THE PRODUCTION FUNCTION APPROACH TO THE BELGIAN OUTPUT GAP, ESTIMATION OF A MULTIVARIATE STRUCTURAL TIME SERIES MODEL

PHILIPPE MOËS*
(NATIONAL BANK OF BELGIUM, RESEARCH DEPARTMENT)

ABSTRACT:

A multivariate structural time series model is applied to the factor inputs of a production function to estimate the Belgian output gap. The number of independent cycles and the frequencies are not restricted a priori. Phase shifts are introduced to allow for leads and lags. Over 1983-2004, a 3.5 years periodicity is found in the cycles. The cycles in the participation and unemployment rates are negligible. Two independent cycles hide behind the cycles of the other variables: hours, TFP and capacity utilization. Phase shifts are significant, with hours leading by as much as 3 quarters and capacity utilization lagging.

JEL CLASSIFICATION: C32, E32.

KEYWORDS: Business cycle, output gap, phase shifts, structural time series models.

INTRODUCTION

For policymakers, the output gap is a useful measure for assessing the amount of slack in the economy. Although the reliability of output gap estimates computed in real time has been questioned by Orphanides and Van Norden (2002) in a monetary policy context, cyclically adjusted budget balances - where budget balances are corrected for the impact of cyclical fluctuations as measured by the output gap - still figure prominently in the stability and convergence programmes updated annually by EU Member States. The recent reform of the Stability and Growth Pact has given an even more prominent role to the cyclically adjusted figures. The history of output gaps is also used to compute "minimum benchmarks" for budget balances ensuring a safety margin against breaching the 3 p.c. reference value under adverse economic circumstances.

Not only in the European Union but in many international organizations and countries alike, the production function approach has emerged as the preferred method for estimating the output gap. In this approach, the output gap is decomposed between factor inputs, which can be further split into several components. As mentioned by Cotis et al. (2005), this decomposition allows a much richer analysis of economic fluctuations and is very useful to monitor the complement to the output gap, i.e. potential output, or its developments, i.e. the growth potential.

Several techniques may be used to compute the cycles of the components. The OECD (Giorno et al., 1995) and the European Commission (Denis et al., 2006) work component by component whereas the ECB (Proietti et al., 2002; Rünstler, 2002) favours multivariate approaches where some commonality is imposed between cycles. In combining the information coming from different variables, it should be possible to derive a better estimate of the business cycle. This will be the case if links exist between factors or components. Rünstler (2002) shows that the combination of information yields more reliable estimates in real time. But it must be kept in mind that the benefits coming from multivariate models would stay an illusion if cycles were not well linked together. A biased estimate would result.

In this paper, a multivariate approach is adopted. Multivariate Structural Time Series (MSTS) models are specified and estimated. They are well grounded in econometric theory (Harvey, 1989). Their starting point is the breakdown of time series into unobserved components: a trend, a cycle and an irregular component, quite appropriate for the purpose at hand. The unobserved components are found by Kalman filtering and the whole system can be estimated by maximum likelihood. This type of model lends itself easily to econometric testing and a research strategy can be designed to look for the best model.

Different specifications are available for trends and cycles. For the trend, we will test two different non nested specifications to check the robustness of the results. There are also many different ways to link cycles. Simplifying, two types of models can be distinguished: models that impose a single common cycle at the outset and symmetric models, which allow one cycle per variable, at least initially:

- The Okun law is very popular in the first type of models (Clark, 1989; Apel and Jansson, 1999; Scott, 2000; Fabiani and Mestre, 2004). It is used to get more precise estimates of the output and unemployment gaps. The Okun law comes from a production function where it is assumed that, over the cycle, unemployment, hours worked, participation rate and productivity move more or less in line. As a result, output and unemployment share a common cycle. Other single cycle models give a central role to variables outside the production function, which are assumed to convey particularly useful information about the cycle, such as capacity utilization (Scott, 2000; Rünstler, 2002; Proietti et al., 2002).[1]

- Symmetric MSTS models that include one cycle per variable are documented in Harvey and Koopman (1997). The cycles are not completely independent as they share some common parameters, mainly a common periodicity. This does not imply that the cycles are common among variables. The idea is to determine the number of truly independent cycles hiding behind the cycles of the variables introduced in the model. If there is only one, then variables share a common cycle. An application of this type of model to the production function can be found in Proietti et al. (2002).

