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The Relationship between the Canadian
Treasury Bill Rate and Expected Inflation
in Canada and in the U.S.A.

by

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ABSTRACT

This paper investigates the relationship between Treasury Bill rates and expected inflation rates in Canada and in the United States during the 1953-1975 period. The tests are based on quarterly data, and use two inflation-interest rate models, in which expected real returns on Treasury Bills are assumed to be either constant, or to follow a random walk. The study indicates that Treasury Bill rates in Canada have provided good assessments of expected inflation in the U.S. rather than in Canada during the entire period under study. This result is consistent with empirical evidence about the predominant influence of U.S. monetary policy and interest rate trends on Canadian monetary policy and interest rate policy, and with the evidence about the efficiency of the U.S. Treasury Bill market during most of the period under study.

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THE RELATIONSHIP BETWEEN THE CANADIAN
TREASURY BILL RATE AND EXPECTED INFLATION
IN CANADA AND IN THE U.S.A.

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On the efficiency of the Canadian Treasury Bill market

The empirical literature analyzing the behavior of nominal returns on U.S. Treasury Bills under conditions of varying rates of inflation has been rapidly expanding in the past few years. Following Fama's work (1975), the main stream of the recent body of empirical work has tested the relationship between interest rates and inflation on an ex-ante basis within the framework of the efficient market hypothesis. The empirical evidence emerging from these tests presents a strong case for market efficiency with regard to the pricing of U.S. Treasury Bills over the last three decades.

This paper extends to Canada the investigation of the relationship between the bill rate and inflation. The hypothesis to be tested is that the nominal Treasury Bill yield in Canada throughout the 1953-1975 period exhibits an inflationary premium which provides a good assessment of expected inflation in the U.S. rather than in Canada.

Very little work has been done to test the relationship between the Canadian Treasury bill rate and inflation within the context of the hypothesis presented above. In a technical report sponsored by the Bank of Canada, Freedman, Longworth and Masson (1977) examined the role of Canadian and U.S. interest rates in the determination of three money market rates in Canada, namely: the term-deposit rate, the rate on finance company paper and the prime loan rate for the period 1963-1975. They observed that U.S. short-term private interest rates⁽¹⁾ played a more significant role in explaining the rate on 90-day

Canadian finance company paper than in explaining the rate on 90-day Canadian non-personal term and notice deposits at chartered banks during that period, whereas the Canadian bank rate and the U.S. prime rate played approximately an equal role in the determination of the Canadian prime rate. In a more detailed study of the same time-span, Fortin (1975) found that Canadian interest rate policy in the 1962-1973 period was primarily influenced by the trend of U.S. interest rates. He attributes this situation to the powerful constraints imposed upon the Bank of Canada by the pegging of the price of the Canadian dollar and the very high interest sensitivity of short-term international capital flows. In a more direct approach, Lee and Mantripragada (1978) analyzed the relationship between yields on 3- and 6- month treasury bills and inflation in Canada for the period 1954-1977 using the framework of Fama's (1975) model. On the basis of the observed high autocorrelation in the residuals, they concluded that the Canadian 3- and 6- month Treasury Bill market is inefficient. Lee and Mantripragada's conclusions can be challenged, however. First, their model assumes a constant expected real rate. This would generate autocorrelation in the residuals if, as was shown later on by Hess and Bicksler, Nelson and Schwert, Garbade and Wachtell, and Fama and Gibbons, the expected real return in fact wanders over time. Secondly, their analysis does not distinguish between sub-periods characterized by wage and price controls, pegged discount rates and fixed versus floating exchange rates. Further on in this paper, it will be shown that these sub-periods represent differences in the behavior of the treasury bill market significant enough to warrant separate analysis.

Finally, Park (1982) examined the efficiency of the Canadian treasury bill market on the grounds that the forward rate is a reflection of all relevant information contained in the past spot rates. He concluded that for the period July 1962 to March 1979, forward rates correctly reflected not only the information on past spot rates but also on future spot rates as well.

