Some years ago Peres [Phys. Rev. A 46, 4413 (1992)] described a gedanken experiment for a pair of spatially spin-j particles in a singlet state and showed using with a dichotomic observable (essentially a parity operator) that Bell’s theorem in the form of the Clauser-Horne-Shimony-Holt (CHSH) inequality is violated by a constant amount (24%) in the limit j → ∞. In this paper we present a scheme for an optical realization of a state that is very close to the spin-j singlet state using two traveling-wave modes of the quantized field using a 50:50 beam splitter with an input number state. A near-singlet states comes about because the binomial output state of the beam splitter can be written as a sum in terms of states in the form |j,m⟩1 ⊗ |j,−m⟩2, each state being associated with a Holstein-Primakoff realization of the su(2) spin algebra in terms of the Bose operators of each of the field modes, where j=N/2, N being the number of photons passing through the beam splitter. The binomial state can violate the CHSH inequality to a greater degree than does the singlet state.

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Some years ago, Peres [1] presented a gedanken experiment involving a spatially separated pair of spin-j particles in the singlet state

$$|Ψ_{SS}⟩ = (2j + 1)^{-1/2} \sum_{m=-j}^{j} (-1)^{j-m} |j,m⟩_1 \otimes |j,-m⟩_2,$$  

(1)

and showed for the dichotomic observable

$$\hat{I} = \exp[iπ(j - \hat{J}_z)] = \sum_{m=-j}^{j} |j,m⟩(-1)^{j-m} |j,m⟩,$$  

(2)

that Bell’s theorem in the form of the experimentally friendly Clauser-Horne-Shimony-Holt (CHSH) inequality [2] is violated by a constant amount in the limit that j → ∞. The value of the constant is 2.481 while 2 is the classical upper bound, giving a 24% violation of the CHSH inequality. Prior to the work of Peres [1], Garg and Mermin [3] showed that for a pair of particles in the singlet state of Eq. (1), Bell’s inequality was violated for measurements of the spin components along nearly all pairs of directions, but that the magnitude of the violations decrease for increasing spin. Garg and Mermin put forward the conjecture that the decrease of the violations, which, in fact, go to zero in the limit of large j, might be due to their use of a slowly varying functions of the spin component m, though earlier Mermin [4] suggested such behavior to be the consequence of approaching the classical limit. But the observable $\hat{I}_j$, where each observer records the quantity $(-1)^{j+m} = ±1$, varies rapidly with m. That the CHSH inequality is violated by a constant amount even as j becomes very large demonstrates that the limit of large j is not necessarily the classical limit [1,5,6].

The work of Ref. [1] is presented as a gedanken experiment. But the natural question arises: Can a spin-singlet state of Eq. (1), or something close to it, be realized in practice? In this paper we show that something very close to the singlet state can be generated in the optical domain using nothing more than a beam splitter with a definite number of photons N injected into one of the input ports, the other containing only the vacuum. The beam splitter generates an su(2), or spin, coherent state where the beam splitter transformation may be given in terms angular momentum operators [7] through the Schwinger realization in terms of two sets of Bose operators [8]. The output of the beam splitter is a superposition of product number states of the form $|n⟩_1 \otimes |N−n⟩_2$. We then show that these number products states are equivalent to product angular momentum states of the form $|j,m⟩_1 \otimes |j,−m⟩_2$, where each of the angular momentum states is associated with a Holstein-Primakoff realization of the angular momentum algebra, one realization for each mode. The full su(2) coherent state in terms of this decomposition has the form

$$|Ψ⟩ = \sum_{m=-j}^{j} B_m^{(j)} |j,m⟩_1 \otimes |j,−m⟩_2,$$  

(3)

which, as it contains a sum over the product states $|j,m⟩_1 \otimes |j,−m⟩_2$ is close in structure to the singlet state of Eq. (1) but where the coefficients $B_m^{(j)}$ differ from that state. In fact, the coefficients we obtain are those of the binomial distribution, well known to be the distribution associated with the su(2) coherent state.

