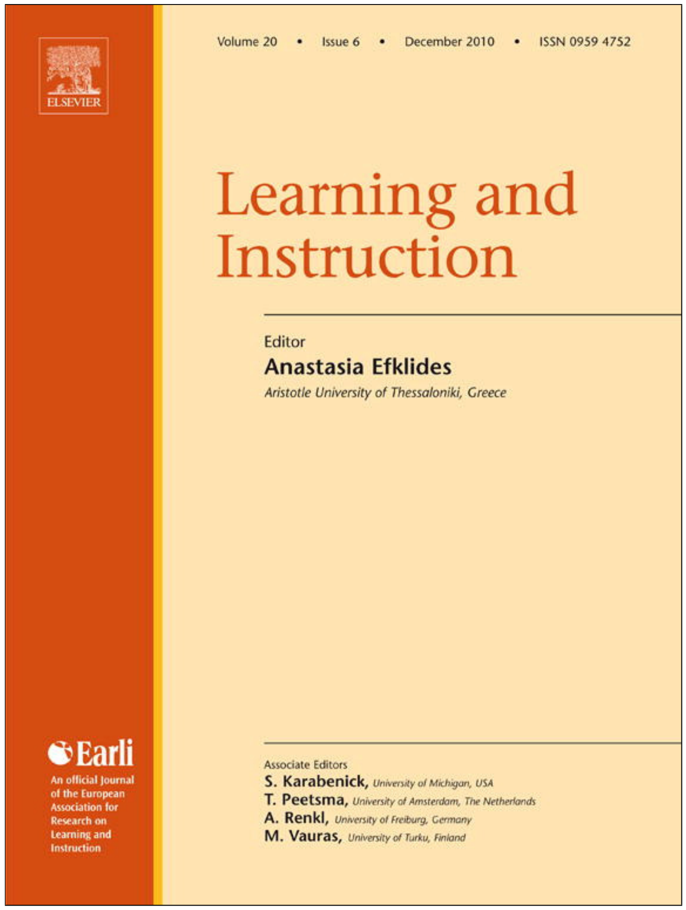
Provided for non-commercial research and education use.
Not for reproduction, distribution or commercial use.



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

http://www.elsevier.com/copyright

Author's personal copy

Learning and Instruction 20 (2010) 521–532

Learning and Instruction

www.elsevier.com/locate/learninstruc

Developmental changes in the comparison of decimal fractions

Laetitia Desmet [a,b,*], Jacques Grégoire [b], Christophe Mussolin [b]

[a] Centre for Research on the Teaching of Mathematics, 5, Rue E. Vandervelde, 1400 Nivelles, Belgium
[b] Department of Psychology, Catholic University of Louvain, 10, Place Cardinal Mercier, 1348 Louvain-la-Neuve, Belgium

Received 5 November 2008; revised 7 May 2009; accepted 26 July 2009

Abstract

Learning about decimal fractions is difficult because it requires an extension of the number concept built on natural numbers. The aim of the present study was to investigate developmental changes in children's misconceptions in decimal fraction processing. A large sample of children from Grades 3 to 6 performed a numerical comparison task on different categories of pairs of decimal fractions. The success rate and the type of error they made varied with age and categories. We distinguished the impact of the value of the digits from the impact of the length of the fractional part on children's pattern of responses. Although both kinds of impact affected the success rate, the digit values had a stronger impact and were mastered later than the length. Our results also showed that a zero just after the decimal point was understood better and earlier than a zero at the end of the fractional part of a number. Cluster analysis was conducted to determine groups of children who answered similarly regarding the response type across the various categories of decimal fractions. To interpret the data the conceptual change framework was used.
© 2009 Elsevier Ltd. All rights reserved.

Keywords: Decimal fractions; Comparison task; Conceptual change

1. Introduction

Learning is traditionally defined as the acquisition of new knowledge. According to the constructivist approach, children have naïve or prior conceptions and they engage them implicitly or intentionally in learning. Some of these prior conceptions are coherent with the new information and constitute a foundation for learning. However, other prior conceptions can impede further learning when the new information is incompatible with the existing knowledge. This leads to systematic errors or misconceptions. In this case, students have to totally reorganise their knowledge and to face a conceptual change.

The reorganisation of knowledge has been the focus of interest for some philosophers and historians of science (Kuhn, 1962; Lakatos, 1970). Kuhn (1962) argued that science is not a cumulative acquisition of knowledge. The revision of a scientific theory depends on conceptual oppositions, which are eventually resolved by a revolution. However, there is frequently a period of crisis, before the revolution takes place, during which some opposing ideas can coexist. Kuhn's (1962) model of the way change occurs in science has been used at an individual level to investigate how concepts evolve in the process of learning (Posner, Strike, Hewson, & Gertzog, 1982). The conceptual change approach was initially related to scientific learning, such as the development of physical or biological concepts (Hewson, 1981; McCloskey, 1983; Posner et al., 1982; Vosniadou, 1994; Vosniadou & Brewer, 1992). Vosniadou (1994, 2002) proposed a theoretical framework according to which: (a) learners have pre-conceptions which can constrain the learning; (b) the process of conceptual change is slow and gradual; and (c) intermediate levels of understanding can be observed. Some misconceptions, at this intermediate level, can be synthetic models (Vosniadou, 1994), which result from learners' attempts to reconcile the new concepts with their pre-existing ideas instead of making a radical reorganisation. Several authors emphasized that prior conceptions are often resistant to teaching (Duit, Roth,

* Corresponding author. Tel.: +32 67 212527; fax: +32 67 220201.
E-mail address: laetitiad@crem.be (L. Desmet).

0959-4752/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.learninstruc.2009.07.004

Author's personal copy

Komorek, & Wilbers, 2001; Mason, 2001; Wiser & Amin, 2001).

The conceptual change model has been applied to other domains than science, such as psychology (Wellman, 2002), general cognitive development (Carey, 1985), and history (Limón, 2002). It has also recently been applied to the learning of mathematics (Vamvakoussi, 2007; Vosniadou & Verschaffel, 2004). In this domain, earlier studies have pointed out some similarities to the conceptual change approach. Bachelard (1938) and Brousseau (1998) suggested the existence of ''epistemological obstacles'' in the learning of mathematics. Fischbein (1987) argued that initial mathematical conceptions do not always support further learning. This fact is now widely recognised (Resnick, 2006). Nevertheless, there are some differences between science and mathematics. Although the extension of the conceptual change model to mathematics learning requires some care, it is commonly agreed that mathematics learning and teaching can gain from the conceptual change approach (Greer, 2004; Greer & Verschaffel, 2007; Vamvakoussi, 2007; Vamvakoussi & Vosniadou, 2004).

In the present study, the theory of conceptual change was applied to a specific domain of mathematics, namely the acquisition of decimal fractions[1]. While several authors (Dole & Sinatra, 1998; Limón & Mason, 2002; Mason, Gava, & Boldrin, 2008; Pintrich, Marx, & Boyle, 1993; Sinatra & Pintrich, 2003) have shown that the affective and motivational characteristics of the learner, as well as various social, instructional and contextual factors, play a role in the adoption of conceptual change, our research was only focused on cognitive aspects of conceptual change (see Vosniadou, 2007 for a discussion).

1.1. Analysis of errors

Anyone who has ever been interested in mathematical teaching or learning knows that acquiring rational numbers is particularly difficult for children. Several studies have indicated that the extension of the number concept to rational or real numbers requires a conceptual change (Lehtinen, Merenluoto, & Kasanen, 1997; Merenluoto & Lehtinen, 2002; Merenluoto & Palonen, 2007; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). Children have to integrate new concepts about these numbers with their prior knowledge based on natural numbers.

The analysis of errors enables us to trace developmental changes in the acquisition of decimal fractions. Previous studies have investigated misconceptions about these numbers using a variety of methodologies (Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985). Sackur-Grisvard and Leonard (1985) presented an ordering task of three decimal fractions to children from Grades 4 to 7. They showed that many children used three implicit and incorrect rules. Rule 1 is to select as the smaller number the one whose fractional part is the smaller whole number (e.g., $12.4 < 12.17$ because $4 < 17$). Rule 2 is to select as the smaller number the one which has more digits in the decimal portion (e.g., 12.94 and $12.24 < 12.7$, because numbers with a two-digit fractional parts are seen as smaller than numbers with one-digit fractional parts). Rule 3 is to select as the smaller number the one whose fractional part begins with zero (which leads to a correct answer), and to apply Rule 1 in other cases (e.g., $3.09 < 3.8$). These authors used the term *rule*. For them, children' use of rule reflects the underlying concepts that guided the construction and selection of the rules.

