JOINT PRODUCTION AND PRICE UNCERTAINTY:
HYPOTHESIS TESTS

MOAWIA ALGHALITH*
(UNIVERSITY OF THE WEST INDIES)

ABSTRACT:
This paper extends the existing estimation methods to allow empirical estimation and hypothesis testing under joint production and price uncertainty. Our approach modifies and expands the use of duality theory. Furthermore, our approach does not require the specification or estimation of the production/cost function. We apply the methodology to the U.S. manufacturing sector.

JEL CLASSIFICATION: D8, D2.

KEYWORDS: estimating equations, hypotheses testing, output uncertainty, price uncertainty, utility.

* Econ. Dept., St. Augustine Trinidad & Tobago, malghalith@gmail.com
INTRODUCTION

In the absence of hedging, there is hardly any literature that provides hypothesis testing under joint production and price uncertainty. Several studies focused on hedging under price uncertainty, including Arshanapalli and Gupta (1996), Rolfo (1980), and Lapan and Moschini (1994). They included price uncertainty but excluded production uncertainty. They derived estimating equations by applying uncertainty analogues of Hotelling’s lemma and Roy’s identity to the indirect expected utility function. However, as will be apparent later in this paper, their method is not directly applicable to the case of multiple sources of uncertainty.

Other studies included both price and output uncertainty in the presence of hedging. For example, Li and Vukina (1998) showed that dual hedging under price and output uncertainty reduces the variance of the income of corn farmers. Rolfo (1980) computed the hedge ratio for cocoa producers. Lapan and Moschini (1994) calculated the same ratio for soya bean farmers.


Unlike the previous literature, this paper has two major contributions. First, our approach modifies and expands the use of duality theory in the sense that duality theory can be utilized when two sources of uncertainty (price and output uncertainty) are present. It is worth noting that the traditional duality theory has been applicable to only one source of uncertainty (price uncertainty). Second, our approach does not require the specification or estimation of the production/cost function. This vastly simplifies the estimation process. In so doing, we devise a method that enables us to empirically estimate a simultaneous price and production uncertainty model; moreover, we develop methods to test important hypotheses regarding the functional form and the attitudes toward risk, such as risk neutrality, separability, and constant absolute risk aversion, CARA.

The paper is organized as follows. Section 1 presents the theoretical model. Section 2 discusses the modification of the duality theory and develops the estimating equations. Section 3 constructs the hypothesis tests. Section 4 discusses the empirical results. The final section provides concluding remarks.
1. THE MODEL

A competitive firm faces an uncertain output price given by \( p = \bar{p} + \sigma \varepsilon \), where \( \varepsilon \) is random with \( E[\varepsilon] = 0 \) and \( \text{Var}(\varepsilon) = 1 \), so that \( E[p] = \bar{p} \) and \( \text{Var}(p) = \sigma^2 \). The level of output realized at the end of the production process is also not known ex ante. Output has both a random and a nonrandom component and is given by \( q \) where \( q \) is random and defined as \( q = y + \theta \eta \) where \( \eta \) is random with \( E[\eta] = 0 \) and \( \text{Var}(\eta) = 1 \), so that \( \text{Var}(q) = \theta^2 \) and the expected value of output is \( y \) (see Alghalith and Dalal (2002) and Lapan and Moschini (1994), among others). We assume that \( \varepsilon \) and \( \eta \) are independent. This assumption is empirically verified in the Conclusion. Costs are known with certainty and are given by a cost function, \( c(y, w) \). While \( y \) represents expected output, it may usefully be thought of as the level of output which would prevail in the absence of any random shocks to output. The firm may be thought of as having \( y \) as its target level of output and committing inputs that would generate this level in the absence of any random shocks. The cost function is then the minimum cost of producing any arbitrary output level \( y \) given the input price vector \( w \). Thus, the profit is \( p = pq - c(y, w) \). The firm is risk-averse and maximizes the expected utility of the profit

\[
\text{Max } E \left[ U(\pi) \right] = E \left[ U(pq - c(y, w)) \right]
\]