We begin with a brief discussion of the su(2) coherent state for a two-mode field. We start with the Schwinger realization of the angular momentum operators given by

$$\hat{J}_x = \hat{a}^† \hat{a}^2, \quad \hat{J}_y = \hat{a}^† \hat{a}^2 1, \quad \hat{J}_z = \frac{1}{2} (\hat{a}^† \hat{a}^1 - \hat{a}^1 \hat{a}^†),$$  

(4)

and an operator $J_0=(\hat{a}^† \hat{a}^2 + \hat{a}^2 \hat{a}^†)/2$ that commutes with all the other operators and is related to the total angular momentum through $\hat{J}^2=J_0(J_0+1)$. The angular momentum states $|J,M⟩$ map onto the two-mode number states $|N⟩_1 \otimes |N⟩_2$, according to

$$|J,M⟩ = |N⟩_1 \otimes |N⟩_2, \quad \text{for} \quad J = \frac{N_1 + N_2}{2}.$$
In terms of the number states, the su(2) coherent states (the spin coherent states) are written as

$$|\xi,J\rangle = \exp(\eta \hat{J}_+ - \eta^* \hat{J}_-)|J,J\rangle$$

$$= (1 + |\xi|^2)^{-J} \sum_{m=-J}^{J} \frac{(2J)!}{(J + M)! (J - M)!} \hat{b}^M |J,J\rangle,$$

where $\xi = (\eta^* / |\eta|) \tan(|\eta|)$ ranges over the entire complex plane. If we assume that the total number of photons in both modes is $N$, the two-mode number state products correspond to the $|J,J\rangle$ angular momentum states according to $|J,J\rangle = |n\rangle_1 \otimes |N-n\rangle_2$, $J = N/2$, $M = n-J$, $n = 0,1,2,\ldots,N$. In terms of the number states, the su(2) coherent state is written as

$$|\xi,J\rangle = (1 + |\xi|^2)^{-N/2} \sum_{n=0}^{N} \frac{N!}{n! (N-n)!} \xi^n |n\rangle_1 \otimes |N-n\rangle_2.$$

(7)

Clearly, the $N$ photons are binomially distributed over the two modes of the field. A state of this sort can be generated with a beam splitter if $N$ photons are injected into one port with only the vacuum at the other [7]. Another possibility for generating such a state is with two parametric down-converters with aligned idler beams [11].

Let us assume, to be definite, that by beam splitting on input $N$ photon and vacuum states, we have available a state of the form of Eq. (7). We now introduce for spin $j$ two new sets of angular momentum operators, in fact, Holstein-Primakoff realizations of the operators, for each mode. These are

$$\hat{J}^+_1 = a_1^\dagger (2j - a_1^\dagger a_1)^{1/2} \hat{a}_1,$n$$

$$\hat{J}^-_1 = a_1^\dagger (2j - a_1^\dagger a_1)^{1/2} \hat{a}_1 - j$$

for mode 1 and

$$\hat{J}^+_2 = a_2^\dagger (2j - a_2^\dagger a_2)^{1/2} \hat{a}_2,$n$$

$$\hat{J}^-_2 = a_2^\dagger (2j - a_2^\dagger a_2)^{1/2} \hat{a}_2 - j$$

for mode 2. We now need a mapping of the single mode number states onto angular momentum states of each of states for the Holstein-Primakoff (HP) realization of the two modes. The mapping is as follows: $|n\rangle_1 \leftrightarrow |j,m\rangle_1$ for $n = j + m$, where we have denoted the bases of the HP realization as $|j,m\rangle_1$ which should not be confused with the two-mode states $|J,J\rangle$ used above. But if we set $j = J = N/2$ we have for mode 1 $|n\rangle_1 = |j,m\rangle_1$ with $m = n - N/2$ and for mode 2 we have $|N-n\rangle_2 = |j,m\rangle_2$ where $m' = N/2 - n = -m$ so that $|N-n\rangle_2 = |j,-m\rangle_2$. With these identifications, Eq. (7) can be written in the suggestive form

$$|\xi,J\rangle = (1 + |\xi|^2)^{-J} \sum_{m=-J}^{J} \frac{2J}{(j+m)!} \xi^{j+m} |j,m\rangle_1 \otimes |j,-m\rangle_2.$$