Sackur-Grisvard and Leonard (1985) also showed that, although the frequency of use of these rules decreases with age, older children still use them for more difficult tasks (i.e., for ordering five decimal fractions). Nesher and Peled (1986) used a comparison task of two decimal fractions in a written questionnaire. Children from Grade 7 to Grade 9 were allowed to answer that the numbers were equal. The percentage of children using Rule 1 or 3 decreased between Grades 7 and 9, while the percentage of children using Rule 2 increased in Grade 8 and then decreased in Grade 9. These authors also suggested that the task of comparing different numbers is not the same as the task of comparing equal ones. All the participants succeeded in identifying the equal numbers.

Resnick et al. (1989) asked participants to compare two decimal fractions and to select the larger one. They established the conceptual sources of the errors, or rules, described by Sackur-Grisvard and Leonard (1985). For them, as for Nesher and Peled (1986), Rule 1 corresponds to an immature conception of decimals, based on a knowledge of natural numbers, whereas Rule 2 stems from the learning of fractions, and can be interpreted as an over-generalisation of a more elaborated idea that a tenth is always bigger than a hundredth.

The results of these descriptive studies are interesting in terms of the pattern of errors exhibited by children. However, further investigation with a more rigorous methodology is needed to investigate the conceptual changes between natural numbers and decimal fractions. First, it is necessary to distinguish errors due to the misconception that the value of the numbers increases with the length of the fractional part (impact of length), from those based on the application of a rule suggesting that the larger number is the one whose fractional part contains the larger digit (impact of the digit values). Rule 1 does not allow us to make this distinction. For example, when children have to choose the bigger of two decimal fractions, 0.03 vs. 0.004, and incorrectly select 0.004, the error could be due either to a misconception based on the fact that 0.004 is longer than 0.03, or on the fact that the digit ''4'' is bigger than the digit ''3''. Previous studies have not been able to differentiate between these two possibilities because they only used numbers varying along two dimensions.

Second, in some previous studies, children were asked to compare decimal fractions by choosing the larger one, or to order decimal fractions from the smallest to the largest. This way of proceeding is problematic because it neglects the possibility that the decimal fractions could be equal.

[1] In the entire article, we consider only positive decimal fractions in the place-value system, e.g., 0.2.

Author's personal copy

Third, some of the rules described above are based on the role of zero. However, it is important to distinguish between a zero placed on the left of a digit in the fractional part of a number (e.g., 0.01) and a zero placed on the extreme right of the fractional part (e.g., 0.10). Finally, it is interesting to analyse children's initial conceptions of decimal fractions before they study them. To do this we would have to study younger participants than previous studies. Decimal fractions are traditionally taught at the end of Grade 4 in Belgium, so we decided to include Grade 3 and Grade 4 children in our study so as to observe these initial conceptions.

1.2. The present study

The aim of the present study was to analyse children's age-related misconceptions about decimal fractions, bearing in mind the four requirements pinpointed above. Children from Grades 3 to 6 performed a comparison task on pairs of decimal fractions. They were asked to select the larger number, or to indicate that the two numbers were equal. The present study enabled the impact of the length of the fractional part of a number to be distinguished from the impact of the value of the digits in the fractional part. Furthermore, the role of the zero was examined, independently of the impact of the values of the other digits.

1.2.1. Hypotheses

It was hypothesised that both length and digit values would have an impact on the comparison of decimal fractions, reflecting misconceptions based on natural numbers. Children were expected to find it difficult to identify both the shorter number of a pair, and the number with the smaller digit value, as the larger (Hypothesis 1). It was also predicted that the digit values would influence performance more strongly than the length difference, since digit values had been learned early and explicitly, while number length is usually implicit knowledge, which is acquired later in childhood (Hypothesis 2). It was also hypothesised that a correct understanding of the role of the zero would only be exhibited by older children (Hypothesis 3), because it implies an elaborate knowledge of place values. Finally, it was expected misconceptions to be observed in the sense of being synthetic models (Hypothesis 4); these would correspond to intermediate levels of understanding between the prior conceptions in Grade 3 and the correct understanding in Grade 6.

2. Method

2.1. Participants

Participants were 156 children of Grade 3 (78 girls and 78 boys; mean age 101.47 months, $SD = 5.28$), 128 of Grade 4 (65 girls and 63 boys; mean age 113.66 months, $SD = 6.73$), 149 of Grade 5 (74 girls and 75 boys; mean age 125.25 months, $SD = 5.38$) and 128 of Grade 6 (62 girls and 66 boys; mean age 137.41 months, $SD = 5.85$). The children came from randomly selected schools of the French-speaking community of Belgium.

2.2. Task and stimuli

A paper-and-pencil comparison task was constructed with eighty pairs of decimal fractions. The pairs of decimals fell into eight different categories (10 pairs in each category). In each pair, the two numbers both had a zero in the unit position, so they only differed in the fractional part.

In the category labelled Neutral Length, both numbers had only a single digit in the fractional part. They only varied in the value of the digits (e.g., 0.1 vs. 0.2), so that the decision as to which was larger was only based on digit values and was not influenced by the length. In the category labelled Neutral Digit Value1, the two numbers involved the same digit, but its position in the decimal differed. The digit was placed first (representing tenths) for the bigger number, but was preceded by a zero in the smaller number (so that it represented hundredths). For pairs like this (e.g., 0.1 vs. 0.01), the numbers only varied in the length of the fractional part, and the correct decision was always incongruent[2] with the length difference and could not be based on the digit values. The items in another category labelled Neutral Digit Value2 were similar to those in the first one, except that the digit in the fractional part represented tenths and thousandths (e.g., 0.1 vs. 0.001), meaning that the length difference was two digits.

In the other categories, the pairs of decimal fractions varied in both the digit value and the length of the fractional part. In the category labelled Congruent Length and Digit Value the pairs contained one number with one and one number with two digits in the fractional part (e.g., 0.1 vs. 0.20). The number with two digits in the fractional part always had a digit in the tenths position which was bigger than the digit in the tenths position of the shorter number, followed by a zero in the hundredths position. So, the larger number was also the longer and the one containing the bigger digit, meaning that the correct decision was congruent with both the digit values and the length difference.

In three other categories, the larger number of the pair always had one digit in the fractional part while the other had two digits, so the correct decision was always incongruent with the length of the fractional part. The arrangement of digit values varied between these categories. In the category labelled Incongruent Length and Congruent Digit Value the correct decision was congruent with the digit values, namely the larger number had the larger digit in the tenths position, and the smaller number was constructed so that a smaller digit occurred in the hundredths position, preceded by a zero (e.g., 0.2 vs. 0.01). In the category labelled Incongruent Length and Digit Value the correct decision was incongruent with digit values, namely the pairs were of the same kind as the previous category, but this time the bigger digit was placed in the smaller number (e.g., 0.1 vs. 0.02). In the category labelled Ending in a Zero, the effect of the digit values depended on the

[2] A dimension was considered as congruent if its processing led to the correct answer, while the reverse is true for the incongruent dimension (Besner & Coltheart, 1979).

Author's personal copy

strategy used by the participants: the number with two digits in the fractional part was the smaller, and this had a zero in the hundredths position and a digit in the tenths position which was smaller than the digit in the tenths position of the shorter number (e.g., 0.2 vs. 0.10). So, the correct decision was congruent with the digit values if the comparison was based only on the digits in the tenths position, but was incongruent if the comparison was based on the whole of the fractional part (with an incorrect role attributed to the zero).

Finally, in the category labelled Ending in a Zero and Equal, the two numbers were equal and had the same digit in the tenths position, but varied in length because one of them ended with a zero (e.g., 0.3 vs. 0.30). Here, the correct decision was incongruent with the length difference and could only be based on correct knowledge of the position value. Moreover, if the children treated the fractional part as a natural number, this would lead to an incorrect response.