HYPOTHESIS:

Based on I. Fisher's work (1930), we can postulate that under the conditions of long-term equilibrium and perfect competition, the nominal interest rate for period t observed at time $t-1$, (Y_{t-1}) , can be written as the sum of the expected real return for period t , $(E(R)_{t-1})$, and a premium equal to the expected inflation rate for that period, $(E(I)_{t-1})$,

$$\text{where } Y_{t-1} = E(R)_{t-1} + E(I)_{t-1} \quad (1)$$

Equation (1), which is purely definitional, represents the equilibrium relationship determining the nominal period by period Treasury Bill rates in the economy⁽²⁾. In testing the efficiency of the U.S. treasury bill market, Fama (1976) hypothesized that the ex-ante real rate is constant over time. This assumption, coupled with the rational expectations hypothesis⁽³⁾, allowed him to test market efficiency by regressing the inflation rate reported at the end of the period on the treasury bill return at the beginning of the period in the following manner:

$$I_t = -\alpha + b_1 Y_{t-1} + \epsilon_t \quad (2)$$

where the ϵ_t are independent random variables, with mean zero.

However, several authors have challenged the hypothesis of a constant ex-ante real rate. Hess and Bicksler (1975), Nelson and Schwert (1977), Gardabe and Wachtel (1978) and Fama and Gibbons (1980) have shown that expected real returns wander over time. These findings suggest that equation (2) should be rewritten as:

$$I_t = b_{o,t} + b_1 Y_{t-1} + \varepsilon_t \quad (3)$$

where $-b_{o,t} = E(R)_{t-1}$ is the expected real rate of return that is supposed to wander over time and where the coefficient b_1 is supposed to equal 1 under the efficiency hypothesis.

Equation (3) can serve as a basis for testing the relationship between nominal Treasury Bill rates and expected inflation rates in Canada during the 1953-1975 period. Of particular importance, however, in conducting these tests for a small open economy subject to international capital flows such as Canada, are the efforts of the monetary authorities to maintain the fluctuations of the Canadian-U.S. dollar exchange rate within narrow limits regardless of the exchange rate system in force. Several studies of Canadian monetary policy have indeed shown that the Bank of Canada had committed itself, during most of the period examined in this paper, to keeping the price of the Canadian dollar within narrow limits in terms of the U.S. dollar, and this even under the floating exchange rate system (Fortin (1975) chapter VI). Following Mundell (1968) we can argue that a small

open economy that pegs its exchange rate in the manner explained above automatically loses all control over both its monetary base and the structure of its interest rates. This indeed seems to have been the case in Canada during most of the period under study. For example, Fortin (1975) has shown that for the 1962-1973 period not only were Canadian interest rates primarily influenced by U.S. interest rates, but also that the almost exclusive determinant of Canadian monetary policy was U.S. monetary policy. Focusing on the Treasury Bill rate in particular, the above evidence, when interpreted in conjunction with the results of the Fama & Gibbons' (1980) study which shows that U.S. Treasury Bill rates have provided good assessments of U.S. inflation rates during the 1953-1977 period, leads to the conclusion that Canadian nominal Treasury bill rates should also provide good assessments of expected U.S. inflation rates during the same time span. A second test can therefore be devised to test this particular hypothesis, as follows:

$$I_{U.S.,t} = b_{0,t} + b_1 Y_{CAN,t-1} + \epsilon_t \quad (4)$$

Finally, the great interdependence between the Canadian and U.S. interest rates reported earlier by Freedman et al (1977) on the one hand, and by Fortin (1975) on the other, leads us to explore the

the influence of the U.S. real rate of return on its Canadian counterpart. Assuming that the real rates in the two countries differ by an expected risk premium ($RP_{o,t}$) that wanders over time, we can explore the link between them using the equation:

$$I_{CAN,t} + E(R_{U.S.})_{t-1} = RP_{o,t} + b_1 Y_{CAN,t-1} + \varepsilon_t \quad (5)$$

where $E(R_{U.S.})_{t-1}$ is obtained from (3) using U.S. data.

