in the index distribution change on different delay times. Since this includes an analysis in time t for a *scalar* Δx , it seems that the findings could be implemented in atmospheric weather low dimensional *vector – models*^{49,50}. Further work in line with the above should be to relate the FPE to an analytical solution, e.g. with a model of the turbulence-like dynamics as was done for financial indices⁵¹ or ionic transport through membranes⁵² through a Beck-Tsallis^{53,54} nonextensive thermodynamics approach.

Acknowledgments

We thank D. Stauffer, C. Nicolis, K. Ivanova, and A. Bunde, for stimulating discussions and comments. Email correspondence with J. Peinke, Ch. Renner and R. Friedrich is specifically appreciated.

Appendix

The coefficients called *drift* and *diffusion* used in the main text pertain to the evolution of the pdf; they are usually describing the motion of particles in fluid mechanics. For example the diffusion coefficient occurs in the Einstein or Langevin equation of Brownian motion, as

$$D = \frac{k_B T}{6\pi\eta a} \quad (11)$$

where T is the bath temperature, η the dynamic viscosity of the fluid and a the diameter of the particle, such that the evolution of the particle is described as a function of time t by $\langle x^2 \rangle = 2Dt$, solution of the Langevin equation

$$M \frac{d^2 x}{dt^2} = -6\pi\eta a \frac{dx}{dt} + R(t), \quad (12)$$

where M is the mass of the particle and $R(t)$ a random force with zero mean.

The Langevin equation is equivalent to the standard diffusion equation for a probability density

$$\frac{dp(x, t)}{dt} = D \frac{\partial^2 p(x, t)}{\partial x^2} \quad (13)$$

for which the solution is a Gaussian

$$p(x, t) = \frac{1}{\sqrt{4\pi Dt}} e^{-x^2/(4Dt)}. \quad (14)$$

The FPE written in the main text contains an extra term to Eq.(14); let it be rewritten here as

$$\frac{d}{dt} p(x, t) = \left[-\frac{\partial}{\partial x} D^{(1)}(x, t) + \frac{\partial}{\partial^2 x^2} D^{(2)}(x, t) \right] p(x, t), \quad (15)$$

from which the mean $\langle x \rangle$ and the variance $\langle \sigma^2 \rangle$ can be defined as usual. Keeping only the linear term in $D^{(1)}$, with a coefficient $D_1^{(1)}$ and the independent $D_0^{(2)}$ and quadratic $D_2^{(2)}$ terms in $D^{(2)}$, from Eq.(9), one easily obtains the evolution of the "particle" as

$$\langle x(t) \rangle = \langle x_0(t) \rangle e^{(2D_1^{(1)} + D_2^{(2)})t} \quad (16)$$

and a similar equation for the variance, but also containing $D_0^{(2)}$, from which one observes that $D^{(1)}$ and $D^{(2)}$ are *true* drift and diffusion coefficients. Note the time scale given by the inverse of $D^{(1)}$ and $D^{(2)}$, i.e., about 1 month.

Knowing that Δx is the NAO index, a difference in pressure, (ΔP) we can roughly rewrite the "official" diffusion coefficient, Eq.(12), as

$$\langle (\Delta P)^2 \rangle = \frac{2k_B T}{6\pi\eta a} t, \quad (17)$$

and "interpret it", suggesting that in further and more precise work, one could develop a model relating the changes in pressure (between Iceland and the Azores) with a temperature field (in principle a temperature gradient, rather than the mean temperature of the bath).

No need to say that the solution of a Brownian particle in a (rotating) bath under a temperature gradient and with a noise force term is indeed what a good weather model is (or should be).