(10)

The original su(2) coherent state for a two-mode field has been decomposed into a sum of product states of the HP realizations of each mode and is similar in form to the singlet state of Eq. (1). It generally lacks the rotational invariance of the singlet state but is still suitable for a test of a Bell inequality. Note that

$$\hat{J}^0_1|\xi,J\rangle = -\hat{J}^0_2|\xi,J\rangle,$$

(11)

where a similar relation obviously holds for the singlet state of Eq. (1) and for any state of the form of Eq. (3). For the singlet state $\hat{J}^0_1|\Psi_{SS}⟩ = -\hat{J}^0_2|\Psi_{SS}⟩$. It is important to remember that the angular momentum operators that define the su(2) coherent state of Eq. (6) are not related in any direct way to the Holstein-Primakoff operators of the two modes. For example, $J_1 \neq J_{1z} + J_{2z}$, where $J_{1z} = (J_1 + J_2)/2$ is determined from Eqs. (4) but $J_{1z}$, are determined from Eqs. (8) and (9). Also note that the states $|J,J\rangle$ are not given in terms of the HP bases using Clebsch-Gordan coefficients, rather we have the direct correspondence

$$|J,J\rangle = |n\rangle_1 \otimes |N-n\rangle_2 = |j,m\rangle_1 \otimes |j,-m\rangle_2,$n$$

$$J = j = N/2, \quad M = m = n - N/2.$$

The joint probability of finding $n$ photons in mode 1 and $N-n$ in mode 2 is given by

$$P_{n,N-n}(|\xi|^2) = (1 + |\xi|^2)^{-N/2} \frac{N!}{n! (N-n)!} |\xi|^{2n},$$

(13)

where we have used Eq. (7). This is equivalent to the probability of detecting the product spin states $|j,m\rangle_1 \otimes |j,-m\rangle_2$ which, from Eq. (10) is given by

$$P_{m,-m}^{(j)}(|\xi|^2) = |\langle j,m | \otimes \langle j,-m | \xi,J\rangle|^2$$

$$= (1 + |\xi|^2)^{2j} \frac{2j}{(j+m)!} |\xi|^{2(j+m)}.$$n

(14)

This can be read as the joint probability of finding $j+m$ photons in mode 1 and $j-m$ in mode 2. The distributions are obviously binomial. In Fig. 1 the joint probability $P_{n,N-n}(|\xi|^2)$ is plotted against $n$ for the choice $|\xi|^2 = 1$ for different total photon number $N$. For this choice of $|\xi|^2$ it is clear that the distribution is symmetric around $n = N/2 - 1$ for $N$ even or about $n = N/2$ for $N$ odd. For other choices of $|\xi|^2$ the distribution looses this symmetry. Furthermore, we have examined the linear entropy for the su(2) coherent state, given by

$$E_{\text{lin}} = 1 - \text{Tr} \hat{\rho}_R^2,$$

(15)

where $\hat{\rho}_R$ is the reduced density operator for one of the output beams of the beam splitter given by
\[ \hat{\rho}_R = \text{Tr}_1 \rho_1(\zeta, j), \zeta, j \]uber mode 1 or mode 2. The linear entropy vanishes for separable states and maximizes for states that are maximally entangled.

In Fig. 2(a) we display the linear entropy as a function of \( \zeta \) for \( N = 5 \). Regardless of the value of \( j \) the entropy is maximum for \( \zeta = 1 \) as is evident from the graph and verified numerically using MATHEMATICA [12]. One can also calculate the von Neumann entropy [13]

\[ E_{\text{vN}} = -\text{Tr}[\hat{\rho}_R \ln \hat{\rho}_R] \]

which, because our state in Eq. (7) already has the form of a Schmidt decomposition [14], can be written as

\[ E_{\text{vN}} = - \sum_{n=0}^{N} \left( \binom{N}{n} \frac{|\zeta|^{2n}}{1 + |\zeta|^2} \ln \left( \binom{N}{n} \frac{|\zeta|^{2n}}{1 + |\zeta|^2} \right) \right). \]