METHODOLOGY:

All the regressions presented in this paper were performed by ordinary least squares, by assuming a constant intercept and by a maximum likelihood method applied on a regression model in which the intercept follows a random walk as developed by Ansley (1980). In the case of simple regressions, the model is defined by

$$y_t = b_{o,t} + b_1 x_t + \varepsilon_t \quad (6)$$

where $b_{o,t} = b_{o,t-1} + \eta_t \quad (7)$

It is assumed that the ε_t and η_t are mutually independent sequences of independent and normally distributed random variables each with mean 0.

Let σ^2 be the common variance of ε_t and $\lambda\sigma^2$ the common variance of η_t .

If the parameter λ equals zero, the intercept remains constant. On the other hand, a large value of λ corresponds to a highly wandering intercept.

However this can also occur if the model is misspecified, since $b_{o,t}$ will then act as a proxy for omitted regressors. Besides b_1 , λ and σ^2 , the model must contain an additional parameter ⁽⁴⁾: the initial value $b_{o,o}$, which corresponds to the constant intercept b_o of the classical regression model in the case where $\lambda=0$. This parameter is not interesting in itself and accordingly will not be mentioned in the results, but its presence in the model is essential when dealing with short series of observations or when λ is far from zero.

Suppose that the data are x_t, y_t for $t = 1, 2, \dots, n$. The estimation method can be represented as follows. Let ∇ be the difference operator. Then (6-7) can be written for $t \geq 2$:

$$\nabla y_t = b_1 \nabla x_t + \nabla \varepsilon_t + \eta_t \quad (8)$$

But $w_t = \nabla \varepsilon_t + \eta_t$ is a first order moving average stochastic process which can be represented by $w_t = a_t - \theta a_{t-1}$ where the a_t are mutually independent. It can be seen that λ and θ are related by $(1 + \theta^2)/\theta = 2 + \lambda$ and that the variance of a_t is σ^2/θ . For $t = 1$, we simply have ⁽⁵⁾:

$y_1 = b_{o,o} + b_1 x_1 + \varepsilon_1 + \eta_1$ so that $w_1 = \varepsilon_1 + \eta_1$. The model is now in a form such that the maximum likelihood estimation of $b_{o,o}, \lambda$ and b_1 can be performed taking into account the autocovariance structure of the w_t . A general purpose optimisation algorithm is used for estimating $b_{o,o}$ and λ whereas b_1 is estimated by a regression without an intercept for each value of $b_{o,o}$ and λ , after the appropriate transformation has been applied on (8) in order to suppress autocorrelation (see Ansley (1980)) ⁽⁶⁾.

Since the classical regression model is imbedded in the regression model with a stochastic intercept, the likelihood ratio test can be used to test the null hypothesis that $\lambda = 0$. The statistic denoted by χ^2_1 is asymptotically distributed like a chi-square with one degree of freedom.

DATA

All the tests in this study are based on quarterly data. Quarterly nominal Treasury Bill yields are represented by the average yields on 3-month Canadian and U.S. treasury bills observed at the end of the third month of each quarter and as published in the "Bank of Canada Review" for the period January 1953 to June 1975. The Canadian series is for the tender of the last Thursday of each month ⁽⁷⁾, whereas the U.S. data is for the tender of the last Monday. Canadian and U.S. inflation rates have been calculated as $(P_t/P_{t-1})-1$ where P_t is the seasonally unadjusted ⁽⁸⁾ consumer price index of the first month of each quarter (t) published by Statistics Canada (during the third week of the following month) and by the Bureau of Labor Statistics (towards the end of the following month) respectively.

The choice of the sampling period deserves comment. The first date of the sampling period, January 1953, coincides roughly with the beginning of the Bank of Canada's active involvement in developing a broader and deeper money market (see Acheson and Chant (1973)). The last observation in the sampling period falls in the month just preceding the establishment of wage and price controls by the Canadian government in October 1975. The period October 1975 - December 1978 during which Canada used wage and

price controls was excluded from the study since the lack of accurate information on economic activity may have hampered the market's efficiency in assessing future inflation rates. Finally, the post-control period did not provide sufficient observations at the time the study was begun, which would permit analysis as a separate sub-period. Therefore the post-control period was not considered.