2.3. Procedure

The 80 pairs of decimal fractions were randomized and presented in the same order to all the participants. The position of the correct response was counterbalanced across the pairs. The child was asked to "circle the larger number or to place an equals sign between them if the numbers are the same in value." The task was administrated collectively to the children in their classrooms during the autumn term. Each child worked individually, in silence but without any time constraint. They all took less than twenty minutes.

3. Results

3.1. Overall analysis

For each participant, the mean rate of correct responses in each category of decimal fraction pairs was calculated. Boys' and girls' performance did not differ significantly, $F(1, 560) < 1$, *ns*. The mean accuracy for each category in each grade is given in Table 1. Cronbach's alpha shows that most of the categories had very good reliability and internal consistency, providing further support for analyses by category rather than by item. The low Cronbach's alpha for the category Neutral Length is explained by its very low variance, nearly all participants succeeding on all items in this category.

Two-way repeated measures ANOVA was conducted on the correct response rates with the Category (1–8) as the within subjects factor and the Grade (3–6) as the between subjects factor. The main effect of Grade was significant, $F(3, 557) = 787.14$, $p < 0.001$, partial $\eta^2 = 0.81$, showing that the correct response rates increased with grade. Post hoc *t*-tests, adjusted with the Bonferroni correction at alpha level 0.05/6 = 0.008, showed that all grades differed significantly from each other ($p < 0.05$ for comparison between Grades 3 and 4; $p < 0.001$ for the other comparisons). The highest improvement was observed between Grades 4 and 5. The main effect of category was also significant, $F(3.67, 2045.40) = 975.88$, $p < 0.001$, partial $\eta^2 = 0.64$.[3] There were lower success rates for the categories Ending in a Zero, Ending in a Zero and Equal, Incongruent Length and Digit Value, Neutral Digit Value1, and Neutral Digit Value2 as compared to the categories Congruent Length and Digit Value, Incongruent Length and Congruent Digit Value, and Neutral Length. Post hoc *t*-tests revealed significant differences between the category Ending in a Zero and the categories Neutral Digit Value1 and Neutral Digit Value2 ($p < 0.01$). Also, the category Ending in a Zero and Equal differed from the category Neutral Digit Value2 ($p < 0.01$). The success rate in the category Congruent Length and Digit Value was significantly higher than in the category Incongruent Length and Congruent Digit Value ($p < 0.001$); the category Neutral Length had the highest success rate and differed significantly from all other categories (in all cases $p < 0.001$).

The interaction between category and grade was also significant, $F(11.02, 2045.40) = 247.12$, $p < 0.001$, partial $\eta^2 = 0.57$. To understand this interaction, two-way repeated measures ANOVAs, with Category as the within subjects factor and Grade as the between subjects factor were conducted, separately for groups of categories that addressed the same question, namely the impact of the length, of the digit value, and of zero. For each ANOVA, the data were the mean rate of correct responses or types of responses, that is, responses including errors. Here after, we focused on double or triple interactions including grade. Main effects or other interactions are reported in Tables 2a, 2b, and 2c.

3.2. The impact of the length of the fractional part

To investigate the impact of the length of a decimal fraction, we compared three categories for which the digit values were always congruent with the correct decision. Variation on performance could thus be due to the length difference which was neutral in category Neutral Length (e.g., 0.1 vs. 0.2), congruent in the category Congruent Length and Digit Value (e.g., 0.1 vs. 0.20), and incongruent in the category Incongruent Length and Congruent Digit Value (e.g., 0.2 vs. 0.01). The interaction between grade and category was significant, $F(3.96, 735.15) = 26.23$, $p < 0.001$, partial $\eta^2 = 0.12$, as illustrated in Fig. 1.

In Grade 3, scores on the categories Neutral Length and Congruent Length and Digit Value did not differ from each other ($p > 0.05$), and were significantly higher than on the category Incongruent Length and Congruent Digit Value (in all cases $p < 0.001$). The pattern was similar in Grade 4; there was no difference between the categories Neutral Length and Congruent Length and Digit Value ($p > 0.05$), and the correct response rates in these two categories were higher than in the category Incongruent Length and Congruent Digit Value (in all cases $p < 0.001$). In Grade 5, the correct response rate was

[3] Mauchly's test indicated that the assumption of sphericity had been violated. Therefore degrees of freedom were corrected using Greenhouse-Geisser estimates.

Author's personal copy

Table 1
Mean rates of correct responses (in percent) in each category by grade.

Category	Example of pair	Cronbach's α	Grade 3	Grade 4	Grade 5	Grade 6	Mean rates
Neutral Length	0.1 vs. 0.2	0.10	99.0	98.6	99.1	99.3	99.0
Congruent Length and Digit Value	0.1 vs. 0.20	0.86	97.8	96.8	93.3	98.5	96.6
Neutral Digit Value1	0.1 vs. 0.01	0.96	5.7	14.9	88.5	98.5	51.9
Neutral Digit Value2	0.1 vs. 0.001	0.98	5.3	15.9	89.5	98.6	52.3
Incongruent Length and Congruent Digit Value	0.2 vs. 0.01	0.94	79.3	78.8	97.7	99.6	88.9
Incongruent Length and Digit Value	0.1 vs. 0.02	0.99	4.1	15.4	87.3	97.3	51.0
Ending in a Zero	0.2 vs. 0.10	0.98	6.3	8.8	79.8	97.5	48.1
Ending in a Zero and Equal	0.3 vs. 0.30	0.98	6.1	8.8	79.7	98.6	48.3
		Means	38.0	42.3	89.3	98.5	

marginally lower in the category Congruent Length and Digit Value than in the category Incongruent Length and Congruent Digit Value ($p = 0.06$); however, correct response rate was significantly lower in the category Congruent Length and Digit Value than in the category Neutral Length ($p < 0.001$). Finally, the categories Neutral Length and Incongruent Length and Congruent Digit Value did not differ significantly from each other ($p > 0.05$). In Grade 6, there was no significant difference between all the categories ($p > 0.05$).

The length difference had a weak but significant impact on children's accuracy even when the digit values were congruent with the correct decision. Third- and fourth-grade children tended to think the longer number was also the larger in magnitude, as expected in Hypothesis 1. Despite an increase in accuracy, fifth-grade children were also influenced by the length, but a reverse effect than the one expected in Hypothesis 1 was observed. They tended to think that the shorter number was the bigger. Finally, sixth-grade children were not influenced by the length of the decimal.

When comparing the categories Neutral Digit Value1 and Neutral Digit Value2 (e.g., 0.1 vs. 0.01 and 0.1 vs. 0.001) it was possible to analyse the impact of the size of the length difference (i.e., one or two digits), independent of the digit values. The Grade × Category interaction was not significant, $F(3, 557) = 1.04$, $p > 0.05$, which means that there was a similar pattern of responses across grades. Two kinds of errors were found for the categories Neutral Digit Value1 and Neutral Digit Value2. Specifically, the incorrect responses corresponded either to "the choice of the longer" number as the bigger (e.g., the selection of 0.01 instead of 0.1), or to "faulty equality" (e.g., 0.1 = 0.01). A 4(grade) × 2(category) × 3(response type: correct, choice of the longer, or faulty equality) repeated measures ANOVA showed that there was a Response × Grade interaction, $F(5.25, 975.27) = 202.11$, $p < 0.001$, partial $\eta^2 = 0.52$. As reported in Table 3, separate analyses within each grade showed that the main effect of response type was significant in all grades (in all cases $p < 0.001$), but the pattern differed across grades.

Specifically, in Grade 3, faulty equality was more frequent than the choice of the longer number which, in turn, was more frequent than the correct answer. The three types of responses differed significantly ($p < 0.001$) from each other. In Grade 4, a similar pattern was observed, but the difference between faulty equality and the choice of the longer number was not significant ($p > 0.05$), although the difference between the correct response and the other two types of responses was significant ($p < 0.001$). In Grade 5, correct responses were significantly more frequent ($p < 0.001$) than faulty equality and the choice of the longer number, whereas the latter two incorrect responses did not differ from each other ($p > 0.05$). The same pattern was observed in Grade 6.

The frequencies of the erroneous choice of the longer decimal when the digit values were congruent or neutral give more evidence to the impact of the length (Hypothesis 1), even in this case "faulty equality" error was much more frequent. Furthermore, it appears that a length difference of one digit

Table 2a
Statistics for the main effects and interactions (without Grade) corresponding to the impact of length.