In Fig. 2(b) we plot the von Neumann entropy as a function of \( \zeta \) for the same value of \( N \) used in Fig. 2(a). We notice that this form of the entropy is also maximized for \( |\zeta| = 1 \). Thus in the balance of the paper, in order to have maximal entanglement between the beams, and in order to get as close to the singlet state as possible with a symmetric joint photon probability distribution, we take \( \zeta = -1 \) so that our state now becomes

\[ |\Psi_{\text{SU(2)}}\rangle = | -1, j \rangle = \left( \frac{1}{\sqrt{2}} \right)^{2j} \sum_{m=-j}^{j} \left( \binom{2j}{j+m} \right)^{1/2} (-1)^{jm} |j,m\rangle_1 \otimes |j,-m\rangle_2. \]

For the special case where \( N = 1 \) we do, in fact, obtain the familiar singlet state for a two spin system

\[ \left| -1, \frac{1}{2} \right\rangle = \frac{1}{\sqrt{2}} \left( \left| \frac{1}{2}, -\frac{1}{2} \right\rangle_1 \otimes \left| \frac{1}{2}, \frac{1}{2} \right\rangle_2 + \left| \frac{1}{2}, \frac{1}{2} \right\rangle_1 \otimes \left| \frac{1}{2}, -\frac{1}{2} \right\rangle_2 \right). \]
\[ | -1 \frac{1}{2} \rangle = \frac{1}{\sqrt{2}}(|0 \rangle_1 \otimes |1 \rangle_2 - |1 \rangle_1 \otimes |0 \rangle_2). \] (20)

The special case of \( N=1 \) has been studied in the context of quantum nonlocality in phase space by Banaszek and Wódkiewicz [15]. For the case with \( j=3/2 \) we have

\[ | -1 \frac{3}{2} \rangle = \frac{1}{2\sqrt{2}} \left[ |0 \rangle_1 \otimes |3 \rangle_2 - |3 \rangle_1 \otimes |0 \rangle_2 \right] \]
\[ \quad + \frac{1}{\sqrt{3}} (|2 \rangle_1 \otimes |1 \rangle_2 - |1 \rangle_1 \otimes |2 \rangle_2), \] (21)

or in terms of the photon number states

\[ | -1 \frac{3}{2} \rangle = \frac{1}{2\sqrt{2}} [ |0 \rangle_1 \otimes |3 \rangle_2 - |3 \rangle_1 \otimes |0 \rangle_2] \]
\[ \quad + \frac{1}{\sqrt{3}} (|2 \rangle_1 \otimes |1 \rangle_2 - |1 \rangle_1 \otimes |2 \rangle_2). \] (22)

We now imagine that the two distant observers of modes 1 and 2 separately apply independent rotations by angles \( \theta_1 \) and \( \theta_2 \), respectively, around the “x” direction, corresponding to independent rotations of two Stern-Gerlach magnets. For an arbitrary state of the form of Eq. (3), we obtain the new state

\[ |\Psi'\rangle = \exp(-i\theta_1 \hat{J}_{1x})\exp(-i\theta_2 \hat{J}_{2x})|\Psi\rangle. \] (23)

Now the observables for each mode are to be the operators

\[ \hat{P}_{1,2}^{(j)} = \exp [i\pi (j - \hat{J}_{1(2)z})]. \] (24)

It is worth pointing out that these operators just parity operators for each of the modes. Parity is an observable that does not seem to have any classical analog. In recent work, one of us, along with Benmousa and Campos [16] have discussed the use of the parity of photon number states for purposes of high resolution (Heisenberg-limited) interferometric measurements of phase shifts. It has also been proposed as an observable for other possible test of local realistic theories involving Bell states given as entangled coherent states [17], for Greenberger-Horne-Zeilinger (GHZ) states that involve a triplet of entangled coherent states [18], and for other system involving continuous variables [19].