REFERENCES

- ¹ Bhattacharya, J., and P.P. Kanjilal, 2000: Revisiting the role of correlation coefficient to distinguish chaos from noise, *Eur. Phys. J. B*, **13**, 399-403.
- ² Provenzale, A., L.A. Smith, R. Vio, and G. Murante, 1992: Distinguishing between low-dimensional dynamics and randomness in measured time series, *Physica D*, **58**, 31-49.
- ³ Rowlands, G., and J.C. Sprott, 1992: Extraction of dynamical equations from chaotic data, *Physica D*, **58**, 251-259.
- ⁴ Penland, C., and L. Matrosova, 1998: Prediction of tropical Atlantic sea surface temperatures using linear inverse modeling, *J. Climate*, **11**, 483-496.
- ⁵ Penland, C., and P.D. Sardeshmukh, 1995: The optimal growth of tropical sea surface temperature anomalies, *J. Climate*, **8**, 1999-2024.
- ⁶ Marshall, J., Y. Kushnir, D. Battisti, P. Chang, A. Czaja, R. Dickson, M. McCartney, R. Saravanan, and M. Visbeck, 2001: North Atlantic Climate Variability: phenomena, impacts and mechanisms, *Inter. Jour. Climat.*, **21** 1863-1898
- ⁷ Hurrell, J.W., Y. Kushnir, and M. Visbeck, 2001: The North Atlantic Oscillation, *Science*, **291**, 603-604.
- ⁸ Palmer, T.N. , 2000: Predicting uncertainty in forecasts of weather and climate, *Rep. Prog. Phys.*, **63** 71-116.
- ⁹ Greatbatch, R. J., 2000: The North Atlantic Oscillation, *Stochastic Environmental Research and Risk Assessment*, **14**, 213-242.

- ¹⁰ Wanner, H., S. Bronnimann, C. Casty, D. Gyalistras, J. Luterbacher, C. Schmutz, D.B. Stephenson, and E. Xoplaki : 2001, North Atlantic Oscillation - concepts and studies, *Surv Geophys* , **22**, 321-382.
- ¹¹ Wallace, J.M., 2000: North Atlantic Oscillation / Annular Mode: Two paradigms ? One Phenomenon, *Quart. J. Royal Met. Soc.*, **126**, 791-805.
- ¹² Ambaum, M. H. P., B. J. Hoskins, and D. B. Stephenson, 2001: Arctic Oscillation or North Atlantic Oscillation?, *J. Climate*, **14**, 3495-3507.
- ¹³ Stephenson, D.B. , V. Pavan, and R. Bojariu, 2000: Is the North Atlantic Oscillation a random walk?, *Int. J. Climatol.*, **20**, 1-18.
- ¹⁴ Hurst, H.E., 1951: Long-term storage capacity of reservoirs, *Trans. Am. Soc. Civil Eng.*, **116**, 770-808.
- ¹⁵ Turcotte, D.L., 1999: *Fractal and Chaos in Geology and Geophysics*, Cambridge Univ. Press, Cambridge, UK.
- ¹⁶ Azbel, M.Ya., 1995: Universality in a DNA Statistical Structure, *Phys. Rev. Lett.*, **75**, 168-171.
- ¹⁷ Hu, K., P. Ch. Ivanov, Zhi Chen, P. Carpena, and H. E. Stanley , 2001: Effect of trends on detrended fluctuation analysis, *Phys. Rev. E*, **64**, 11 114 - 11 121.
- ¹⁸ Pelletier, J.D., 1997: Analysis and modeling of the natural variability of climate, *J. Climate*, **10**, 1331-1342.
- ¹⁹ Talkner, K., and R.O. Weber, 2000: Power spectrum and detrended fluctuation analysis: Application to daily temperatures, *Phys. Rev. E* , **62**, 150-160.

- ²⁰ Fernandez, I., C.N. Hernandez, and J.M. Pacheco, 2003: Is the North Atlantic Oscillation just a pink noise?, *Physica A*, **323**, 705-714.
- ²¹ Hasselmann, K., 1976: Stochastic climate models. Part 1: Theory, *Tellus*, **28**, 473-485
- ²² Frankignoul, C., and K. Hasselmann, 1977: Stochastic climate models. Part II: application to sea-surface temperature anomalies and thermocline variability, *Tellus*, **29**, 289-305.
- ²³ Leith, C.E., 1978: Predictability of climate, *Nature*, **276**, 352-355.
- ²⁴ Risken, H., 1984: *The Fokker-Planck Equation*, Springer-Verlag, Berlin.
- ²⁵ Jones, P.D., T. Jónsson, and D. Wheeler, 1997: Extension to the North Atlantic Oscillation using early instrumental pressure observations from Gibraltar and South-West Iceland, *Int. J. Climatol.*, **17**, 1433-145.
- ²⁶ Jones, P.D., K.R. Briffa, T.P. Barnett, and S.F.B. Tett, 1998: High resolution paleoclimatic records for the last millenium: interpretation, integration and comparison with general circulation model control-run temperatures, *The Holocene*, **8**, 455-471.
- ²⁷ Luterbacher, J., C. Schmutz, D. Gyalistras, E. Xoplaki, and H. Wanner, 1999 : Reconstruction of monthly NAO and EU indices back to AD 1675, *Geophys. Res. Lett.*, **26**, 2745-2748.
- ²⁸ Schmutz, C., J. Luterbacher, D. Gyalistras, E., Xoplaki, and H. Wanner, 2000 : Can we trust proxy-based NAO index reconstructions?, *Geophys. Res. Lett.*, **27**, 1135-1138.
- ²⁹ Malamud, B.D., and D.L. Turcotte, 1999: Self-affine time series: Measures of weak and strong persistence, *J. Stat. Plann. Infer.* **80**, 173-196.