Effect/interaction	*df*	*F*	*p*	partial η^2	Post hoc *t*-tests
Categories Neutral Length, Congruent Length and Digit Value, and Incongruent Length and Congruent Digit Value (CR)					
Category	1.32, 735.15	62.66	<.001	0.10	Category Incongruent Length and Congruent Digit Value < Category Congruent Length and Digit Value < Category Neutral Length
Grade	3, 557	22.01	<0.001	0.10	Grade 3 = Grade 4 < Grade 5 = Grade 6
Categories Neutral Digit Value1 and Neutral Digit Value2 (CR)					
Category	1, 557	1.51	*ns*	<0.01	–
Grade	3, 557	616.77	<0.001	0.77	Grade 3 < Grade 4 < Grade 5 < Grade 6
Categories Neutral Digit Value1 and Neutral Digit Value2 (types of errors)					
Response	1.75, 975.27	132.23	<0.001	0.19	CR (52.1%) > FE (29.7%) > CL (18.1%) (all *)
Response × Category	1.60, 890.69	4.31	<0.05	0.01	Difference FE − CL was larger in Category Neutral Digit Value1 (30%; 18%) than Neutral Digit Value2 (29%; 19%)

CR = Correct response; FE = Faulty equality; CL = Choice of the longer number; CS = Choice of the shorter number. *$p < 0.001$.

Author's personal copy

Table 2b
Statistics for the main effects and interactions (without Grade) corresponding to the impact of digit values.

Effect/interaction	*df*	*F*	*p*	partial η^2	Post hoc *t*-tests
Categories Neutral Digit Value1, Incongruent Length and Congruent Digit Value, and Incongruent Length and Digit Value (CR)					
Category	1.32, 735.06	761.16	<0.001	0.58	Category Neutral Digit Value1 = Category Incongruent Length and Digit Value < Category Incongruent Length and Congruent Digit Value
Grade	3, 557	518.56	<0.001	0.74	Grade 3 < Grade 4 < Grade 5 < Grade 6
Categories Neutral Digit Value1, Incongruent Length and Congruent Digit Value and Incongruent Length and Digit Value (types of errors)					
Response	1.51, 843.35	1147.17	<0.001	0.67	CR (65%) > CL (24.7%) > FE (10.3%) (all *)
Grade	3, 557	1.75	*ns*	<0.01	–
Category		<1			
Response × Category	2.09, 1166.06	425.33	<0.001	0.43	CR (52%) > FE (30%) > CL (18%) in Category Neutral Digit Value1, while CR > CL > FE in Categories Incongruent Length and Congruent Digit Value (89%; 11%; 0%) and Incongruent Length and Digit Value (54%; 46%; 0%).

C = Coherent, CR = Correct response; FE = Faulty equality; CL = Choice of the longer number; CS = Choice of the shorter number. *$p < 0.001$.

was sufficient to have an impact, and that a larger length difference did not result in a greater effect, or lead to a different type of response.

3.3. The impact of the digit value

The impact of the digit value was analysed by comparing the categories Neutral Digit Value1 (e.g., 0.1 vs. 0.01), Incongruent Length and Congruent Digit Value (e.g., 0.2 vs. 0.01), and Incongruent Length and Digit Value (e.g., 0.1 vs. 0.02). The Category × Grade interaction was significant, $F(3.96, 735.10) = 188.11$, $p < 0.001$, partial $\eta^2 = 0.50$, as illustrated in Fig. 2. Separate analyses were conducted within each grade to understand this interaction.

Specifically, in Grade 3, the main effect of category was significant, $F(1.23, 189.91) = 663.30$, $p < 0.001$, partial $\eta^2 = 0.81$, indicating that the category Incongruent Length and Congruent Digit Value yielded more correct responses than either the Neutral Digit Value1 or Incongruent Length and Digit Value (in all cases $p < 0.001$); the latter two categories did not differ between them ($p > 0.05$). Although the correct response rates were increased in the other grades, the same pattern of responses was observed in Grade 4, $F(1.15, 146.35) = 298.24$, $p < 0.001$, partial $\eta^2 = 0.70$, and Grade 5, $F(1.63, 240.81) = 17.04$, $p < 0.001$, partial $\eta^2 = 0.10$. In Grade 6, the main effect of category was also significant, $F(1.78, 226.55) = 3.38$, $p < 0.05$, partial $\eta^2 = 0.03$, but the effect size was low.

In brief, third- and fourth-grade children had similar patterns of response, despite a slight increase in accuracy with age, and were strongly affected by digit values. They were generally unable to succeed (below 20% of correct responses) when the digit values were incongruent with the correct decision, supporting Hypothesis 1. Fifth-grade children succeeded well in all the three categories, but digit values still had an impact on their decision. Sixth-grade children did not seem to be influenced by the digit values.

Although the categories Neutral Digit Value1 and Incongruent Length and Digit Value did not differ with respect to the mean rate of correct responses, further analyses were conducted on types of errors with Category (Neutral Digit Value1, Incongruent Length and Congruent Digit Value, and Incongruent Length and Digit Value) and Response (correct, choice of the longer number, and faulty equality) as within subjects factors and grade as between subjects factor. The Response × Grade interaction was significant, $F(4.54, 843.35) = 350.45$, $p < 0.001$, partial $\eta^2 = 0.65$, as well as the Response × Category × Grade interaction, $F(6.28, 1166.06) = 112.57$, $p < 0.001$, partial $\eta^2 = 0.38$. Separate analyses were conducted for each grade to examine this double interaction. As reported in Table 4, the main effect of type of response was significant for each category and in all grades (in all cases $p < 0.001$). However, the pattern of responses differed across categories and grades.

Regarding the type of response, the Response × Category interaction was significant in Grade 3, $F(1.63, 251.99) = 458.34$, $p < 0.001$, partial $\eta^2 = 0.75$, in Grade 4, $F(1.86, 235.76) = 131.30$, $p < 0.001$, partial $\eta^2 = 0.51$, and in Grade 5, $F(1.76, 260.88) = 10.12$, $p < 0.001$, partial $\eta^2 = 0.06$, but not in Grade 6, $F(1.49, 188.95) = 1.28$, $p > 0.05$, partial $\eta^2 = 0.01$.

Further support to the impact of digit values (Hypothesis 1) was given by the fact that, when the digit values were neutral,

Table 2c
Statistics for the main effects and interactions (without Grade) corresponding to the impact of zero.

Effect	*df*	*F*	*p*	partial η^2	Post hoc *t*-tests
Category Ending in a Zero (types of errors)					
Response	1.00, 558.37	727.75	<0.001	0.57	CR, here CS (51.1%) = CL (48.8%) > FE (0.1%) (both *)
Grade	3, 557	1.74	ns	<0.01	–
Category Ending in a Zero and Equal (types of errors)					
Response	1.30, 726.31	715.99	<0.001	0.56	CR, here equal (48.3%) = CL (49.3%) > CS (0.2%) (both *)
Grade	3, 557	2.13	ns	0.01	–

CR = Correct response; FE = Faulty equality; CL = Choice of the longer number; CS = Choice of the shorter number. *$p < 0.001$.

Author's personal copy

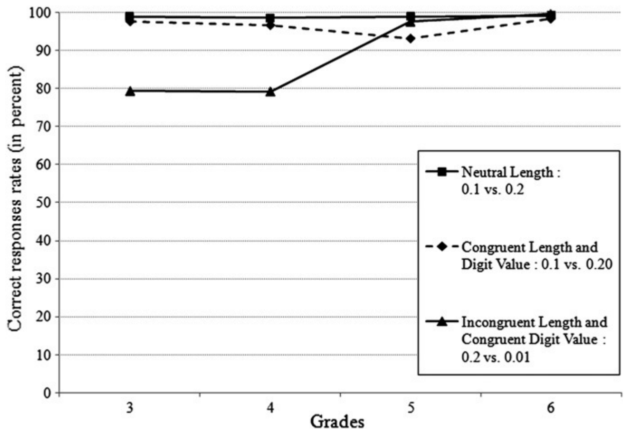


Fig. 1. Mean rates (in percent) of correct response by grade in categories Neutral Length, Congruent Length and Digit Value, and Incongruent Length and Congruent Digit Value.