We define the correlation between the values of the parity obtained by the two observers as the expectation value of their product taken with the state of Eq. (23):

\[ C(\theta_1, \theta_2) = \langle \Psi' | \hat{P}_{1}^{(j_1)} \hat{P}_{2}^{(j_2)} | \Psi' \rangle \]
\[ = \exp(i\theta_1 \hat{J}_{1x}) \exp(-i\theta_2 \hat{J}_{2x}) \exp(-i\theta_1 \hat{J}_{1x}) \exp(-i\theta_2 \hat{J}_{2x}) \]
\[ \times \exp(-i\pi (\hat{J}_{1x} \cos \theta_1 + \hat{J}_{1y} \sin \theta_1)) \times \exp(-i\pi (\hat{J}_{2x} \cos \theta_2 + \hat{J}_{2y} \sin \theta_2)) |\Psi\rangle. \] (25)

where we have used the operator relation

\[ \exp(i\theta_1 \hat{J}_{1x}) \exp(-i\theta_2 \hat{J}_{2x}) = \hat{J}_{1x} \cos \theta + \hat{J}_{1y} \sin \theta. \] (26)

It is straightforward to show that

\[ \exp[-i\pi (\hat{J}_{1x} \cos \theta + \hat{J}_{1y} \sin \theta)] \]
\[ = \exp(-i\pi \frac{\hat{J}_{1x}}{2}) \exp(-i(2 \theta_2 \hat{J}_{2y}) \exp(-i\pi \frac{\hat{J}_{1x}}{2}), \] (27)

so that after some manipulations we may write

\[ C(\theta_1, \theta_2) = \sum_{m'} \sum_{m''} B_{m'}^{j_1} B_{m''}^{j_2} d_{m',m''}^{(j_1 j_2)} \]
\[ \times \Theta(\theta_2 - \theta_1), \] (28)

where the \( d \)'s are reduced rotation matrix elements defined by

\[ d_{m',m}^{(j)} (\beta) = \langle j, m' | \exp(-i\beta \hat{J}_y) | j, m \rangle \] (29)

and given as [20]

\[ d_{m',m}^{(j)} (\theta) = \sum_{k} (-1)^{j-m+m'} \]
\[ \times \exp \left[ \frac{\pi}{2} \sin \theta \right] \exp \left[ \frac{\pi}{2} \cos \theta \right] \] (30)

where the sum over \( k \) is taken whenever arguments of the factorials in the denominator are non-negative. In what follows we consider two possible states, the first being the spin-singlet state of Eq. (1) with the coefficients

\[ B_{m}^{j} = (2j+1)^{-1/2} (-1)^{j+m} \] (spin-singlet state), (31)

and for the \( su(2) \) coherent state of Eq. (18) where

\[ B_{m}^{j} = \left( \frac{1}{\sqrt{2}} \right)^{2j} \left( \frac{2j}{j+m} \right)^{1/2} (-1)^{j+m}, \] [su(2) coherent state]. (32)

The CHSH form of Bell’s inequality reads [21]

\[ S = |C(\theta_1, \theta_2) + C(\theta_2, \theta_3) + C(\theta_3, \theta_4) - C(\theta_4, \theta_1)| \leq 2. \] (33)

Violations of this inequality indicate nonclassical correlations between the particles that cannot be explained by local realistic alternatives to quantum mechanics.

In the case of the singlet state, its rotational invariance allows one to rewrite Eq. (23) as

\[ |\Psi_{ss}^{j_1} \rangle = \exp(-i\theta_1 \hat{J}_{1x}) \exp(-i\theta_2 \hat{J}_{2x}) |\Psi_{ss} \rangle \]
\[ = \exp(-i\theta_1 \hat{J}_{1x}) \exp(-i\theta_2 \hat{J}_{2x}) |\Psi_{ss} \rangle \] (34)

which is possible because \( \hat{J}_{1x} |\Psi_{ss} \rangle = \hat{J}_{2x} |\Psi_{ss} \rangle \). The correlation functions in the case of the singlet state depend only on the difference angle \( \theta = \theta_2 - \theta_1 \) and as shown by Peres [1] are given by
The CHSH inequality, derived from local hidden variable theories, is not available. Nevertheless, calculations that follow.

verified this result numerically as a check on the numerical calculations that follow.

