## Spatial Focusing Of Electromagnetic Waves Using The UWB Time Reversal Method

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#### Abstract

Time Reversal techniques enable to concentrate the transmitted power on a certain area, where the receiver is located. In this paper, a closed-form solution for the spatial energy of a time-reversed channel is developed. The theoretical results are compared with experimental measurements at 6.85 GHz with a bandwidth of 7.5 GHz to assess the validity of the UWB TR method.

## 1 Introduction

The popularity of wireless communication technologies has increased the need for reliable, high-speed communications. To cope with the need for high-data rate transmissions, new technologies have been proposed which use large bands of the RF spectrum, such as ultra-wideband (UWB) or 60 GHz communications. These techniques enable to concentrate the transmitted power on a certain area, where the receiver is located. Focusing technologies present multiple advantages such as reduction of the amount of electromagnetic power absorbed by people's bodies, higher reliability of the communication by lowering the rate of interference, etc.

In this paper a new analytic formalism based on physical properties of electromagnetic waves is developed which allows to characterize the quality of spatial focusing Time Reversal. Furthermore a study of the optimization and the compromise between different wave's parameters has been done. Finally, MISO and MIMO experiments are presented to assess the validity of the theoretical results.

## 2 Time Reversal Modeling

The Time Reversal (TR) method is a two-step transmission method. First, the receiver sends a pilot broadcast signal to the transmitter. The transmitter deduces the channel impulse response  $h(\tau)$  by sampling the received (known) signal. The second step consists of the transmission itself: the transmitter filters the signal with the Time Reversal version of the impulse response  $h^*(-\tau)$ . In that case, and if the channel remains static, the receiver will get the transmitted signal through the equivalent channel  $h(\tau)*h^*(-\tau)$ as described in [1, 2, 3]. In this paper, we will consider the total energy over the bandwidth of the signal by integrating the previous formula which has already been used in [4, 5]. Then, to insert a plane wave expansion, we use the Parseval equality (see [6]). If the transmitted signal has a unit energy, the energy of the received signal will be the following:

$$E(\overrightarrow{r}) = \int |h_{\overrightarrow{r}}(\tau) * h^*_{\overrightarrow{r_0}}(-\tau)|^2 d\tau$$

$$= \int_{\Delta f} |H_{\overrightarrow{r}}(f)H^*_{\overrightarrow{r_0}}(f)|^2 df$$
(1)

where  $\Delta f$  is the bandwidth,  $\overrightarrow{r}$  is the position of the evaluated energy, and  $\overrightarrow{r_0}$  is the position where the energy is being focused (the receiver's position). The transfer function, in a local area, can be developed as follows:

$$H_{\overrightarrow{r}}(f) = \sum_{i=0}^{N} a_i e^{j\phi_i} e^{-j\omega\tau_i} e^{-j\overrightarrow{\beta_i}} \overrightarrow{r}$$
(2)

where  $\phi_i$ ,  $\tau_i$  and  $\beta_i$  are respectively the phase, delay and wave number of the  $i^{th}$  plane wave.

First, we make a change of coordinates system by choosing  $\overrightarrow{r_0}$  as the center. That implies  $\overrightarrow{r_0} = \overrightarrow{0}$  and  $\overrightarrow{r} = \overrightarrow{\Delta r}$ . By integrating the energy over the band  $[f_0 - \Delta f/2, f_0 + \Delta f/2]$ , a closed-form expression can be obtained for the spatial distribution of the energy:

$$E(\overrightarrow{r}) = \sum_{i=1}^{N} \sum_{h=1}^{N} \sum_{l=1}^{N} \sum_{m=1}^{N} \sum_{m=1}^{N} \alpha_{ihlm} (\cos(\Phi_1^{ihlm} + 2\Psi_1^{ihlm} f_0) \operatorname{sinc}(\Psi_1^{ihlm} \Delta f) + \cos(\Phi_2^{ihlm} + 2\Psi_2^{ihlm} f_0) \operatorname{sinc}(\Psi_2^{ihlm} \Delta f))$$
(3)

where  $\alpha_{ihlm} = a_i a_h a_l a_m$  and the followings:

$$\begin{aligned}
\Phi_1^{ihlm} &= \phi(i) - \phi(h) - \phi(l) + \phi(m) \\
\Phi_2^{ihlm} &= \phi(i) - \phi(h) + \phi(l) - \phi(m) \\
\Psi_1^{ihlm} &= \pi(\tau(i) - \tau(h) - \tau(l) + \tau(m) + \Delta \overrightarrow{1_{lm}} \frac{\Delta \overrightarrow{\tau}}{c}) \\
\Psi_2^{ihlm} &= \pi(\tau(i) - \tau(h) + \tau(l) - \tau(m) - \Delta \overrightarrow{1_{lm}} \frac{\Delta \overrightarrow{\tau}}{c}).
\end{aligned} \tag{4}$$

It is shown that the focal spot depends on the number of waves N, their angle of incidence, magnitude, phase and delay, the central frequency and the bandwidth. This closed-form solution allows to define the best combination of parameters in order to improve the focusing skills of the TR by studying them one by one and then by selecting the most affecting ones.