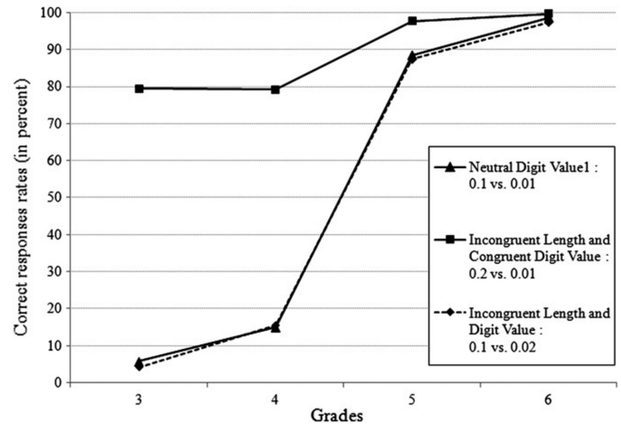


Fig. 2. Mean rates (in percent) of correct response by grade in categories Neutral Digit Value1, Incongruent Length and Congruent Digit Value, and Incongruent Length and Digit Value.

third- and fourth-grade children tended to think that the numbers were equal. When the digit values were incongruent, they committed "choice of the longer" errors (e.g., the selection of 0.02 instead of 0.1), indicating that their decision were based on the digit magnitude, irrespective of its place value. Fifth-grade children succeeded well in all the three categories, but still committed errors in the incongruent category.

It was also predicted (Hypothesis 2) that the impact of the digit values on performance would be stronger than the impact of the length. To test this hypothesis, the correct response rate for the incongruent condition was subtracted from the congruent ones for each factor separately. The impact of digit values was computed with the difference between Incongruent Length and Congruent Digit Value category and Incongruent Length and Digit Value category. The impact of the length was obtained by the difference between Congruent Length and Digit Value category and Incongruent Length and Congruent Digit Value. For each grade, paired sample *t*-tests showed that impact of digit value was stronger than impact of the length ($p < 0.001$ for Grades 3, 4 and 5; $p < 0.01$ for Grade 6), supporting Hypothesis 2.

3.4. The impact of zero

Some categories with a zero in the tenths position have been already described as part of the investigation of the impact of digit values (categories Neutral Digit Value1, Incongruent Length and Congruent Digit Value, and Incongruent Length and Digit Value). In what follows, the categories Incongruent Length and Congruent Digit Value, and Incongruent Length and Digit Value were not taken into account, because the impact of the zero cannot be distinguished from the impact of the digit values. Instead the focus

Table 3
Mean rates (in percent) and *F* values of the type of response in the two categories with neutral digit value by grade.

	df	*F*	CR	CL	FE
Category Neutral Digit Value1 (0.1 vs. 0.01)					
Grade 3	1.28, 198.56	74.79	5.7_a	31.0_b	63.1_c
Grade 4	1.70, 216.28	17.04	14.9_a	34.6_b	50.3_b
Grade 5	1.43, 212.07	413.47	88.5_a	3.8_b	7.7_b
Grade 6	1.09, 138.78	5391.01	98.5_a	1.0_b	0.4_b
Category Neutral Digit Value2 (0.1 vs. 0.001)					
Grade 3	1.25, 193.55	63.02	5.3_a	33.3_b	61.4_c
Grade 4	1.74, 221.26	12.95	15.9_a	36.3_b	47.8_b
Grade 5	1.44, 214.18	461.11	89.5_a	3.7_b	6.9_b
Grade 6	1.01, 128.29	3518.33	98.6_a	1.3_b	0.1_b

CR = Correct response; CL = Choice of the longer number; FE = Faulty equality. For each category, the frequencies in the same row that do not share subscripts differ at $p < 0.001$ regarding the post hoc tests.

Table 4
Mean rates (in percent) and *F* values of the type of response in the categories Neutral Digit Value1, Incongruent Length and Congruent Digit Value, and Incongruent Length and Digit Value by grade.

	df	*F*	CR	CL	FE
Category Neutral Digit Value1 (0.1 vs. 0.01)					
Grade 3	1.28, 198.56	74.79	5.7_a	31.0_b	63.1_c
Grade 4	1.70, 216.28	17.04	14.9_a	34.6_b	50.3_b
Grade 5	1.43, 212.07	413.47	88.5_a	3.8_b	7.7_b
Grade 6	1.09, 138.78	5391.01	98.5_a	1.0_b	0.4_b
Category Incongruent Length and Congruent Digit Value (0.2 vs. 0.01)					
Grade 3	1.02, 157.58	259.74	79.3_a	20.1_b	0.6_c
Grade 4	1.00, 127.15	194.52	79.1_a	20.7_b	0.1_c
Grade 5	1.00, 148.84	2591.82	87.5_a	12.3_b	0.1_b
Grade 6	1.00, 127.00	49565.66	98.0_a	2.0_b	0.0_b
Category Incongruent Length and Digit Value (0.1 vs. 0.02)					
Grade 3	1.04, 161.49	1423.56	4.1_a	95.3_b	0.5_a
Grade 4	1.01, 127.61	102.87	26.8_a	72.5_b	0.3_c
Grade 5	1.01, 148.66	372.31	87.5_a	12.3_b	0.1_c
Grade 6	1.00, 127.00	2916.21	98.0_a	2.0_b	0.0_b

CR = Correct response; CL = Choice of the longer number; FE = Faulty equality. For each Category, the frequencies in the same row that do not share subscripts differ at $p < 0.001$ regarding the post hoc tests.

Author's personal copy

was on the categories Neutral Digit Value1 (e.g., 0.1 vs. 0.01), Ending in a Zero (e.g., 0.2 vs. 0.10) and Ending in a Zero and Equal (e.g., 0.3 vs. 0.30). Differences between categories were expected depending on the position of the zero within the fractional part. In the category Neutral Digit Value1, children who did not properly understand the concept of place value as it relates to the zero would tend to consider the two decimals as equal, because the digit values were identical. Conversely, the decimals in the category Ending in a Zero and Equal would be seen as unequal if the child erroneously concluded that the zero at the end of the number modified the overall magnitude of the fractional part. In the same way, the participants could attribute an erroneous role to the zero in the category Ending in a Zero, leading to an incorrect choice of the larger number, even when the digit values are congruent with the correct decision.

In the category Neutral Digit Value1, third- and fourth-grade children frequently committed errors suggesting that they considered either that the zero had no effect (faulty equality) or that it contributed to the length of the decimal (choice of the longer number). In both cases, the digits were processed without taking into account the place-value of the zero. However, from Grade 5 onwards, children understood the use of the place-holder zero on the left (see Table 4).

In the categories Ending in a Zero and Ending in a Zero and Equal, there were three possible types of responses. The first was choice of the longer number, which corresponded to an error in both the above categories. The second corresponded to the choice of the shorter number of the pair, which gave the correct response in the category Ending in a Zero, but an error in the category Ending in a Zero and Equal. The last possible response was the ‘‘equal’’ answer, which gave the correct response in the category Ending in a Zero and Equal, but a faulty equality in the category Ending in a Zero. Two-way repeated measures ANOVA was conducted on the frequencies of these responses within each category, with Response (choice of the longer number, choice of the shorter number, and faulty equality) as the within subjects factor and Grade (3–6) as the between subjects factor.

In the category Ending in a Zero, the Response × Grade interaction was significant, $F(3.01, 558.37) = 468.28$, $p < 0.001$, partial $\eta^2 = 0.72$. As shown in Table 5 the types of responses differed between them in all grades (in all cases $p < 0.001$), but the choice of the longer number was prevalent in Grades 3 and 4, whereas the choice of the shorter number (correct response) in Grades 5 and 6 (in all cases $p < 0.001$). In the category Ending in a Zero and Equal, grade also interacted significantly with the type of response, $F(3.91, 726.31) = 560.35$, $p < 0.001$, partial $\eta^2 = 0.75$. The types of responses differed in all grades (in all cases $p < 0.001$).

In brief, third- and fourth-grade children erroneously considered the decimal fractions ending in a zero as being bigger. Fifth-grade children performed much better with these two categories, but not all of them were yet convinced that the zero on the right had no impact. Only sixth graders consistently understood that the zero ending the decimal part had no impact. These results support Hypothesis 3.