Examination of the sampling period reveals three broad sub-periods which are clearly outlined in the analysis. The first sub-period extends from the second quarter of 1953 to the third quarter of 1962. During this time the Bank of Canada intervened actively to peg its floating discount rate at $1/4\%$ above the average weekly tender rate of 91-day treasury bills. To facilitate the implementation of this policy, the Bank of Canada may have tried to stabilize the treasury bill rate during those years (see Park (1982)). The second sub-period goes from the fourth quarter of 1962 to the third quarter of 1970. The discount rate of the Bank of Canada was no longer pegged, but the exchange rate between the Canadian and U.S. dollar was "fixed". Finally, the third sub-period, which extends from the fourth quarter of 1970 to the third quarter of 1975, witnessed the change from a fixed to a floating exchange rate between the Canadian and the U.S. dollar. The results of the analysis will be presented first for the period as a whole, and then for the three sub-periods described above.

RESULTS

Table 1 sets out the results of the test of the efficient Treasury

TABLE 1

Period (quarters)	Number of observations	Least square estimates of the regression $I_{CAN,t} = b_0 + b_1 Y_{CAN,t-1} + \epsilon_t$					Maximum likelihood estimates of the regression $I_{CAN,t} = b_0 + b_1 Y_{CAN,t-1} + \epsilon_t$					
		b_0 ($\pm SE$)	b_1 ($\pm SE$)	R^2 (F)	DW	s_e	λ	0	b_1 ($\pm SE$)	R^2	s_e	X^2_1
2/1953- 3/1975	90	-1.17 ($\pm .81$)	1.09 ($\pm .18$)	.293 (36.5)	1.36	3.10	.0602	.783	.39 ($\pm .23$)	.582	2.40	25.9*
2/1953-3/1962	38	-.02 (± 1.13)	.49 ($\pm .38$)	.043 (1.62)	2.30 (1)	2.73	10^{-9}	1	.50 ($\pm .40$)	.043	2.76	0
4/1962-3/1970	32	.99 (± 1.39)	.44 ($\pm .27$)	.081 (2.64)	2.46 (1)	2.05	10^{-10}	1	.43 ($\pm .28$)	.081	2.08	0
4/1970-3/1975	20	.41 (± 2.43)	1.30 ($\pm .44$)	.328 (8.77)	.92	3.45	.416	.530	.17 ($\pm .64$)	.746	2.18	7.19*

* significant at the 5% level

(1) The null hypothesis of no autocorrelation cannot be rejected at the 5% level of significance

Bill market hypothesis for Canada within the framework of equation (3). Besides the statistics mentioned in the methodology section and the standard error of the coefficients ($\pm SE$), the results also include the coefficient of determination (R^2), the residual standard deviation (s_e), the Fisher goodness of fit statistic (F) with $(2, n-2)$ degrees of freedom, the number of observations (n) and the Durbin-Watson statistic (D.W.).

A quick look at the table reveals that the results obtained with O.L.S. are rather weak, either because the R^2 is low ⁽⁹⁾ or because of residual autocorrelation. On the other hand, the regression with the stochastic intercept seems to give a better fit for the whole 1953-1975 period and for the third sub-period as well. This is not surprising in view of the sharp increase in the rate of inflation that took place in the third sub-period. The table also exhibits a value for θ close to 1 in two cases out of four ⁽¹⁰⁾. As for the estimates of b_1 , they provided no support for the hypothesis tested ⁽¹¹⁾, either for the entire sampling period or for the sub-periods under analysis ⁽¹²⁾.