- ³⁰ Koscielny-Bunde, E., A. Bunde, S. Havlin, H. E. Roman, Y. Goldreich, and H.-J. Schellnhuber, 1998: Indication of a universal persistence law governing atmospheric variability *Phys. Rev. Lett.*, **81**, 729-732.
- ³¹ Ivanova, K., and M. Ausloos, 1999: Application of the Detrended Fluctuation Analysis (DFA) method for describing cloud breaking *Physica A* , **274**, 349-354.
- ³² Ivanova, K., M. Ausloos, E.E. Clothiaux, and T.P. Ackerman, 2000: Break-up of stratus cloud structure predicted from non-Brownian motion liquid water and brightness temperature fluctuations, *Europhys. Lett.*, **52**, 40-46.
- ³³ Mandelbrot, B.B., 1982: *The Fractal Geometry of Nature*, Freeman, San Francisco.
- ³⁴ Pecseli, H.L., 2000: *Fluctuations in Physical Systems*, Cambridge Univ. Pres, Cambridge, UK.
- ³⁵ Friedrich, R., J. Peinke, and Ch. Renner, 2000: How to quantify deterministic and random influences on the statistics of the foreign exchange market, *Phys. Rev. Lett.*, **84**, 5224-5228.
- ³⁶ Hänggi, P., and H. Thomas, 1982: Stochastic processes : time evolution, symmetries and linear response, *Phys. Rep.*, **88**, 207-319.
- ³⁷ Gardiner, C.W., 1983: *Handbook of Stochastic Methods*, Springer-Verlag, Berlin.
- ³⁸ Grasso, J.R., and D. Sornette, 1998: Testing self-organized criticality by induced seismicity, *J. Geophys. Res.*, **103**, 29965-29987.
- ³⁹ Kolmogorov, A.N., 1941a: Local structure of turbulence in an incompressible viscous fluid for very large Reynolds numbers, *Doklady Akad. Nauk SSSR*, **30**, 299-303.

- ⁴⁰ Kolmogorov, A.N., 1941b: On degeneration of isotropic turbulence in an incompressible viscous liquid, *Doklady Akad. Nauk SSSR*, **31**, 538-541.
- ⁴¹ Parisi, G., and U. Frisch, 1985: in *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics*, Ghil M., Benzi R. and Parisi G., Eds. North Holland, New York.
- ⁴² Frisch, U., 1995: *Turbulence*, Cambridge Univ. Press, Cambridge UK.
- ⁴³ Johnson, S.D., D.S. Battisti, and E.S. Sarachik, 2000: Empirically Derived Markov Models and Prediction of Tropical Pacific Sea Surface Temperature Anomalies, *J. Climate*, **13**, 3-17.
- ⁴⁴ Monetti, R.A., S. Havlin, and A. Bunde, 2003: Long-term persistence in the sea surface temperature fluctuations, *Physica A*, **320** 581-589.
- ⁴⁵ Josey, S. A., E.C. Kent, and B. Sinha, 2001 : Can a State of the Art Atmospheric General Circulation Model Reproduce Recent NAO Related Variability at the Air-Sea Interface?, *Geophys. Res. Lett.*, **28**, 4543-4546.
- ⁴⁶ Fraedrich, K. and C.-D. Schönwiese, 2002: Space-Time Variability of the European Climate in *The Science of Disasters*, A. Bunde, J. Kropp, H.J. Schellnhuber (eds.), Springer, Berlin, pp.105-139.
- ⁴⁷ Jónsson, T. and M. Miles, M. 2001: Anomalies in the seasonal cycle of sea level pressure in Iceland and the North Atlantic Oscillation, *Geophys. Res. Lett.*, **28**, 4231-4234.
- ⁴⁸ Czaja, A., and C. Frankignoul, 2002: Observed impact of Atlantic SST anomalies on the North Atlantic Oscillation, *J. Climate*, **15**, 606-623.