Table 5
Mean rates (in percent) and F values by grade of the type of response in the categories ending in a zero.

	df	*F*	CR	CL	CS
Category Ending in a Zero (0.2 vs. 0.10)					
Grade 3	1.00, 156.03	1582.79	6.3_a	93.5_b	0.1_c
Grade 4	1.00, 127.40	219.79	8.8_a	91.0_b	0.2_c
Grade 5	1.00, 148.00	271.17	79.8_a	20.2_b	0.0_c
Grade 6	1.01, 128.24	3331.89	97.5_a	2.4_b	0.1_b
Category Ending in a Zero and Equal (0.1 vs. 0.10)					
Grade 3	1.20, 185.83	1384.10	6.1_a	92.0_b	1.0_c
Grade 4	1.10, 140.12	565.77	8.8_a	89.3_b	1.1_c
Grade 5	1.33, 197.05	186.86	79.7_a	15.2_{b*}	4.6_{c*}
Grade 6	1.08, 137.54	6488.17	98.6_a	0.5_b	0.9_b

CR = Correct response; CL = Choice of the longer number; CS = Choice of the shorter number. For each category, the frequencies in the same row that do not share subscripts differ at $p < 0.001$ (*$p < 0.05$) regarding the post hoc tests.

3.5. Cluster analysis on the type of responses across categories

A two-step cluster analysis was conducted to determine groups of children who answered in a similar fashion. This two-step cluster analysis was conducted on the type of responses given to the eighty items (i.e., pairs of decimal fractions). For each category, there were three types of responses: the correct response, and two types of errors. Depending on the category, the possible errors were the following: (a) faulty equality; (b) choice of the longer number; (c) choice of the smaller number: (d) choice of the shorter number (see Table 6). In a first step, pre-clusters were identified. In a second step, the pre-clusters were grouped into the desired number of clusters by hierarchical clustering. A five-cluster solution emerged. The cluster analysis was repeated using random start points and yielded approximately the same solution, supporting the five-cluster solution.

The type of response was analyzed for each child. In order to know if the three types of responses appeared in equal proportions for the ten items of a category or if a type of responses was significantly more frequent than others, chi-square tests were used. It appeared that a type of response could be considered as dominant when it appeared at least seven times on the ten items, $\chi^2(2, N = 10) = 6.21$, $p < 0.05$. In all other cases, responses were considered as incoherent. Table 6 reports the frequencies of the response type for each category as a function of cluster membership.

Cluster 1 comprised children who based their decision on the digit values (the larger digit or the whole fractional part), regardless of the place values. In Cluster 2, children tended to be influenced by the length of the fractional part, frequently selecting the longer number; these children were more incoherent in the category Incongruent Length and Congruent Digit Value, indicating an interaction between the digit value and the length. Cluster 3 corresponded to children who gave correct responses in most cases but were slightly hesitant. Children of Cluster 4 were nearly expert. They responded correctly in all categories except for those ending in a zero (e.g., 0.2 vs. 0.10 and 0.3 vs. 0.30), where they misunderstood

Author's personal copy

Table 6
Frequencies of types of responses for each category as a function of cluster membership.

Category Neutral length (0.1 vs. 0.2)				
	CR	CM	FE	IN
Cluster 1 ($n = 168$)	168	0	0	0
Cluster 2 ($n = 85$)	85	0	0	0
Cluster 3 ($n = 31$)	31	0	0	0
Cluster 4 ($n = 43$)	43	0	0	0
Cluster 5 ($n = 231$)	231	0	0	0
Category Congruent Length and Digit Value (0.1 vs. 0.20)				
	CR	CS	FE	IN
Cluster 1 ($n = 168$)	168	0	0	0
Cluster 2 ($n = 85$)	85	0	0	0
Cluster 3 ($n = 31$)	18	8	0	5
Cluster 4 ($n = 43$)	41	0	0	2
Cluster 5 ($n = 231$)	230	0	0	1
Category Neutral Digit Value1 (0.1 vs. 0.01)				
	CR	CL	FE	IN
Cluster 1 ($n = 168$)	0	0	150	18
Cluster 2 ($n = 85$)	0	74	4	7
Cluster 3 ($n = 31$)	23	1	1	6
Cluster 4 ($n = 43$)	22	3	2	16
Cluster 5 ($n = 231$)	231	0	0	0
Category Incongruent Length and Congruent Digit Value (0.2 vs. 0.01)				
	CR	CL	FE	IN
Cluster 1 ($n = 168$)	162	3	0	3
Cluster 2 ($n = 85$)	31	38	0	16
Cluster 3 ($n = 31$)	30	0	0	1
Cluster 4 ($n = 43$)	38	1	0	4
Cluster 5 ($n = 231$)	231	0	0	0
Category Incongruent Length and Digit Value (0.1 vs. 0.02)				
	CR	CL	FE	IN
Cluster 1 ($n = 168$)	0	168	0	0
Cluster 2 ($n = 85$)	0	85	0	0
Cluster 3 ($n = 31$)	20	5	0	6
Cluster 4 ($n = 43$)	23	16	0	4
Cluster 5 ($n = 231$)	231	0	0	0
Category Ending in Zero (0.2 vs. 0.10)				
	CR	CL	FE	IN
Cluster 1 ($n = 168$)	4	158	0	6
Cluster 2 ($n = 85$)	1	84	0	0
Cluster 3 ($n = 31$)	21	6	0	4
Cluster 4 ($n = 43$)	0	40	0	3
Cluster 5 ($n = 231$)	230	0	0	1
Category Ending in Zero and Equal (0.3 vs. 0.30)				
	CR	CL	CS	IN
Cluster 1 ($n = 168$)	6	150	0	12
Cluster 2 ($n = 85$)	0	84	0	1
Cluster 3 ($n = 31$)	17	2	5	7
Cluster 4 ($n = 43$)	0	41	0	2
Cluster 5 ($n = 231$)	230	0	0	1

CR = Correct response, CM = Choice of the smaller, FE = Faulty equality, IN = Incoherent , CS = Choice of the shorter, CL = Choice of the longer.

the meaning of the zero. Finally, Cluster 5 comprised participants who gave correct responses in all categories.

The cluster membership distribution in each grade differed significantly from the expected distribution as shown by a chi-square analysis (see Table 7). In Grade 3, $\chi^2(4, N = 156) = 123.75$, $p < 0.01$, the great majority of children (64%) belonged to Cluster 1, while 27% of them belonged to Cluster 2. In Grade 3, there were very few children who belonged to more advanced clusters, namely 1% to Cluster 3, 8% to Cluster 4, and 0% to Cluster 5. In Grade 4, $\chi^2(4, N = 128) = 64.92$, $p < 0.01$, the majority of children (48%) also belonged to Cluster 1, while 16% of them belonged to Cluster 2. Some fourth-grade children were nearly expert (16% to Cluster 4), hesitant expert (3% to Cluster 3) or expert (3% to Cluster 5). In Grade 5, $\chi^2(4, N = 149) = 146.50$, $p < 0.01$, Cluster 1 and 2 had almost disappeared. Most of the children by far were expert (72% to Cluster 5) or have a lot of correct responses (13% to Cluster 3; 7% to Cluster 4). In Grade 6, $\chi^2(4, N = 128) = 297.08$, $p < 0.01$, almost all the children were expert (Cluster 5: 95%).

Table 7
Frequencies per grade of each cluster.

	Grade 3	Grade 4	Grade 5	Grade 6
Cluster 1 ($n = 168$)	64	48	5	0
Cluster 2 ($n = 85$)	27	30	3	0
Cluster 3 ($n = 31$)	1	3	13	5
Cluster 4 ($n = 43$)	8	16	7	0
Cluster 5 ($n = 231$)	0	3	72	95

A large majority of third- and fourth-grade children based their decision exclusively on the digit values and belonged to Cluster 1, while about a quarter of them belonged to Cluster 2 and tended to be influenced by the length of the fractional part, frequently selecting the longer number. These results provided further support to Hypotheses 1 and 2 related to the impact of both length and digit values on children's performance. The responses of nearly expert children (Cluster 4) and hesitant expert (Cluster 3) confirm Hypothesis 4, related to the existence of intermediate levels of understanding.