2. ESTIMATING EQUATIONS

The maximization problem implies the existence of an indirect expected utility function \( V \), such that

\[
V(\bar{p}, \sigma, \theta, w, B) = E \left[ U \left( p(y^* + \theta \eta) - c(y^*, w) + B \right) \right]
\]

where \( y^* \) is the optimal value of \( y \) and \( B \) is a shift parameter; such shift parameters have been used in theoretical work by Dalal (1990) and have been exploited for empirical estimation by Appelbaum and Ullah (1997). Let \( \pi^* \) represent the value of \( \pi \) corresponding to \( y^* \). The envelop theorem applied to (1) implies

\[
\frac{\partial V}{\partial \bar{p}} = V_p = y^* E \left[ U'(\pi^*) \right] + \theta E \left[ U'(\pi^*) \eta \right]
\]

and

\[
\frac{\partial V}{\partial B} = V_B = E \left[ U'(\pi^*) \right],
\]

so that (2) and (3) imply

\[
\frac{V_p}{V_B} = y^* + \frac{\theta E \left[ U'(\pi^*) \eta \right]}{V_B}.
\]
In a model without output uncertainty, (4) would provide a basis for deriving an estimating equation for \( y^* \), since we would have \( \eta = 0 \), and then
\[
y^* = \frac{V_p}{V_B}.
\]
However, since \( \eta \) is random, this doesn't work, and hence a different procedure is needed to circumvent this problem.

Consider this approximation around the arbitrary point of expansion \( \hat{\pi} \)
\[
U'(\pi^*); U'(\hat{\pi}) + U^*(\hat{\pi})(\pi^* - \hat{\pi})
\]
Multiplying through by \( \eta \) and taking expectations of both sides,
\[
E[U'(\pi^*)\eta]; U'(\hat{\pi})E[\eta] + U^*(\hat{\pi})E[\pi^*\eta] = U^*(\hat{\pi})\theta \bar{\rho}.
\]
(5)

Since \( \hat{\pi} \) is a constant, \( U^*(\hat{\pi}) \) is a parameter which can be estimated. Letting \( \beta \equiv U^*(\hat{\pi}) \) we can approximate \( E[U'(\pi^*)\eta] \) by \( \beta \theta \bar{\rho} \), substituting this into (4) yields
\[
y^* = \frac{V_p - \beta \theta^2 \bar{\rho}}{V_B}.
\]
(6)

In order to get expressions for \( V_p \) and \( V_B \), we need to have an expression for \( V \). Since the form of the indirect expected utility function is not known, we approximate it by a second-order Taylor's series expansion about the arbitrary point \( A = (\hat{p}, \hat{\sigma}, \hat{\theta}, \hat{w}, \hat{B}) \); letting subscripts denote partial derivatives and taking the partial derivatives of \( V \) with respect to \( \bar{p} \) and \( \bar{B} \), respectively, we obtain
\[
V_p(\bar{p}, \sigma, \theta, \omega, B) = V_p(A) + \sum_i V_{pi}(A)(w_i - \hat{w}_i) + V_{pp}(A)(\bar{p} - \hat{p})
\]
\[
+ V_{pa}(A)(\sigma - \hat{\sigma}) + V_{pB}(A)(\theta - \hat{\theta}),
\]
(7)

\[
V_B(\bar{p}, \sigma, \theta, \omega, B) = V_B(A) + V_{pB}(A)(\bar{p} - \hat{p})
\]
\[
+ \sum_i V_{Bi}(A)(w_i - \hat{w}_i) + V_{\sigma B}(A)(\sigma - \hat{\sigma})
\]
\[
+ V_{\sigma \theta}(A)(\theta - \hat{\theta}).
\]
(8)

Using (7) and (8), we can rewrite (6) as
\[
y^* = \frac{V_p(A) + \sum_i V_{pi}\hat{w}_i + V_{pp}\hat{\sigma} + V_{pa}\hat{\theta} - \beta \theta^2 \bar{\rho}}{V_B(A) + V_{pB}\hat{\rho} + \sum_i V_{Bi}\hat{w}_i + V_{\sigma B}\hat{\sigma} + V_{\sigma \theta}\hat{\theta}},
\]
(9)

where tildes symbolize deviations from the point of approximation and all the first and second partial derivatives of \( V \) are evaluated at the point of expansion, \( A \) and \( B \) is set equal to its initial value of 0. All the derivatives of \( V \) as well as \( \beta \) in (9) are the parameters to be estimated. However, for estimation purposes, some normalization is required since
is homogeneous of degree zero in all the parameters. A convenient normalization is $V_B(A) = 1$. Thus (9) becomes

$$y^* = \frac{V_p(A) + V_{pp}\bar{p} + \sum V_{pi}\hat{\omega}_i + V_{p\sigma}\bar{\sigma} + V_{p\theta}\bar{\theta} - \beta \theta^2 \bar{p}}{1 + V_{p\bar{p}}\bar{p} + \sum V_{bi}\hat{\omega}_i + V_{\sigma\sigma}\bar{\sigma} + V_{\theta\theta}\bar{\theta}}$$  \hspace{1cm} (10)

3. Hypothesis Testing

We will use (10) to develop hypothesis tests for Pope's (1980) separable utility function, risk neutrality, and CARA.