- ⁴⁹ Grabowski, W.W., and P.K. Smolarkiewicz, 1999: CRCP: a cloud resolving convection parameterization for modeling the tropical convecting atmosphere, *Physica D*, **133**, 171-178.
- ⁵⁰ Ragwitz, M., and H. Kantz, 2000: Detecting non-linear structure and predicting turbulent gusts in surface wind velocities, *Europhys. Lett.*, **51**, 595-601.
- ⁵¹ Ausloos, M. and K. Ivanova, 2003: Dynamical model and nonextensive statistical mechanics of a market index on large time windows, *Phys. Rev. E* , **68**, 046122.
- ⁵² Ausloos, M., K. Ivanova and Z. Siwy, 2004: Searching for self-similarity in switching time and turbulent cascades in ion transport through a biochannel. A time delay asymmetry, *Physica A* , **336**, 319-333.
- ⁵³ Tsallis, C., 1988: Possible generalization of Boltzmann-Gibbs statistics *J. Stat. Phys.*, **52**, 479-.
- ⁵⁴ Beck, C., 2001: Dynamical foundations of nonextensive statistical mechanics, *Phys. Rev. Lett.* **87**, 180601.

Figure Captions

Figure 1 Time evolution of the monthly averaged NAO index fluctuations from January 1825 to November 2002 (2135 points) available on the web sites *http : //www.cru.uea.ac.uk/ftpdata/nao.dat* and updated at *http : //www.cru.uea.ac.uk/timo/projpages/nao_update.htm*)

Figure 2 Integration of the NAO index fluctuations signal shown on Figure 1

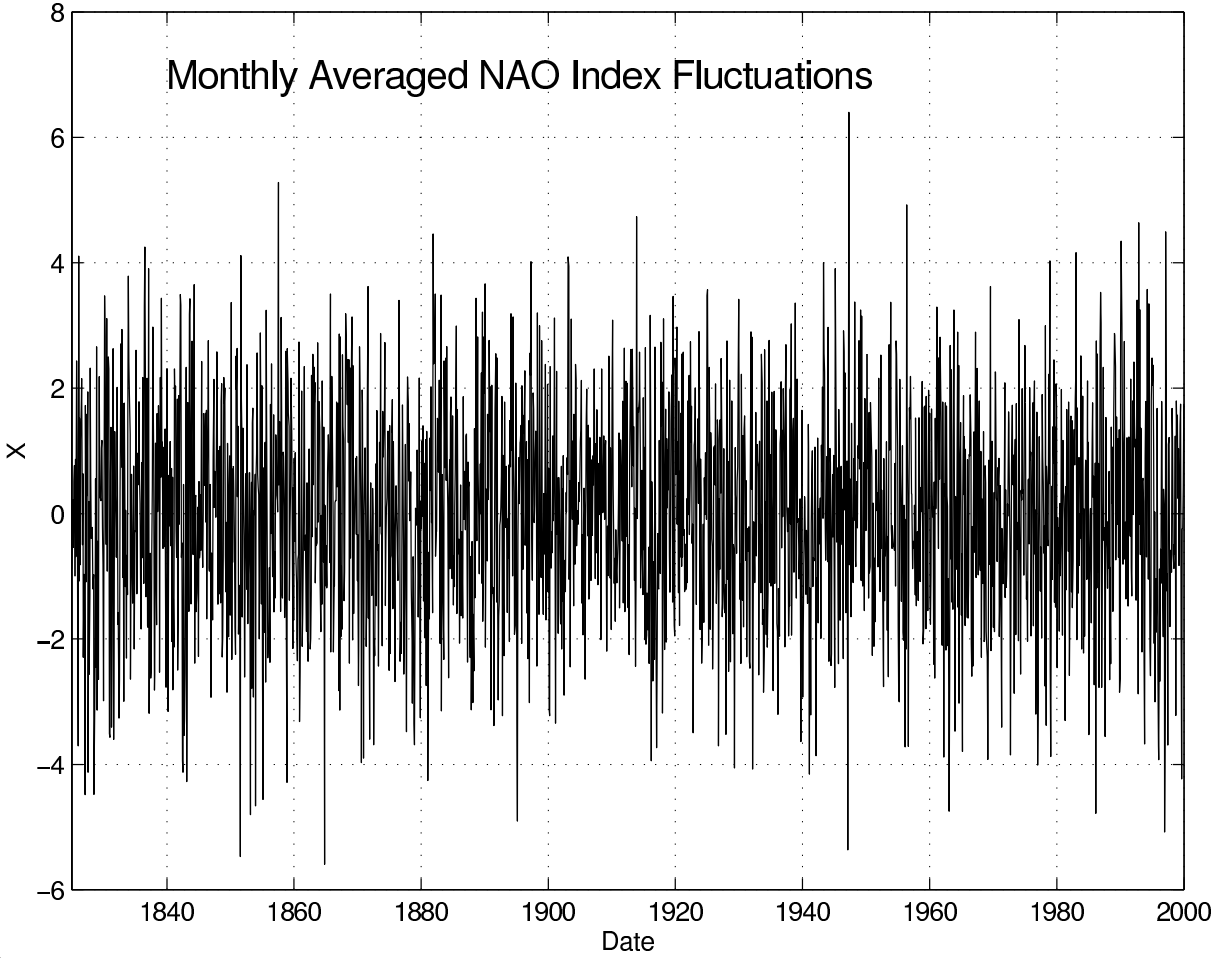
Figure 3 Rescaled range analysis of the NAO index signal. Inserts: Rescaled range analysis of the NAO index signal shortened by 5% at the beginning of the data series (left upper panel) or at the end (right lower panel)