4. Discussion

The present study investigated performance of children from Grade 3 to 6 in comparing pairs of decimal fractions, and how they evolved during learning. To the best of our knowledge, the current study is the first attempt to disentangle the impact of digit values from the impact of the length of the fractional part. The results showed that both length and digit values had an impact on the comparison of decimal fractions (Hypothesis 1). The results provided also some pieces of evidence that the digit values influenced performance more strongly than the length difference in young children (Hypothesis 2). In addition, it appeared that a correct understanding of the role of the zero as a placeholder was only exhibited by older children (Hypothesis 3). Finally, intermediate levels of understanding between the prior conceptions about decimal fractions and the correct understanding were observed (Hypothesis 4).

Here after, the results relating to the initial conceptions of third- and fourth-grade children are discussed, and then the conceptions of fifth-grade and sixth-grade children. Results regarding the impact of the zero as a placeholder are discussed at the same time.

4.1. Initial conceptions about decimal fractions

Considering that the teaching of decimal fractions traditionally begins at the end of Grade 4 in Belgium, the analysis

Author's personal copy

of children's performance at the beginning of Grade 3 and 4 has allowed us to investigate the initial conceptions about decimal fractions. These initial conceptions were not investigated in the past despite the fact that they have theoretical implications regarding the conceptual change theory and practical implications for mathematics teaching.

The results of the present study showed that children without formal knowledge of decimal fractions were mainly influenced by knowledge they use to process natural numbers, leading to a low level of correct responses. They were strongly affected by digit values in the fractional part: they only succeeded when the digit values were congruent with the correct decision. The length difference had also a weak but significant impact on their accuracy: they tended to think the longer number was also the larger in magnitude. Third and fourth graders also attributed to the zero the same role as for natural numbers. They erroneously judged that a zero in the tenths position had no impact and that a zero at the end of the fractional part had an impact on the number magnitude.

Cluster analysis has provided further evidence of the impact of both length and digit values on children's performance. A large majority of third and fourth graders based their decision exclusively on the digit values and belonged to Cluster 1, while about a quarter of them belonged to Cluster 2 and tended to be influenced by the length of the fractional part, frequently selecting the longer number. These findings also tend to support that digit values have more impact than length in younger children (Hypothesis 2).

4.2. Evolution of the conceptions about decimal fractions

Grade 5 children succeeded well in all categories, but were still affected by the digit values and the length of the decimal fraction. Fifth-grade children also had some difficulty with the role of the zero. They usually answered correctly with pairs which had a zero in the tenth position, although the digit values still had some impact. However, they still tended to think that a number with a zero at the end of the fractional part was larger than one without it. In fact, these pairs were the most difficult for fifth graders. This suggests that a zero placed just after the decimal point is understood earlier than a zero at the end of the fractional part of the number. In brief, our results show that natural number conceptions still had an impact even for these older children who had more experience with decimal fractions.

However, they tended to choose the shorter number as the larger, the opposite pattern to younger children. It has previously been postulated that this effect is based on fractions knowledge (Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985), and can be interpreted as an overgeneralization such as "a tenth is always bigger than a hundredth". In line with this view, we might suggest that this pattern arises because fifth-grade children have more experience with fractions than younger children, especially with the conversion between fractions and decimals. Another possibility is that these results reflect an overgeneralization by children who have learned that the longer decimal is not always the bigger and that decimal fractions differ from the natural numbers.

Cluster analysis has shown that the great majority of fifth-grade children were expert (Cluster 5) or had a lot of correct responses but were still hesitant for all pairs (Cluster 3) or treated erroneously only the pairs ending in a zero (Cluster 4). The results observed in Grade 5 could be interpreted as an interaction between the knowledge about the natural numbers and the knowledge about rational numbers. When confronted with decimal fractions (expressed in the place-value system), children seem to apply both kinds of knowledge to the task.

By Grade 6, a ceiling effect was observed. Children showed only a weak facilitation effect for congruent digits, probably due to persistent conceptions based on natural numbers, and the length of the decimal had no effect on their performance. The progression between Grades 5 and 6 corresponded to an improvement in the most difficult categories, that is, sixth graders consistently understood that the zero ending the fractional part had no impact. This suggests that the understanding of decimal fractions becomes more precise at this stage of education. Furthermore, almost all the sixth-grade children were expert (Cluster 5) in this comparison task of decimal fractions.

4.3. Theoretical and practical implications

It has been proposed in the past that the selection of the number of which the fractional part is the smaller whole number as being the smaller number (e.g., $12.4 < 12.17$ because $4 < 17$) corresponds to an immature conception of decimal fractions, based on a knowledge of natural numbers (Rule 1 for Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985). On the one hand, the current study fit well with the idea that the misconceptions of young children derive from their familiarity with natural numbers. The choice of the longer decimal as the larger is most probably due to the fact that this strategy consistently yields the correct answer with natural numbers. In the same way, the impact of digit values is consistent with the children treating the fractional part as a natural number. On the other hand, whereas previous studies have not distinguished between the effect of digit values and of length, our findings demonstrate that both factors have an impact on children's conception about decimal fractions, before and during their formal learning. In addition, our cluster analysis showed that some children were exclusively influenced by digit value (Cluster 1) while others were more influenced by the length (Cluster 2).

With respect to fact that another misconception leads to the selection of the shorter number (Rule 2 for Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985), some children in Grade 5 tended to consider the shorter number as the larger number, resulting in a reverse effect of length relative to other children. It has been postulated that this rule is based on fractions knowledge, and can be interpreted as an overgeneralization such as "a tenth is always bigger than a hundredth". In line with this view, we might suggest that this pattern arises because fifth-grade children had more

Author's personal copy

experience with fractions than younger children. Nevertheless, this erroneous strategy did not appear to characterize a particular group of children as the cluster analysis showed. This does not fit well with the frequency of this misconception reported in previous studies (Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985). The apparent conflict between our results and earlier studies is probably due to differences between countries in the organization of the curriculum. In countries where fractions are taught before decimals, such as Israel (Nesher & Peled, 1986; Resnick et al., 1989) and the United States (Resnick et al., 1989), the frequency of this misconception was higher than in countries where decimal fractions expressed in the place-value system are taught before other fractions, such as France (Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985). In countries where the teaching of decimal fractions expressed in the place-value system usually occurs at least a year after the teaching of fractions, we may suppose that it acts upon children's conception of decimal fractions. However, it is important to note that studies reporting a high frequency of the selection of the shorter number as being the bigger (Nesher & Peled, 1986; Resnick et al., 1989) involved explicit comparisons between fractions and decimal fractions. This could force the participants to translate from the place-value system to fractions, leading to unusual errors.

The impact of the zero within the fractional part could be related to the conception that the smaller number is the one of which the fractional part begins with zero (Rule 3 for Nesher & Peled, 1986; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985). However, it appeared that this conception depends strongly on the digits used. Young children in Grades 3 and 4 were able to answer that the number which had a zero just after the decimal point was smaller when the digit value was congruent with the correct answer, but not when it was incongruent.

To conclude, the various conceptions described above need to be refined in the light of our results. It is necessary to control for and/or to manipulate the digit values and the length so that we are sure about the children's conceptions. To that end, it is useful to be exhaustive in the kind of decimal fraction pairs being compared.

Considering that conceptual change theory has postulated that (a) learners have pre-conceptions which can constrain their learning; (b) the process of conceptual change is slow and gradual; and (c) intermediate levels of understanding can be observed (Vosniadou, 1994, 2002), the results are in line with the application of this approach to mathematics education. The present study showed that initial conceptions of decimal fractions prior the formal teaching were based on both the digit values and the length of the fractional part. Cluster analysis showed that some young children have initial conceptions based exclusively on digit values (Cluster 1), while others are more influenced by the length (Cluster 2). It also appears that conceptions based on the length were less resistant to the teaching than those based on the digit values. However, both conceptions constrain the learning of decimal fractions. It was also observed that the learning of decimal fractions was slow and gradual and that intermediate levels of understanding exist. Before succeeding in the comparison of all pairs of decimal fractions (Cluster 5), some children have correct responses for some pairs, but not for the other more difficult pairs (Cluster 4), or are incoherent, sometimes succeeding in the comparison, sometimes not (Cluster 3). This could be related to synthetic models (Vosniadou, 1994), that is, to learners' attempts to reconcile the new concepts with their pre-existing ideas instead of making a radical reorganisation.