We favour the second approach where a common cycle is not imposed at the outset on the data. If the common cycle restriction is invalid, the estimated cycle would be biased. As a consequence, the Okun law will not be imposed on the system. Unemployment, hours worked, participation rate and total factor productivity will all be present with their own cycle (at least initially). Capacity utilization will be added to the system in order to provide extra information but it will be one variable among others, introduced in a symmetric way. It will not receive a dominant status in the determination of "the" cycle. The existence of a single common cycle will however be put to the test.

A final issue is the question of leading or lagging cycles. It is often dealt with in an ad hoc way, introducing lags where they are believed to exist. The symmetric MSTS models could not deal with lags initially. Rünstler (2004) showed how the models could be refined to overcome the problem. The leads and lags are estimated simultaneously with the other parameters, without introducing any a priori restriction. Applications can be found in Koopman and Azevedo (2004) and Azevedo et al. (2006). We will test for the presence of leading or lagging cycles in our models.

The rest of the paper is organised as follows. The next section describes the production function approach and introduces the MSTS model. Section 2 will present the results. Section 3 will check for the existence of a single common cycle and test some restrictions on the lag structure. Section 4 will come back to the results for the unemployment rate.

---

[1] In principle, capacity utilisation could be introduced into the production function. But it is only measured in the manufacturing sector and it is not clear whether the responses refer to the level of the capital stock or to the output level.

1. THE PRODUCTION FUNCTION APPROACH TO THE OUTPUT GAP

The production follows a standard Cobb-Douglas function with constant returns

$$Y_t = TFP_t \left(L_t HOURS_t \right)^\alpha K_t^{1-\alpha}$$

with $\quad L_t = PWA_t PART_t \left(1 - UR_t \right)$

where PWA is the working age population, PART is the participation rate and UR is the unemployment rate. The labour share α is set at .65, the standard used by the European Commission. TFP, the total factor productivity, is computed as a residual. If a variable is missing in the production function, this will affect TFP.

We assume that each element is the sum of three components: a trend, a cycle and an irregular component. The two elements, ln(K) and ln(PWA), are considered acyclical and made of a single component, the trend.

$$\text{For} \quad z_t = \begin{pmatrix} \ln HOURS_t \\ \ln PART_t \\ \ln TFP_t \\ \ln(1 - UR_t) \end{pmatrix} \equiv \begin{pmatrix} LHOURS_t \\ LPART_t \\ LTFP_t \\ UR\min_t \end{pmatrix}$$

$$z_t = \mu_t + C_t + \varepsilon_t \quad \text{with } \varepsilon_t \sim \text{NID}\left(0, \Sigma_\varepsilon\right) \tag{1}$$

where μ is the vector of trend components, C, the vector of cycle components and ε, the vector of irregular components.

The output gap is the weighted sum of the cycle components:

$$C_t^{LY} = C_t^{LTFP} + \alpha \left(C_t^{LPART} + C_t^{UR\min} + C_t^{LHOURS} \right) \tag{2}$$

To allow for connections between variables and more especially between their cycles, we model the components from equation (1) along the lines of Multivariate Structural Time Series (MSTS) models. In these models the cycles are assumed to share some common properties (see below) and the number of truly independent cycles may be lower than the number of variables in the system. This introduces strong commonalities between cycles. In these commonalities lies the econometric benefit of a multivariate approach to the output gap.

It is possible to add extra variables to the system to improve the measurement of cycles further. In this paper, capacity utilisation (LDUC, taken in logarithm) is added to vector z_t as it is considered a good -although imperfect- proxy of the business cycle. It is worth noting that contrary to what is usually done, LDUC is treated as any other variable

meaning that a stochastic trend may appear in LDUC. In Belgium, capacity utilization may be trending (see chart 1).[2] Just-in-time technology improvements could permanently raise the level of capacity utilization.