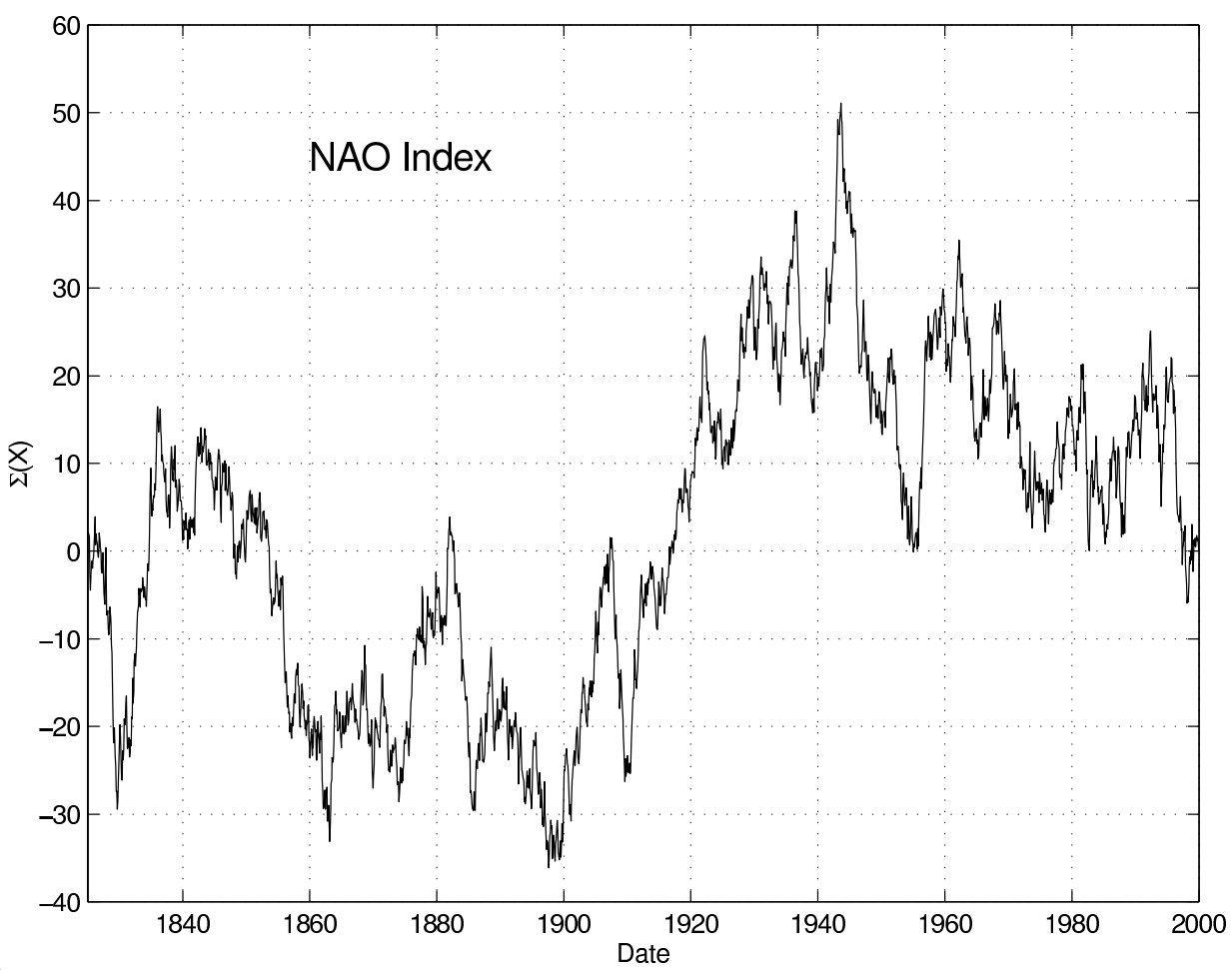
Figure 4 Detrended fluctuation function of the (integrated) NAO index signal. Inserts: Detrended fluctuation function of the (integrated) NAO index signal after shortening by 5% at the beginning of the data series (left upper panel) or at the end (right lower panel)