4.4. Conclusion

The present study has focused on decimal fractions expressed in the place-value system. From a theoretical point of view, in mathematics, the set of decimals is not of the greatest interest. Nevertheless, it is the more convenient system for calculating with rational numbers and it is commonly used in daily life, for example to handle money. The results of the present study suggest that to help children to understand that conceptions based on natural numbers cannot automatically be applied to decimal fractions various pairs of decimal fractions should be presented to them. In particular, children should work with decimal fractions that are congruent, incongruent and neutral with respect to digit values and length. They should also work with decimal fractions including a zero just after the decimal point or at the end of the fractional part.

More generally, the learning of decimal fractions represents a situation among others in which a mathematical entity, here the number system, is extended. Previous studies have applied the conceptual change theory to other mathematical reorganizations, like extensions of number systems (Merenluoto & Lehtinen, 2002; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004) or extension of a mathematical symbol and the related operation (Prediger, 2008; Vlassis, 2004). Conceptual change in mathematics learning could appear each time when the new knowledge to be acquired comes in conflict with what is already known. In these cases, it is important for the maths teacher to be careful of the prior conceptions of the students and to not expect immediate results after the instruction.

Further research in mathematics education would gain from the conceptual change approach and from the combination of this approach with other established theoretical frameworks in this field (see Prediger, 2008) like ''epistemological obstacles'' (Brousseau, 1998).

Acknowledgments

The first author is supported by the Centre for Research on the Teaching of Mathematics, Nivelles, Belgium. The second author is senior researcher, full professor at the Catholic University of Louvain, Belgium. The third author is supported by a grant from the Catholic University of Louvain (FSR, ARC), Belgium.

Author's personal copy

The authors would like to thank the journal's editor, Anastasia Efklides, and three anonymous reviewers for their pertinent comments and suggestions. Special thanks go to Nathalie Lefèvre for her statistical assistance.

References

Bachelard, G. (1938). *La formation de l'esprit scientifique*. [*The formation of the scientific mind*]. Paris: Vrin.

Besner, D., & Coltheart, M. (1979). Ideographic and alphabetic processing in skilled reading of English. *Neuropsychologia, 17*, 467–472.

Brousseau, G. (1998). *La théorie des situations didactiques*. [*The theory of didactical situations*]. Grenoble, France: La Pensée Sauvage.

Carey, S. (1985). *Conceptual change in childhood*. Cambridge, MA: MIT Press.

Dole, J. A., & Sinatra, G. M. (1998). Reconceptualizing change in the cognitive construction of knowledge. *Educational Psychologist, 33*(2/3), 109–128.

Duit, R., Roth, W. M., Komorek, M., & Wilbers, J. (2001). Fostering conceptual change by analogies – between Scylla and Charybdis. *Learning and Instruction, 11*, 283–304.

Fischbein, E. (1987). *Intuition in science and mathematics*. Dordrecht, The Netherlands: Kluwer.

Greer, B. (2004). The growth of mathematics through conceptual restructuring. *Learning and Instruction, 14*, 541–548.

Greer, B., & Verschaffel, L. (2007). Nurturing conceptual change in mathematics education. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Re-framing the conceptual change approach in learning and instruction* (pp. 319–328). Amsterdam: Elsevier.

Hewson, P. W. (1981). A conceptual change approach to learning science. *European Journal of Science Education, 3*, 383–396.

Kuhn, T. S. (1962). *The structure of scientific revolutions*. Chicago: The University of Chicago Press.

Lakatos, I. (1970). *Criticism and the growth of knowledge*. New York: Cambridge University Press.

Lehtinen, E., Merenluoto, K., & Kasanen, E. (1997). Conceptual change in mathematics: from rational to (un)real numbers. *European Journal of Psychology of Education, 12*(2), 131–145.

Limón, M. (2002). Conceptual change in history. In M. Limón, & L. Mason (Eds.), *Reconsidering conceptual change: Issues in theory and practice* (pp. 259–289). Dordrecht, The Netherlands: Kluwer.

Limón, M., & Mason, L. (2002). *Reconsidering conceptual change: Issues in theory and practice*. Dordrecht, The Netherlands: Kluwer.

McCloskey, M. (1983). Naïve theories of motion. In D. Gentner, & A. L. Stevens (Eds.), *Mental models* (pp. 299–324). Hillsdale, NJ: Erlbaum.

Mason, L. (2001). Instructional practices for conceptual change in science domain. [Special issue]. *Learning and Instruction, 11*, 4–5.

Mason, L., Gava, M., & Boldrin, A. (2008). On warm conceptual change: the interplay of text, epistemological beliefs, and topic interest. *Journal of Educational Psychology, 100*(2), 291–309.

Merenluoto, K., & Lehtinen, E. (2002). Conceptual change in mathematics: understanding the real numbers. In M. Limón, & L. Mason (Eds.), *Reconsidering conceptual change. Issues in theory and practice* (pp. 233–258). Dordrecht, The Netherlands: Kluwer.

Merenluoto, K., & Palonen, T. (2007). When we clashed with the real numbers: complexity of conceptual change in number concept. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Re-framing the conceptual change approach in learning and instruction* (pp. 247–263). Amsterdam: Elsevier.

Nesher, P., & Peled, I. (1986). Shifts in reasoning. The case of extending number concepts. *Educational Studies in Mathematics, 17*, 67–79.

Pintrich, P. R., Marx, R. W., & Boyle, R. B. (1993). Beyond cold conceptual change: the role of motivational beliefs and classroom contextual factors in the process of conceptual change. *Review of Educational Research, 63*, 167–199.

Posner, G. J., Strike, K. A., Hewson, P. W., & Gertzog, W. A. (1982). Accommodation of a scientific conception: towards a theory of conceptual change. *Science Education, 66*, 211–227.

Prediger, S. (2008). The relevance of didactic categories for analysing obstacles in conceptual change: revisiting the case of multiplication of fractions. *Learning and Instruction, 18*, 3–17.

Resnick, L. B. (2006). The dilemma of mathematical intuition in learning. In J. Novotná, H. Moraová, M. Krátvá, & N. Stehlíková (Eds.), *Proceedings of the 30th conference of the international group for the psychology of mathematics education, Vol. 1* (pp. 173–175). Prague: PME.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: the case of decimal fractions. *Journal for Research in Mathematics Education, 20*(1), 8–27.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organization in the process of learning a mathematical concept: the order of positive decimal numbers. *Cognition and Instruction, 2*(2), 157–174.

Sinatra, G. M., & Pintrich, P. R. (Eds.). (2003). *Intentional conceptual change*. Mahwah, NJ: Erlbaum.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*, 503–518.

Vamvakoussi, X. (2007). Extending the conceptual change approach to mathematics learning: an introduction. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Re-framing the conceptual change approach in learning and instruction* (pp. 239–246). Amsterdam: Elsevier.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: a conceptual change approach. *Learning and Instruction, 14*, 453–468.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in 'negativity'. *Learning and Instruction, 14*, 469–484.

Vosniadou, S. (1994). Capturing and modelling the process of conceptual change. [Special issue]. *Learning and Instruction, 4*, 45–69.

Vosniadou, S. (2002). Exploring the relationships between conceptual change and intentional learning. In G. Sinatra, & P. R. Pintrich (Eds.), *Prospects and problems for modes of intentional conceptual change* (pp. 377–406). Mahwah, NJ: Erlbaum.

Vosniadou, S. (2007). The cognitive-situative divide and the problem of conceptual change. *Educational Psychologist, 42*(1), 55–66.

Vosniadou, S., & Brewer, W. F. (1992). Mental models of the earth: a study of conceptual change in childhood. *Cognitive Psychology, 24*, 535–585.

Vosniadou, S., & Verschaffel, L. (2004). Extending the conceptual change approach to mathematics learning and teaching. *Learning and Instruction, 14*, 445–451.

Wellman, H. M. (2002). Understanding the psychological world: developing a theory of mind. In U. Goswani (Ed.), *Blackwell handbook of childhood cognitive development* (pp. 167–187). Oxford, UK: Blackwell.

Wiser, M., & Amin, T. (2001). "Is heat hot?" Inducing conceptual change by integrating everyday and scientific perspectives on thermal phenomena. *Learning and Instruction, 11*, 331–356.