**Separable Utility Function.** This has the form $U(\pi) = a\pi - b(\pi - \pi)^2$, so that $U'(\pi) = a - 2b(\pi - \pi)$ and $E[U'(\pi)] = a$ (a constant). This implies that $V_B$ is a constant since $E[U'(\pi)] = V_B$ and thus

$$V_{Bp} = V_{Bi} = V_{Bo} = V_{B\sigma} = V_{B\theta} = 0.$$  \hspace{1cm} (11)

**Risk Neutrality.** In this case $U' = 0$. This implies that $U$ is constant and hence is $E[U'(\pi)]$. Thus $V_B$ is constant, and all its partial derivatives are equal to zero. Thus (11) applies. In addition, $E[U(\pi)] = a + bE[\pi]$, implying that $E[U'(\pi)]$ is independent of both price risk and output risk. This implies $V_{\sigma} = V_{\theta} = 0$, and hence all the partial derivatives of $V_{\sigma}$ and $V_{\theta}$ are also 0. Therefore, $V_{p\sigma} = V_{p\theta} = 0$. Furthermore, since $\beta$ is equal to $U^*(\hat{\pi})$, we must also have $\beta = 0$. Thus, the parameter restrictions implied by risk neutrality are

$$V_{Bp} = V_{Bi} = V_{Bo} = V_{p\sigma} = V_{p\theta} = \beta = 0.$$  \hspace{1cm} (12)

**Constant Absolute Risk Aversion.** Recall that $V_B = E[U'(\pi^*)]$; differentiating with respect to $\sigma$ yields

$$V_{Bo} = -kE\left[U'(\pi)(p - c_\gamma)\frac{\partial y^*}{\partial \sigma} + q\epsilon\right],$$

since with CARA $U''(\pi) = -kU'(\pi)$. From the first-order condition, $E[U'(\pi)(p - c_\gamma)] = 0$, and thus

$$V_{Bo} = -kE[U'(\pi)q\epsilon] = -kV_{o\sigma}.$$  \hspace{1cm} (13)

Now $E[U'(\pi)q\epsilon] = \frac{1}{\sigma}E[U'(\pi)q(p - \bar{p})] = \frac{1}{\sigma}Cov(U'(\pi)q, p) < 0$. When evaluated at the point of expansion, (13) becomes

$$V_{Bo}(A) = -kV_{o\sigma}(A)$$  \hspace{1cm} (14)
and since $V_\sigma(A)$ is negative, it is clear that (14) implies $V_{B\sigma}(A)$ is positive. Thus constant absolute risk aversion implies $V_{B\sigma}(A) > 0$. We can test for $V_{B\sigma}(A) = 0$; if we cannot reject this hypothesis then we will reject CARA.

4. EMPIRICAL RESULTS

We need to generate data series for $y^*, p, \alpha, \theta$ in order to estimate (10). Since these are not directly observable we have to generate these values from observable data. There is some arbitrariness in the method chosen to do so, since there is no unambiguously “best” approach. Some empirical studies have adopted an extremely simple approach such as Arshanapalli and Gupta (1996), who used a simple moving average process, while others use much more complex methods.

In order to generate a series of expected prices, we use an expanded version of the method developed by Chavas and Holt (1996) where the price at time $t$ is considered as a random walk with a drift. Thus,

$$p_t = \delta + \alpha p_{t-1} + \epsilon_t,$$

where $p_t$ is the price at time $t$, $p_{t-1}$ is the previous year's market price, $\delta$ is a drift parameter, and $\epsilon_t$ is a random variable with $E[\epsilon_t] = 0$. Hence

$$E_t[p_t] = \delta + \alpha p_{t-1}.$$

Similarly, to generate a series for $y^*$, we use the method developed by Lapan and Moschini (1994), and model output at time $t$ by

$$q_t = \phi + \varphi q_{t-1} + u_t,$$

where $q_t$ is the output at time $t$, $q_{t-1}$ is the previous year's output, and $u$ is an error term with $E[u] = 0$. Hence,