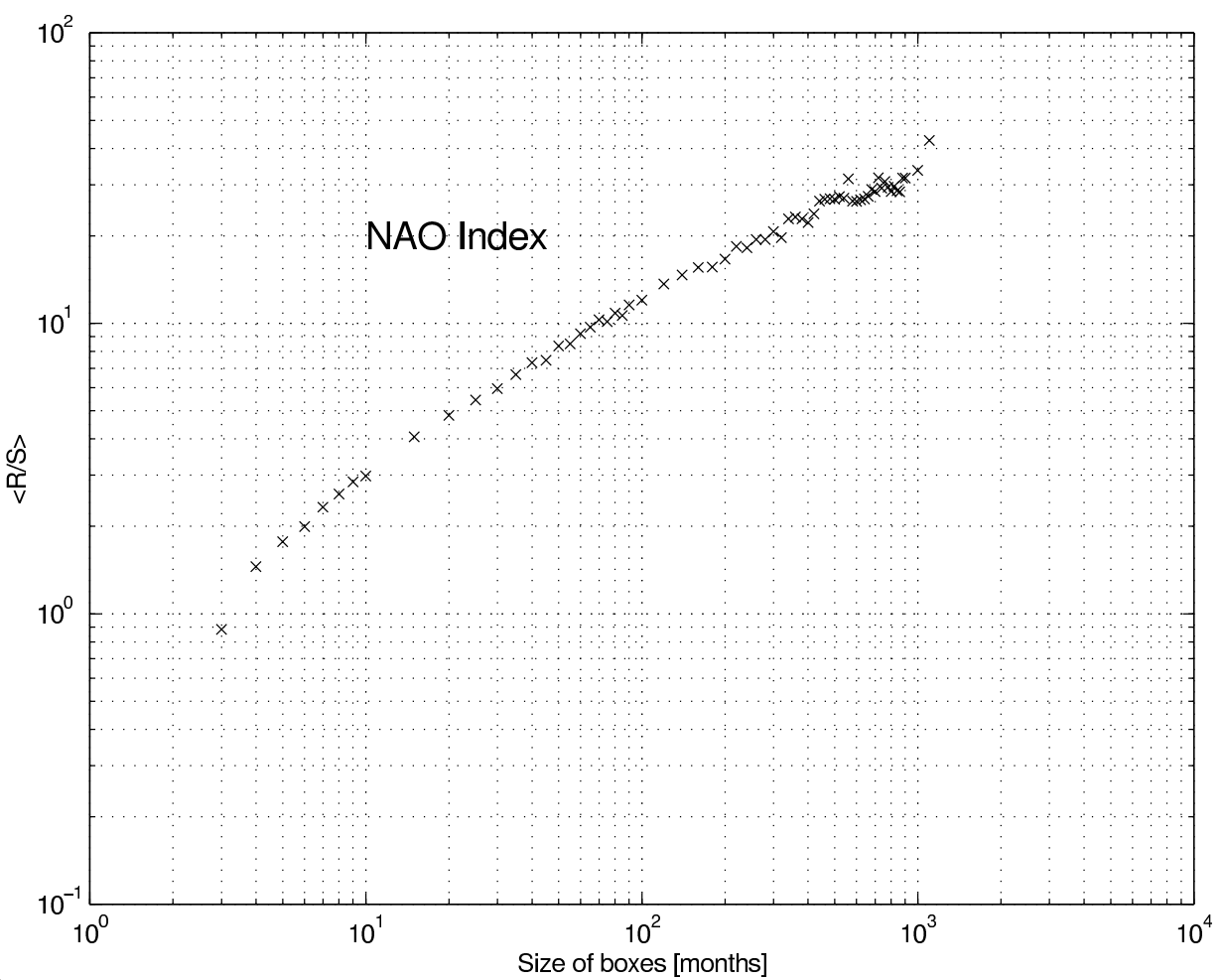
Figure 5 Kramers-Moyal coefficients (a) $M^{(1)}$ and (b) $M^{(2)}$ estimated from the empirical conditional density probability of the NAO distribution values. The solid curves represent a linear and a quadratic fit, respectively, excluding large events