The CHSH inequality, derived from local hidden variable theory, in this case takes the form

\[ S = |C(\theta_1 - \theta_2) + C(\theta_2 - \theta_3) + C(\theta_3 - \theta_4) - C(\theta_4 - \theta_1)| \leq 2, \]

Taking \(\theta_1 - \theta_2 = \theta_2 - \theta_3 = \theta_3 - \theta_4 = x/(2j + 1)\) one can show that

\[ \lim_{j \to \infty} S = \frac{3 \sin x}{\sin} - \frac{3 \sin x}{3x}, \]

a result that is independent of \(j\) and which maximizes for \(x=1.054\) yielding a saturation value of \(S=2.481\). We have verified this result numerically as a check on the numerical calculations that follow.

For the case of the su(2) coherent state, a closed form for the correlation function is not available. Nevertheless, treating the function \(S=S(\theta_1, \theta_2, \theta_3, \theta_4)\) as the function of the four angles that it is, we have maximized it numerically using MATHEMATICA [12] for increasing values of the spin \(j\). The results are given in Table I for values of \(j\) up to \(j=10\). The special case \(j=1/2(N=1)\) yields the maximal violation of the CHSH inequality allowed by quantum mechanics, \(S=2\sqrt{2}\), as we would expect. But rather unexpectedly, the violations of the inequality, after a low at spin \(j=2\), the value of \(S\) at that value still being rather high at \(S=2.5193\), increases with increasing \(j\). We show this in Fig. 3 where we plot the obtained maximum values of \(S\) as a function of \(j\). Interestingly, all the maximum values of \(S\) obtained with our binomial state, at least out to \(j=29/2\), exceed the value of 2.481 allowed by the corresponding spin-singlet states.

There are some obstacles that need to be overcome in implementing our proposed scheme. The first is with respect to the problem of generating traveling wave field in a photon number state \(|N\rangle\) to inject into the beam splitter which, in turn, generates the state of Eq. (18). One such possible scheme to accomplish this has recently been discussed by Sanaka [22]. This is an interesting proposal involving the extraction of arbitrary photon number states from a coherent state. It is related to the quantum scissors device described by Pegg and co-workers [23] for the production of superposition states of the vacuum and single photon number state.

Another problem is with respect to the issue of implementing the rotations found in Eq. (23). In the case of a true spin system these are just rotations of the Stern-Gerlach magnets. But in our optical version things are not so clear. The operator \(\hat{J}_z\) in one of the modes that we shall generically call the \(a\) mode, has the form

\[ \hat{J}_z = \frac{1}{2}\hat{J}_+ + \frac{1}{2}\hat{J}_- = \frac{1}{2}[(\hat{a}^\dagger(2j-\hat{a}^\dagger\hat{a})^{1/2} + (2j-\hat{a}^\dagger\hat{a})^{1/2}\hat{a}], \]

and it is not too clear how one could fully realize this operator in an optical context. But in the case when \(j\) is large, we may expand it as

\[ \hat{J}_z = \sqrt{j}\left[\hat{a} + \hat{a}^\dagger - \frac{1}{2}(\hat{a}^\dagger\hat{a}^2 - \hat{a}^\dagger\hat{a} + \ldots)\right]. \]

If \(j\) is large enough such that the higher order terms may be dropped we are left with the approximation \(\hat{J}_z = \sqrt{j}(\hat{a} + \hat{a}^\dagger)\), and thus the operator for a rotation about the \(x\) axis by an angle \(\theta\) is approximated by \(\exp(-it\hat{J}_x) = \hat{D}(-it\sqrt{j})\) where

### Table I. Maximum values of \(S\) for increasing \(j=N/2\) along with the corresponding angle settings. These results were obtained numerical using MATHEMATICA.

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<th>(j)</th>
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<th>(\theta_2)</th>
<th>(\theta_3)</th>
<th>(\theta_4)</th>
<th>(S)</th>
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\[ \hat{D}(-i\theta j) = \exp[-i\theta j(\hat{a} + \hat{a}^\dagger)] \quad (40) \]

is the displacement operator. This approximation holds assuming that states with high quantum numbers near \( n = 2j \) are not significantly populated, a condition that holds for our states as is indicated in Fig. 4. The displacement operator of Eq. (40) can be realized, as has been shown by Paris [24] with a beam splitter where the first port is the \( a \) mode and the second, the \( b \) mode, contains a strong coherent field. If we take the beam splitter transformation to be represented by the unitary operator [7]