CHART 1. PRODUCTION FUNCTION APPROACH, THE DATA
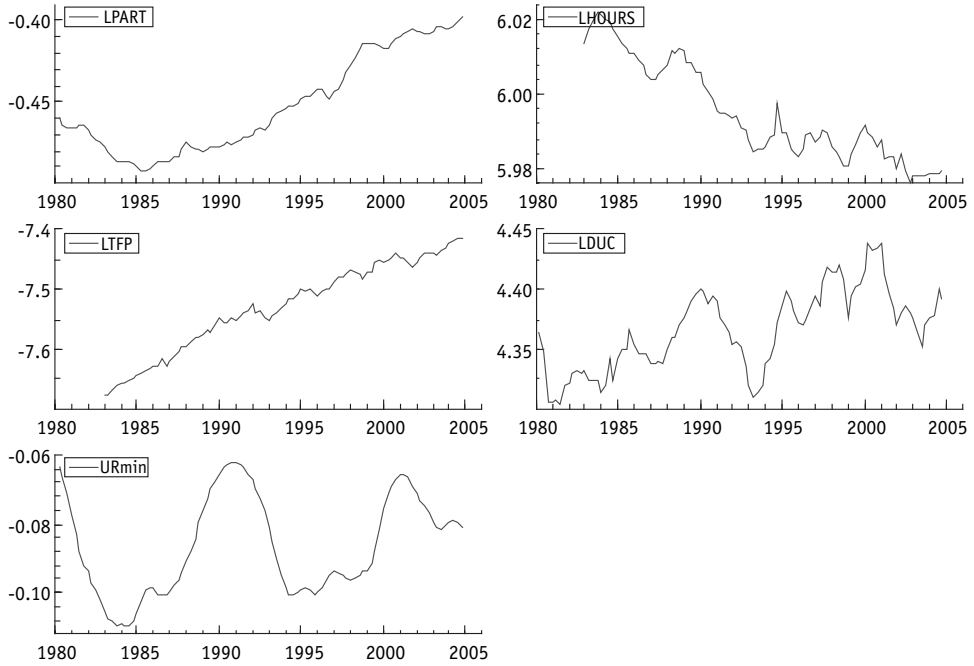
But there is another reason to introduce a trend in LDUC. Often in the literature, capacity utilization gets no trend component and its cycle, close to the variable itself, is introduced in the other variables to account for their cycle. In such a setting, the introduction of capacity utilization compels the other cycles to replicate its movements closely and the measured output gap is often quite close to capacity utilization. The introduction of a stochastic trend together with the possibility to have different cycles will help prevent such an outcome.

Regarding the data used in this paper, the unemployment rate is the harmonized rate of unemployment. Hours are based on published data from the National Accounts Institute over 1995-2002 and on NBB's own calculations for the other periods. All data are seasonally adjusted.

---

[2] Repeated Chow breakpoint tests suggest that a break occurred in the level of capacity utilization around 1994.

1.1. Specification of the multivariate structural time series model

Trends and cycles in (1) adopt standard specifications (see Harvey, 1989, or Harvey and Koopman, 1997). To introduce phase shifts between cycles, we will follow the approach suggested by Rünstler (2004).

The trends are defined as integrated random walks (IRW)

$$\mu_{t+1} = \mu_t + \beta_t$$
$$\beta_{t+1} = \beta_t + \xi_t \qquad \xi_t \sim \text{NID}\left(0, \Sigma_\xi\right) \tag{3a}$$

This specification, where the (N x 1) vector of slopes β follows a random walk process, produces smooth trends. It implies that the variables are I(2) if the variances are different from zero. We also tested the damped slope (DS) specification introduced by Proietti et al. (2002) that produces relatively smooth trends although the variables remain I(1):

$$\mu_{t+1} = \mu_t + \beta_t + m$$
$$\beta_{t+1} = \theta\beta_t + \xi_t \qquad \xi_t \sim \text{NID}\left(0, \Sigma_\xi\right) \tag{3b}$$

m is a vector of drift constants and θ is a diagonal matrix of damping factors with $0 < \theta_{ii} < 1$. When θ_{ii} is close to zero, the corresponding trend will be close to a RW with drift.

For the cycles present in C, we first define N independent or "structural" cycles Ψ (and their "companion" cycles Ψ^*) with the standard trigonometric form

$$\begin{pmatrix} \psi_{t+1} \\ \psi_{t+1}^* \end{pmatrix} = \rho\left(\begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \otimes I_N\right)\begin{pmatrix} \psi_t \\ \psi_t^* \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa_t^* \end{pmatrix} \qquad \begin{pmatrix} \kappa_t \\ \kappa_t^* \end{pmatrix} \sim \text{NID}\left(0, I_2 \otimes \Sigma_\kappa\right) \tag{4}$$

with damping factor $0 < \rho < 1$, frequency $0 < \lambda < \pi$ and Σ_k diagonal to preserve independence. The cycle periodicity is equal to $2\pi/\lambda$. All the structural cycles share a common frequency parameter λ and a common damping factor ρ. But the two parameters are estimated rather than being imposed a priori.[3]