Table 2 shows the results of the regression equation (4) in which we postulate that Canadian nominal quarterly Treasury Bill rates anticipate the U.S. inflation rate quarter after quarter. The results obtained by O.L.S. indicate an improvement in the R^2 statistic when compared to those in Table 1. However, this observation does not apply to the first sub-period in which the U.S. inflation rate was nearly constant. More

TABLE 2

Period (quarters)	Number of observations	Least square estimates of the regression $I_{US,t} = b_0 + b_1 Y_{CAN,t-1} + \epsilon_t$					Maximum likelihood estimates of the regression $I_{US,t} = b_0 + b_1 Y_{CAN,t-1} + \epsilon_t$					
		b_0 ($\pm SE$)	b_1 ($\pm SE$)	R^2 (F)	DW	s_ϵ	λ	θ	b_1 ($\pm SE$)	R^2	s_ϵ	X^2_1
2/1953-3/1975	90	-1.98 ($\pm .58$)	1.26 ($\pm .13$)	.521 (95.6)	1.18	2.22	.0738	.763	.89 ($\pm .17$)	.738	1.65	30.4**
2/1953-3/1962	38	-.55 ($\pm .78$)	.70 ($\pm .27$)	.162 (6.97)	1.82 (1)	1.88	10^{-11}	1	.69 ($\pm .27$)	.162	1.90	0
4/1962-1970	32	-2.67 ($\pm .93$)	1.17 ($\pm .18$)	.585 (42.3)	2.03 (1)	1.37	10^{-8}	1	1.17 ($\pm .19$)	.585	1.40	0
4/1970-3/1975	20	-1.31 (± 1.70)	1.52 ($\pm .31$)	.576 (24.4)	1.64 (1)	2.42	.184	.653	1.12 ($\pm .47$)	.733	1.97	1.24

* significant at the 5% level

(1) The null hypothesis of no autocorrelation cannot be rejected at the 5% level of significance

importantly, the value of b_1 is not significantly different from 1 at a 5% level of significance, thus leading us to accept the hypothesis tested for the entire 1953-1975 period as well as for the three sub-periods. The regression intercept is negative throughout, indicating a positive real rate of return; but it is only significant when the entire sampling period is considered probably on account of the small number of observations in the sub-periods.

When the results of the regression with the stochastic intercept are considered, the relationship stands out more clearly. Even though the results for the first and second sub-periods were the same under both methods, the goodness of fit is nonetheless superior with the maximum likelihood method as opposed to the O.L.S. method. Moreover the observations made with respect to b_1 in the preceding paragraph are still valid under the maximum likelihood method with b_1 values even showing a minor improvement. As regards the first two sub-periods, it appears that a model with a constant expected real rate yields results that are as good as those obtained using a model with a stochastic intercept. Note also that the value of $b_{0,t}$ fluctuates within a wide range in the maximum likelihood regression: from a low of -2 (at the end of the first sub-period) to a high of 3.5 (at the end of the third sub-period). This probably reflects active government intervention to peg the Canadian dollar.

There is a natural explanation for the phenomenon revealed in Table 2. As long as the predominant influence of U.S. monetary and interest rate policy persists in Canada, and given that the U.S.

Treasury Bill rate provides a good assessment of expected inflation in the U.S. during most of the period under study, it is quite appropriate for the Canadian Treasury Bill market to anticipate the inflation rate in the U.S. rather than at home. This argument is reinforced when the correlation coefficients between U.S. and Canadian inflation rates are considered. As Table 3, shows there is no conclusive evidence of any systematic lagged impact of U.S. inflation rates on Canadian rates.

In the relationship between expected real returns in Canada and the U.S., Table 4 presents the results of regression equation (5), using the O.L.S. and the maximum likelihood methods ⁽¹³⁾. In the first and second sub-periods during which the Bank of Canada "officially" intervened in the Bill market and in the exchange market, the results are the same using both methods. However the results of the third sub-period and of the whole sampling period seem to be better with O.L.S. Indeed, in these last two instances the value of the coefficient b_1 is not significantly different from 1 when using the O.L.S. method. But the same does not hold true with the maximum likelihood method.

On the whole, the evidence presented in Table 4 regarding a link between the Canadian and U.S. real rates seems inconclusive. The least that can be said about the above rates is that they were not equivalent during the period 1953-1975. Moreover, in the regression with the stochastic intercept the value of the χ^2_1 statistic is highly significant when the whole period 1953-1975 is considered, indicating that the differential real rate of return between Canada and the U.S. is not constant, but rather wanders over time.