$$E[q_t] = y_t^* = \phi + \varphi q_{t-1}.$$

To generate a series for $\sigma$ we will also use Chavas and Holt’s method:

$$\sigma^2_t = \sum_{j=1}^{3} \omega_j \left(p_{t-j} - E_{t-j} p_{t-j}\right)^2,$$

where the weights $\omega_j$ are 0.5, 0.33 and 0.17. This is done to reflect the idea of declining weights. The price variance is thus measured as the weighted sum of squared deviations of the previous prices from their expected values. Similarly,

$$\theta^2_t = \sum_{j=1}^{3} \omega_j \left(q_{t-j} - y_{t-j}^*\right)^2,$$
We used annual time series data (for the period 1947-2000; N=54) pertaining to the aggregate manufacturing sector of the U.S. The aggregate manufacturing output ($y^*$) is produced using four inputs: materials ($m$), energy ($e$), capital ($k$), labor ($l$), with prices given, respectively, by $w_m$, $w_e$, $w_k$, and $w_l$. Gross output price is given by $p$. Gross output price and quantity data are taken directly from the worksheets of the U.S. Department of Commerce, Bureau of Economic Analysis. The quantity and the price of each input are derived or taken from Department of Census, Bureau of Economic Analysis. The data set is constructed using Berndt and Wood's methods (see Berndt and Wood (1986) for a detailed description). Since the focus of this study is the U.S. aggregate manufacturing sector, we ruled out panel data. Rewriting the estimating equations to explicitly introduce the 4 input prices we will be using, (10) becomes

$$y^* = \frac{V_p(A) + V_{pe} \tilde{p} + V_{pm} \tilde{w}_m + V_{pl} \tilde{w}_l + V_{pk} \tilde{w}_k + V_{pm} \tilde{w}_m + V_{pk} \tilde{w}_k + V_{pa} \tilde{\sigma} + V_{pa} \tilde{\theta} - \beta\theta^2 \tilde{p}}{1 + V_{pl} \tilde{p} + V_{pm} \tilde{w}_m + V_{pl} \tilde{w}_l + V_{pl} \tilde{w}_k + V_{pk} \tilde{w}_k + V_{sa} \tilde{\sigma} + V_{sa} \tilde{\theta}}; \quad (15)$$

Before we proceeded with the estimation, we generated data series for $\varepsilon$ and $\eta$ (using Chavas and Holt's method) and tested the independence assumption and we strongly accepted the null hypothesis that $\varepsilon$ and $\eta$ are independent. We also performed outlier analysis. We used a non-linear least squares regression to estimate (15). The results are presented in Table 1. The non-linear least squares regression yields maximum likelihood estimates and thus we can use the likelihood ratio test to test the hypotheses.

We first tested for risk neutrality and we strongly rejected it. The results of the estimation appear in Column 5 of Table 1. Then, we tested for separability. The results of estimation appear in Column 4 of Table 1. The hypothesis is rejected at .05 significance level. To test for CARA, we need to implement an indirect test since we have an inequality restriction ($V_{B\sigma} > 0$). Thus we will first test for $V_{B\sigma} = 0$. If we accept this hypothesis, we will reject CARA. We cannot reject the hypotheses and hence we reject CARA. The results are reported in Column 3 of Table 1. The model's fit is excellent as indicated by the values of $\chi^2$ (see Table 1); the significance level at which the alternative hypothesis, that each parameter equals 0, would be rejected is .005. Thus we present this model as our final estimating form.

**CONCLUSION**

We empirically estimated a simultaneous price and production uncertainty model. In so doing, we developed methods to test important hypotheses regarding the functional form and the attitudes toward risk, such as risk neutrality, separability and CARA. We strongly rejected risk neutrality; this indicates that uncertainty exists in the U.S. manufacturing sector. We also rejected common functional forms of the utility: separability and CARA. The unrestricted model seems to fit the data well. This implies that a more general form of preferences describes the manufacturing sector. This result is consistent with the theoretical foundations of decisions under uncertainty, since these forms of the utility function (separability and CARA) are very restrictive. They are usually employed in theoretical and empirical literature for convenience. Moreover, our results are intuitive in the sense that the manufacturing sector is expected to exhibit more sophisticated preferences. The generality of the preferences is a major strength of this paper.
REFERENCES