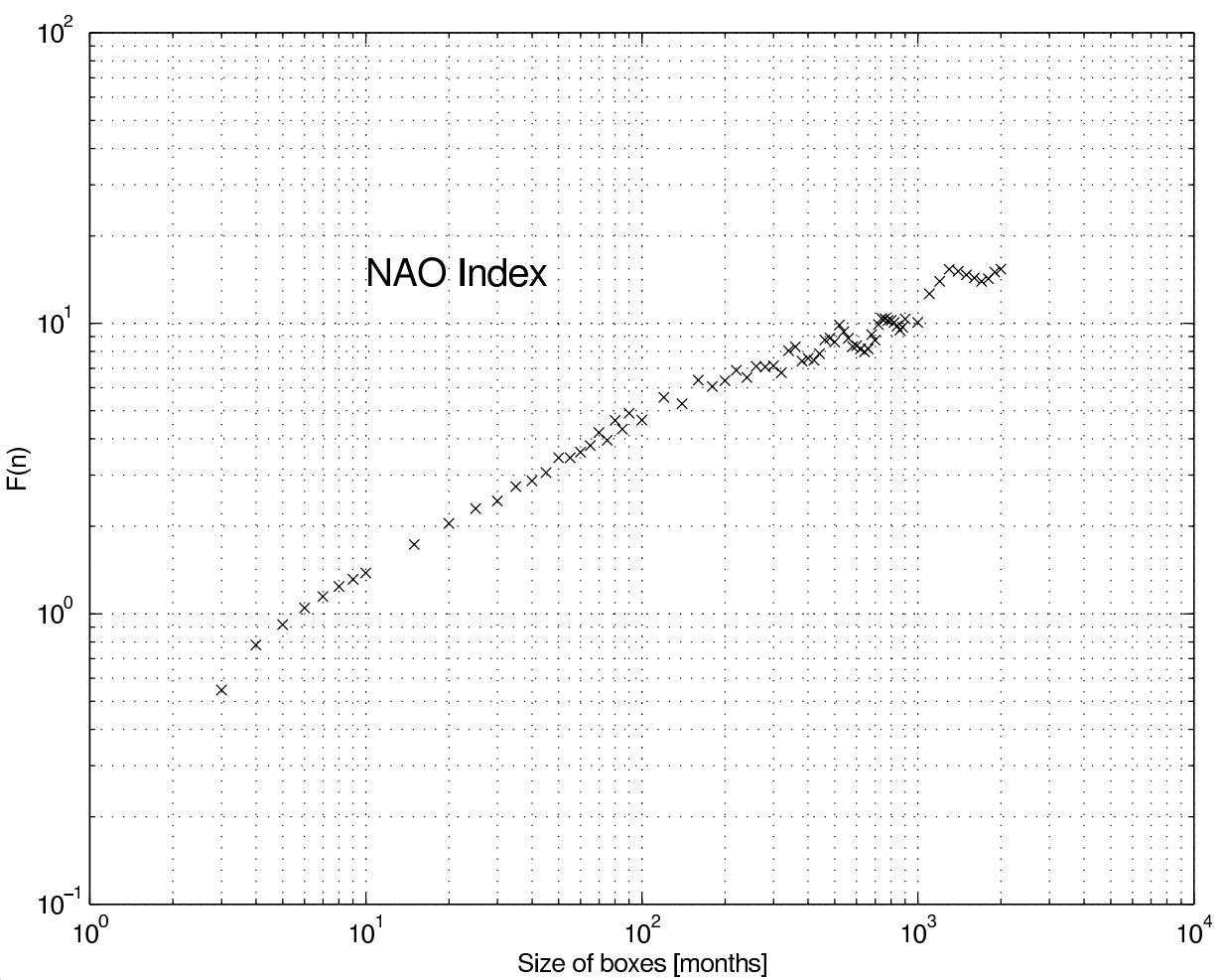
Figure 6 Raw data (symbols), and theoretical (Gaussian) pdf (solid line) comparing the NAO fluctuation distribution functions for various time lags, $\Delta t = 16, 8, 4, 2, 1$ months (or for $\tau = 0,1,2,3,4$). The case $\Delta t = 400$ months is also "shown" for comparison