TABLE 3

CORRELATION COEFFICIENTS BETWEEN CANADIAN AND U.S. INFLATION RATES FOR THE PERIOD 1953 - 1975

Periods (quarters)	Number of observations	Correlation coefficients between $I_{US,t}$ and					Correlation coefficients between $I_{US,t}$ and				
		$I_{CAN,t}$	$I_{CAN,t+1}$	$I_{CAN,t+2}$	$I_{CAN,t+3}$	$I_{CAN,t+4}$	$I_{CAN,t}$	$I_{CAN,t-1}$	$I_{CAN,t-2}$	$I_{CAN,t-3}$	$I_{CAN,t-4}$
2/1953 - 3/1975	90	.78	.61	.51	.57	.47	.78	.57	.53	.61	.51
2/1953 - 3/1962	38	.52	.40	-.09	.08	.05	.52	-.14	-.03	.46	.08
4/1962 - 3/1970	32	.62	.05	.17	.30	.23	.62	.38	.28	.28	.35
4/1970 - 3/1975	20	.73	.55	.44	.51	.30	.73	.71	.51	.44	.44

TABLE 4

Period (quarters)	Number of observations	Least square estimates of the regression ** $I_{CAN,t} + E(R_{US,t-1}) = RP_o + b_1 Y_{CAN,t-1} + \epsilon_t$				Maximum likelihood estimates of the regression ** $I_{CAN,t} + E(R_{US,t-1}) = RP_o + b_1 Y_{CAN,t-1} + \epsilon_t$				
		b_o ($\pm SE$)	b_1 ($\pm SE$)	R^2 (F)	DW	s_ϵ	λ	θ	R^2	s_ϵ χ^2_1
2/1953-3/1975	90	.80 ($\pm .70$)	.99 ($\pm .16$)	.314 (40.2)	1.78 ⁽¹⁾	2.69	.0220	.862	.464	2.39 9.61*
2/1953-3/1962	38	1.08 (± 1.13)	.49 ($\pm .39$)	.043 (1.62)	2.30 ⁽¹⁾	2.73	10^{-10}	1	.043	2.76 0
4/1962-3/1970	32	3.23 (± 1.39)	.44 ($\pm .27$)	.081 (2.64)	2.46 ⁽¹⁾	2.05	10^{-10}	1	.081	2.08 0
4/1970-3/1975	20	4.09 (± 2.04)	.94 ($\pm .37$)	.263 (6.43)	1.20 ⁽²⁾	2.90	.187	.573	.573	2.28 2.80

* significant at the 5% level

** The reader should be reminded that estimates of $ER_{US,t-1}$ are obtained from equation (3) using U.S. data.

(1) The null hypothesis of no autocorrelation cannot be rejected at the 5% level of significance

(2) This value falls within the zone of indeterminacy

CONCLUSION

This paper has examined the relationship between nominal Treasury Bill rates in Canada and expected inflation rates in both Canada and the U.S. during the period 1953-1975. All the tests have been conducted on the basis of quarterly data and have used two interest rate models in which expected real rates are either constant or follow a random walk. The study shows that the pricing of Canadian Treasury Bills does not seem to have taken into account expected inflation in Canada, but rather that Treasury Bill rates in Canada have provided good assessments of expected inflation rates in the U.S. for the entire period under study. This result is consistent with existing evidence on the predominant influence of U.S. monetary and interest rate policies, on Canadian monetary policy and interest rate structure and with the evidence concerning the efficiency of the U.S. Treasury Bill market during most of the period examined.

As for the relationship between real rates in Canada and the U.S., the evidence suggests that these two rates are not equivalent and that the differential between them has tended to wander over time during the period examined in the study.

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FOOTNOTES:

1. More specifically, interest rates on three-month U.S. certificates of deposit, on 90-day U.S. commercial paper, on 90-day U.S. finance company paper, and on prime business loans at U.S. banks.
2. In her Ph.D. dissertation, A. Mandron argues that equation (1) is theoretically incorrect on the grounds that the premium for expected inflation $[E(I)_{t-1}]$ introduces a bias in the equation. She maintains that this premium should be replaced by a parameter linked to the rate of inflation but smaller in value than $[E(I)_{t-1}]$. However, since there is no empirical evidence as to the size of the bias introduced by $[E(I)_{t-1}]$, we shall still use equation (1) and interpret the results with caution. See A. Mandron (15) chapter 2.
3. If expectations are rational, then:

$$I_t = E(I_t/n_t) + \epsilon_t$$

where n_t is the set of information available at time t , and ϵ_t is a random variable with zero mean.