# 3 Parameters study

Our goal is now to study the influence of all the parameters in equation (3). Practically, the delays and phases can not be exactly known. They will be considered as random variables and the aim will be to define mean values and describe the conditions to impose on these ones for which the model is suited. For the other parameters, we will try to define optimal values.

### 3.1 Delay spread and environment conditions

In this part, we study the phase and delay spread and their link with the environment. In the previous part, we made an implicit hypothesis: in order to discriminate two waves separated from a definite delay time  $\tau$  in equation (2), the receiver needs to be wide band one.

The most important assumption in this model is to consider that it is possible to find some delays<sup>\*</sup> which are spaced with at least  $1/\Delta f$ . Otherwise, we have to consider a narrow band system for which this model is not suited. We can also show that if this assumption is verified the phases  $\phi$  have no influence on the focusing skills. This assumption imposes a condition on the environment depending on the bandwidth used. The delay spread of probability density function has to be several times greater than  $1/\Delta f$ . It is easily confirmed for indoor communications and ultra wide band systems.

### 3.2 Angular spread

It has been proved in beamforming methods, and it is also true here as we can see in the equations above, that the angular spread has a great influence on the focusing quality. On the figure 1, we have represented, in a system made up of three waves, the radius variation at a relative energy of -1.5 dB by keeping the principal wave and moving the other two.

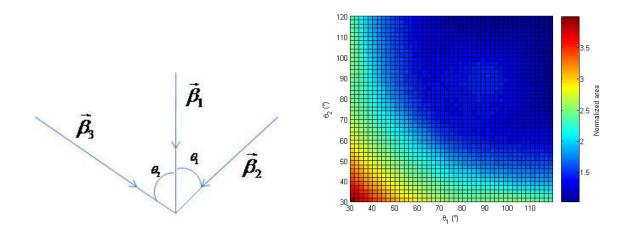


Figure 1: Radius variation term of the angular incidence of the secondary waves with  $f_0 = 6.85$  GHz and  $\Delta f = 7.5$  GHz.

As can be seen on figure 1, the focus area can be several times greater than the optimal case when the angular spread has been decreased. The focus area, at  $\theta_1 = 30^{\circ}$ 

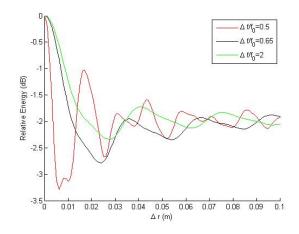
<sup>\*</sup>At least three waves to allow 2D focusing.

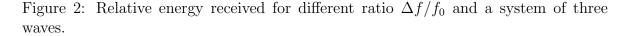
and  $\theta_2 = 30^{\circ}$ , is about four times higher than the case at  $\theta_1 = 120^{\circ}$  and  $\theta_2 = 120^{\circ}$ . We can infer that an optimal focus quality is obtained when the angular spread is maximized. This simulation has been calculated for systems made up of different numbers of wave N and the best quality is always obtain for an uniform angular arrival.

#### 3.3 Center frequency and bandwidth

In this section and the followings, we will define the energy threshold at -1.5 dB to characterize the focus area. Center frequency and band width are coupled parameters and it is difficult to study them independently. It is also important to keep in mind the condition  $\Delta \tau > 1/\Delta f$  for practical experiments or situations.

As can be seen in equation (2), there will be more coherent terms at the focus point and incoherent terms everywhere else when the integration band is increased. It can be proven that using a wider band brings down the secondary energy peaks.





As can be seen on figure 2, we can find the minimum bandwidth and the optimal central frequency for which all the secondary peaks are below the threshold energy and show that it is obtained by a rough relationship:  $\Delta f_{thres}=0.644f_0$ , the radius of the focus area is equal to the physical limit of TR:  $0.18\lambda_0$  where  $\lambda_0=c/f_0$ . To improve the discrimination between the principal and the secondary peaks, the band has to be widen.

In practical cases, the frequency band will be fixed<sup>†</sup> and the central frequency would be optimized by choosing  $f_0 = \Delta f / 0.644$ .