## APPENDIX

### Table 1. Estimation results

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unrestricted</th>
<th>$V_{B\sigma} = 0$</th>
<th>Separability</th>
<th>Risk N.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_p^{-} (A)$</td>
<td>623.12</td>
<td>623.1</td>
<td>625.32</td>
<td>603.78</td>
</tr>
<tr>
<td></td>
<td>(6.275)</td>
<td>(6.6)</td>
<td>(5.530)</td>
<td>(5.601)</td>
</tr>
<tr>
<td>$V_{pp}$</td>
<td>-331.18</td>
<td>-790.01</td>
<td>-684.78</td>
<td>-857.71</td>
</tr>
<tr>
<td></td>
<td>(992.77)</td>
<td>(1072.36)</td>
<td>(138.36)</td>
<td>(121.67)</td>
</tr>
<tr>
<td>$V_{pcs}$</td>
<td>-3646.17</td>
<td>-1106.92</td>
<td>-33.64</td>
<td>-121.75</td>
</tr>
<tr>
<td></td>
<td>(1632.37)</td>
<td>(631.36)</td>
<td>(215.21)</td>
<td></td>
</tr>
<tr>
<td>$V_{p\theta}$</td>
<td>-1.249</td>
<td>1.311</td>
<td>-1.507</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.757)</td>
<td>(2.133)</td>
<td>(.4807)</td>
<td></td>
</tr>
<tr>
<td>$V_{pm}$</td>
<td>-510.56</td>
<td>-757.19</td>
<td>435.81</td>
<td>123.87</td>
</tr>
<tr>
<td></td>
<td>(924.83)</td>
<td>(983.8)</td>
<td>(98.58)</td>
<td>(103.77)</td>
</tr>
<tr>
<td>$V_{pe}$</td>
<td>-45.72</td>
<td>261.81</td>
<td>-66.71</td>
<td>-29.91</td>
</tr>
<tr>
<td></td>
<td>(243.54)</td>
<td>(307.92)</td>
<td>(15.23)</td>
<td>(11.93)</td>
</tr>
<tr>
<td>$V_{pk}$</td>
<td>300.68</td>
<td>293.41</td>
<td>-10.35</td>
<td>-33.88</td>
</tr>
<tr>
<td></td>
<td>(190.18)</td>
<td>(243.33)</td>
<td>(41.66)</td>
<td>(57.06)</td>
</tr>
<tr>
<td>$V_{pl}$</td>
<td>1042.79</td>
<td>1423.41</td>
<td>672.97</td>
<td>978.46</td>
</tr>
<tr>
<td></td>
<td>(991.54)</td>
<td>(10115)</td>
<td>(103.08)</td>
<td>(78.85)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-2024E-01</td>
<td>-.247</td>
<td>.1584E-02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.1281E-01)</td>
<td>(.1472)</td>
<td>(.7777E-02)</td>
<td></td>
</tr>
<tr>
<td>$V_{Bp^{-}}$</td>
<td>.8085</td>
<td>-.2099</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.484)</td>
<td>(1.654)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{B\sigma}$</td>
<td>-4.806</td>
<td>(.207)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{B\theta}$</td>
<td>.1912</td>
<td>.5755</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.2773)</td>
<td>(.3352)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{Bm}$</td>
<td>-1.365</td>
<td>-1.863</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.226)</td>
<td>(1.36)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{Be}$</td>
<td>.4508</td>
<td>.3707</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.2995)</td>
<td>(.3725)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{Bk}$</td>
<td>.4863</td>
<td>.4894</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.2676)</td>
<td>(.2807)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{Bl}$</td>
<td>.4180</td>
<td>1.2783</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.496)</td>
<td>(.028)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-L</td>
<td>-121.747</td>
<td>-123.069</td>
<td>-129.449</td>
<td>-140.5</td>
</tr>
<tr>
<td># of rest.</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>2.64</td>
<td>3.84</td>
<td>15.4</td>
<td>37.5</td>
</tr>
<tr>
<td>crit. $\chi^2$</td>
<td>14.07</td>
<td>18.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Table 2. Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>$q$</th>
<th>$w_m$</th>
<th>$w_k$</th>
<th>$w_e$</th>
<th>$w_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVG</td>
<td>1.061</td>
<td>640.01</td>
<td>1.059</td>
<td>1.657</td>
<td>0.897</td>
<td>0.920</td>
</tr>
<tr>
<td>SD</td>
<td>1.032</td>
<td>339.707</td>
<td>1.163</td>
<td>4.955</td>
<td>0.857</td>
<td>1.254</td>
</tr>
</tbody>
</table>