4. No mention of this parameter has been made by Fama and Gibbons (1980) and the Ansley's paper (1980) is not very clear in this regard.

5. This implies that the process w_t is non-stationary since the variance of w_1 differs from the variance of the other w_t . This fact is not mentioned by Ansley (2). It can be seen, however, that with a slight modification of the procedure, the non-stationarity of w_t can be dealt with. It is a special case of a more general algorithm for time-dependent time series models presented by Mlard (16), and which extends the Ansley algorithm.
6. More precisely, the regressant values are $y_1 - \bar{y} - b'_{o,o}$ for $t = 1$ and Δy_t for $t > 1$ and the regressor values are $x_1 - \bar{x}$ for $t = 1$ and Δx_t for $t > 1$, where $\bar{x}$ and $\bar{y}$ are the arithmetic means of the x_t and y_t , respectively; $b'_{o,o}$ is related to $b_{o,o}$ by the equation $b_{o,o} = b'_{o,o} + \bar{y} - b_1 \bar{x}$ and is the very parameter which enters into the optimization algorithm.
7. A time series analysis of the average yield is possible even though the Bank of Canada is known to place a "floor" bid at the weekly auction of the treasury bills, because the floor bid does not vary systemically with the U.S. Treasury Bill rate or with the predicted inflation rate in Canada.
8. As a result, residual autocorrelation of order 4 has been observed in many of the models, and is smaller for the US than for the Canadian inflation. Seasonally adjusted data might have introduced short-term distorsion. We have adopted the same attitude as Fama and Gibbons in this regard.

9. Fama and Gibbons argue that the coefficient of determination can be difficult to interpret since the stochastic process for the inflation is non-stationary. In any case, the R^2 should not be used to compare the sub-periods. As can be seen from the column the s_e , the magnitude of the residual standard deviations are similar in such a way that the discrepancies between the R^2 statistics are mainly due to different amplitude of variation of the dependent variable.
10. This strange behavior is now well known in time series maximum likelihood estimation, see Ansley and Newbold (3), Cryer and Ledolter (4).
11. One should be cautious when interpreting this result, since equation (1) which is the theoretical basis of this test incorporates a potential bias as explained in footnote (2).
12. The values of χ^2_1 are sometimes negative in these cases because the optimization algorithm does not attain the true optimum. They are reported as zero in the table.
13. At least two other approaches could be thought of to conduct this test. The first approach is based on estimates of $(-b_{o,t})$ obtained separately for Canada and the U.S. using equation (3). As was shown before however, estimates of $(-b_{o,t})$ are not valid for Canada and furthermore the series are highly autocorrelated (the D.W. statistic of the regression for the whole 1953-1975 period for example is as low as 0.02).

The second approach uses the following regression equation:

$$\frac{1 + Y_{CAN,t-1}}{1 + I_{CAN,t}} - 1 = b_0 + b_1 \left(\frac{1 + Y_{US,t-1}}{1 + I_{US,t}} - 1 \right) + \epsilon_t$$

An example of the estimated regression equation for the whole 1953-1975 period is:

$$\frac{1 + Y_{CAN,t-1}}{1 + I_{CAN,t}} - 1 = \begin{matrix} 0.16 \\ (\pm 0.27) \end{matrix} + \begin{matrix} 0.93 \\ (\pm 0.13) \end{matrix} \left(\frac{1 + Y_{US,t-1}}{1 + I_{US,t}} - 1 \right) + \epsilon_t$$

$$D.W. = 1.97; R^2 = 0.37; F = 51.7; s_e = 2.37$$

It should be noted, however, that in this approach the expected real rates of return (b_{ot}) defined in equation (3) are replaced by approximations that may introduce errors in the variables. This in turn could invalidate O.L.S. estimates of the coefficients.