#### **3.4** Number of waves N

This model can be applied to ray tracing softwares but we have not yet define the practical limit of the plane wave expansion in equation (2). First, we can show that the focus radius is increased at most one percent by adding waves but on the contrary, the discrimination is higher. So in practical cases, it will be hard to decrease the number of waves by changing the environment while increasing them would be easier by

<sup>&</sup>lt;sup>†</sup>To have the highest resolution, all the frequency band available will be used.

obstructing the line-of-sight (see 4. Measurement Campaign). We have proven that after a dozen of waves uniformly shared out around the receiver's location, the spatial energy distribution almost does not vary.

### 3.5 Conclusion

The parameters can be categorized into physical and system resources. The bandwidth and central frequency have to be optimized while they are fixed by the devices used. The physical parameters such as the number of wave and the angular spread do not have the same influence on the energy distribution. In conclusion, in order to have the best resolution the angular diversity will be increased by using a MISO system as can be seen on figure 3. The number of waves will influence the discrimination and in practical situations, will be the highest possible.

# 4 Measurement Campaign

#### 4.1 Measurement setup

Experimental measurements were performed to assess the performances of the TR method. The channel frequency responses were measured with a four-port VNA. Three transmit base station were considered, and the receiver was moved on a 10 cm square grid with an automatic positioning device. Since all base stations are in line-of-sight, the channel can be reduced to a channel composed of three waves incident on the receiver. The TR is applied as a posterior treatment.

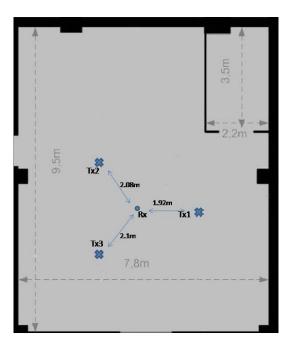


Figure 3: Schematic top view of the experimental unit. The crosses represent the positions of the base stations, the dot represents the center of the grid.

The experiment has been done on a bandwidth from 3.1 GHz to 10.6 GHz by a step of 7.5 MHz that implies in our calculations:  $f_0 = 6.85$  GHz and  $\Delta f = 7.5$  GHz.

In order to reduce the statistical error we have not five snapshots per measurement. Two kinds of experiments have been realised, the first created a three waves system by applying the configuration of figure 3. The second one had the same configuration except that the line-of-sight has been obstructed to create more strong waves. The results presented in the following part have been done in the second configuration.

#### 4.2 Measurement results

An example of the spatial distribution of the energy by applying TR is given in Figure 5. It can be seen that the energy is concentrated on the focal spot that has a radius of about  $0.2\lambda$ . The location of the focal spot can be modified, and similar results are observed when changing the receiver's position. On Figure 5, a focalization example with TR for multiple receivers is shown (multiple focal spots).

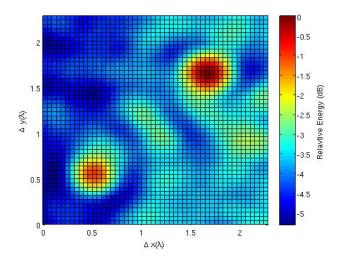


Figure 4: Relative energy received for three base stations. The signal is sent to two locations simultaneously.

The obstruction of the line-of-sight is affecting the channel by creating artificially more strong waves than the three ones described above. As predicted, the radius of the focus area is not modified but the discrimination is increased. These results can be compared with the theoretical conclusions and assess the validity of our TR model.

### 4.3 Comparison with theory

By using a ray tracing model, we could evaluate approximate values of the different wave parameters in order to compare the results with the experiment. This model has been applied in the configuration of the figure 3 with the same system resources as the experiment.

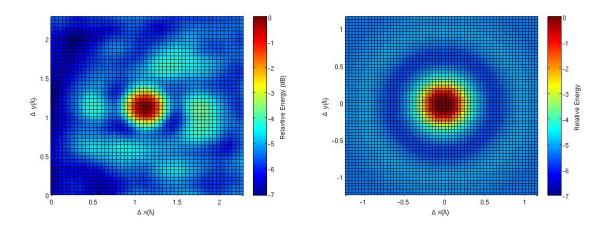


Figure 5: Relative energy received for three base stations. Comparison between, on the left, the experiment result and on the right, the theorical calculations.

As can been seen, the energy level and the area around the focal spot fit with the theorical experiment. We have to remind that it is a closed-form model so the energy far from the focus point is not well described.

## 5 Conclusion

UWB Experiments proved the validity of the energy distribution in a closed area around the reveiver position described by our modeling. It has also predicted the physical limit  $(0.2\lambda)$  of the radius of the focal spot with the system resources available ( $f_0 = 6.85GHz$ and  $\Delta f = 7.5GHz$ ).

We have also applied TR in different environments to testify our model and the results were successfull. We have also shown that the evolution of the energy distribution follows our equations if the parameters described at section 3, are varying.

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