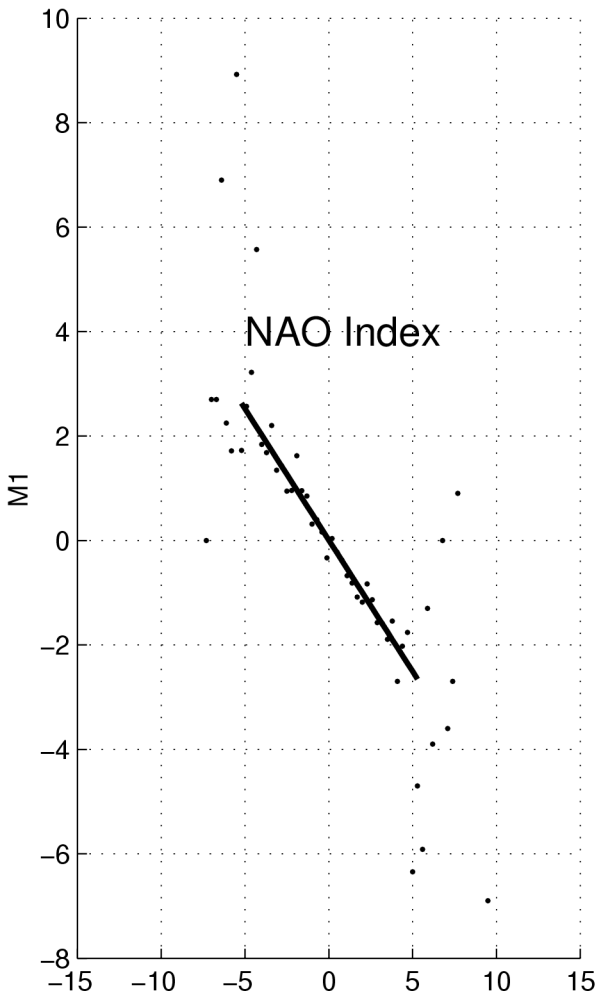
Monthly Averaged NAO Index Fluctuations



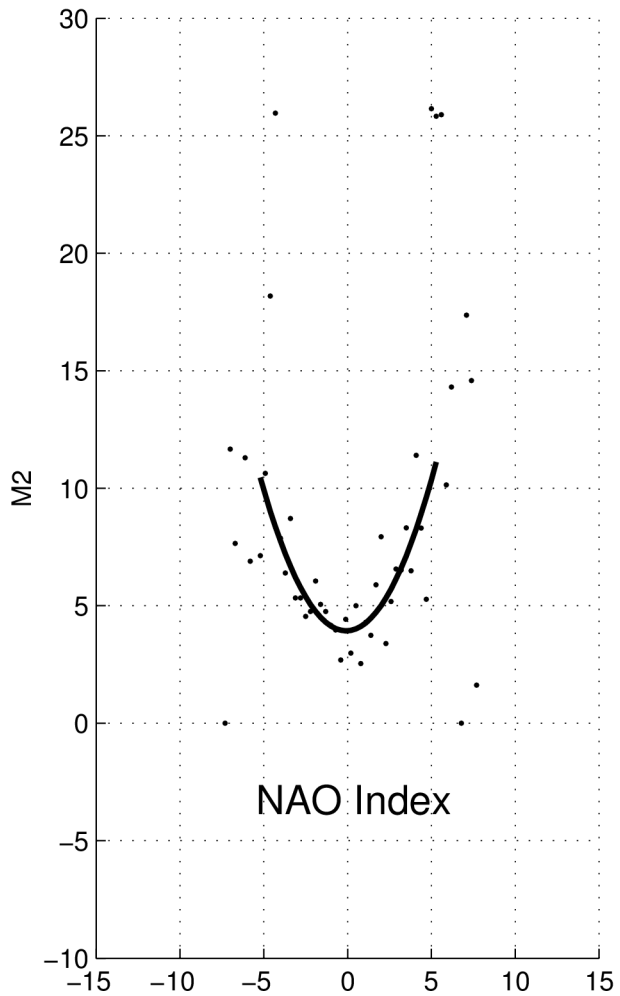








(a) Δx [NAO Units]



(b) Δx [NAO Units]