\[ \hat{U}_{\text{BS}} = \exp[-i\hat{a}\hat{b} + \hat{a}^\dagger \hat{b}^\dagger], \quad (41) \]

where \( \tau \) is related to the transmissivity \( t \) according to \( t = \cos^2 \tau \), we can, assuming that the \( b \) mode is in the strong coherent state \( |\beta\rangle_b \), make the “parametric approximation” of replacing the operators \( \hat{b} \) and \( \hat{b}^\dagger \) by the \( c \)-number amplitudes \( \beta \) and \( \beta^* \), respectively. Thus we have the approximation that

\[ \hat{U}_{\text{BS}} \approx \exp[-i\tau(\hat{a} \beta^* + \hat{a}^\dagger \beta)] = \hat{D}(-i\tau \beta^*). \quad (42) \]

If we are able to adjust the phase of the strong coherent states so that \( \beta \) is real then the beam splitter transformation becomes

\[ \hat{U}_{\text{BS}} = \exp[-i\tau(\beta \hat{a}^\dagger + \hat{a})] = \hat{D}(-i\tau \beta). \quad (43) \]

A more refined approximation to Eq. (43) that takes into account the \( 2\pi \) periodicity in the exact Heisenberg equations of motion is [25]

\[ \hat{U}_{\text{BS}} \approx \exp[-i\beta \sin(\hat{a} + \hat{a}^\dagger)] = \hat{D}(-i\beta \sin \tau). \quad (44) \]

The parametric approximation holds under the conditions that

\[ |\beta| \to \infty, \quad \sin \tau \to 0, \quad |\beta| \sin \tau = \text{constant}, \quad (45) \]

without making any assumption as to the state of the signal in the \( a \) mode. The limit \( \sin \tau \to 0 \) represents a beam splitter whose transmittance approaches 100%. Equation (44) agrees with our Eq. (40) if we make the identification \( \beta \sin \tau = \theta j \). The rotational transformations implemented in the manner described here are performed on each of the output beams of our beam splitter as indicated in Fig. 4.

Finally, there is the issue of parity detection. The obvious way to detect parity is to count the photons and raise \(-1\) to that power and then average. Of course, the obvious problem with this idea is the efficiency and inability to resolve photon counts at the level of a single photon of the currently, and readily, available photon detectors. While there have been innovations recently that have led to progress, the available detectors are sensitive only to low photon numbers. Another possible technique for detecting parity is the use of a switching devices that are sensitive to photon number parity. Yurke and Stoler [26] have described a four-wave mixer operator in a nonlinear regime as a device where the output paths of photons are parity dependent. And Gerry et al. [16(a)] have shown that a Mach-Zehnder interferometer with Kerr media in both arms, a symmetric nonlinear interferometer, can perform the same task. For both devices, it is only necessary to determine the output paths of photons the measure parity and thus the photodetectors themselves can be insensitive to photon number. A possible objection to these approaches is that large third-order susceptibilities are needed. But this may be countered by the recent developments in electromagnetically induced transparency [27] (EIT) which have the ability to greatly enhance these nonlinearities. Enhanced nonlinear susceptibilities will be key to a great variety of interactions required for various optical versions of quantum information processing [28]. Another prospect for parity measurements, also requiring enhanced nonlinear susceptibilities, is that of a quantum nondemolition measurement of parity. This prospect will be discussed in detail elsewhere [29].

In summary, we have discussed an optical realization of a gedanken experiment for probing violations of a Bell inequality in the limit of large spin. An su(2) coherent state obtained from a 50:50 beam splitter with an input number \( N \) and vacuum states can be understood as a two-particle entangled state similar to a spin-singlet for spin \( j = N/2 \). The main difference is that in the beam splitter output state the distribution over the number, or spin, states is a binomial
and that the state is not rotationally invariant. We find that not only are the violations of the Bell inequality are greater than in the case of the spin-singlet state as discussed by Peres [1], but that they even increase with increasing $j$.

[29] C. C. Gerry, A. Benmoussa, and R. A. Campos (to be published).

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