The interdependence between the C cycles present in the variables follows from

$$C_{t+1} = F\psi_t + F^*\psi_t^* \tag{5}$$

---

[3] In the frequency domain, imposing identical parameters between cycles is tantamount to imposing common band pass filters to extract cycles. Specification (4) comes close to the specification of "Butterworth" band pass filters (see Harvey and Trimbur, 2003). Frequency bands will depend on ρ and λ but also on the variances involved. Here, the variances are left free and the common parameters ρ and λ will be estimated.

$$\text{with } F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ F_{21} & 1 & 0 & 0 & 0 \\ F_{31} & F_{32} & 1 & 0 & 0 \\ F_{41} & F_{42} & F_{43} & 1 & 0 \\ F_{51} & F_{52} & F_{53} & F_{54} & 1 \end{pmatrix} \text{ and } F^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ F_{21}^* & 0 & 0 & 0 & 0 \\ F_{31}^* & F_{32}^* & 0 & 0 & 0 \\ F_{41}^* & F_{42}^* & F_{43}^* & 0 & 0 \\ F_{51}^* & F_{52}^* & F_{53}^* & F_{54}^* & 0 \end{pmatrix} \quad (6a)$$

With $F^* = 0$, (5) is equivalent to the "similar cycles" specification of Harvey and Koopman (1997). F is a Cholesky matrix that mixes the independent structural cycles Ψ together to produce "similar" C cycles that may be correlated. However, Rünstler (2004) observed that no phase shifts were present between similar cycles: the highest correlation between cycles is found *before* introducing leads or lags. This is unfortunate if we believe that some variables are leading of lagging. To allow for phase shifts (leads or lags between C cycles), it is necessary to give some weight to the cycles Ψ^*, introducing an F* matrix different from zero. With $F^* \neq 0$, (5) is equivalent to the "Choleski decomposition" of Rünstler (2004).

What is the link between the structural cycles and the C cycles? Looking at individual cycles (we omit t for simplicity), C_1, the cycle of the first variable, will be equal to Ψ_1. The first structural cycle is the cycle of the first variable in the system. This structural cycle may be present in the other variables as well if $F_{i,1} \neq 0$. If $F^*_{i,1} \neq 0$, it will be shifted in variable i. The second structural cycle Ψ_2 is the only other structural cycle present in C_2 so it will be the residual cycle on that variable. Again, it may be present in the remaining variables, with or without shift. We may continue the reasoning until the last structural cycle is found.

The term "structural" does not mean that an economic interpretation is given to the cycles. It is immediate that a different ordering of the variables will produce other structural cycles although the C cycles remain unaffected. This is always the case with Cholesky decompositions. In this analysis, "structural" points to the independence existing between cycles in (4), not to an economic interpretation.

As mentioned before, the number of structural cycles may be lower than N, the number of variables. This will be the case if the C cycles are strongly interrelated. In the limit case, a single common cycle may be shared by all the variables, with or without phase shifts. Then the F and F* matrices will have an N x 1 dimension. We do not impose a priori restrictions on the number of structural cycles governing the system. Neither do we define some variables as necessarily leading or lagging. All variables are treated symmetrically and the phase shifts emerge freely from the estimation of matrix F*. In appendix 1, the state-space form of the model is given for the damped slope specification.

1.2. A PARTICULAR CASE, THE ADDITIVITY OF LEADS AND LAGS

It is worth noting that with more than one structural cycle, it is often impossible to place the variables on a unique time line defining in a coherent way all the bilateral leads (or lags) between variables. The bilateral leads will instead be given by a matrix that does not necessarily verify the additivity property. That is, if variable Y1 leads variable Y2 by 2 periods and Y2 leads Y3 by 3 periods, the lead of Y1 on Y3 will not necessarily equal 5.

Although somewhat disturbing, the lack of additivity is a welcome property because the "problem" is faced in practice: when computing leads based on maximum correlations between variables (cross-correlogram) there is usually no unique time line, the results depending on the reference variable.

A special case of additivity is when there is a single common cycle plus idiosyncratic cycles on all variables (but one). It follows that the lack of additivity is an indication that there is more than one common cycle among variables.

We will test additivity by imposing constraints on the F* matrix of shifts. The idea is simple: if the same timing is found on each structural cycle, additivity will apply. To impose a common timing, the relative weights of all the companion cycles, responsible for the phase shifts, must be equal. For a given variable i,

$$\frac{F^*_{ij}}{F_{ij}} = \frac{F^*_{ik}}{F_{ik}} = \phi_i \quad \text{or} \quad F^*_{ij} = \phi_i F_{ij}$$

for each structural cycle j (k) present in i.[4] Or in matrix form,

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ F_{21} & 1 & 0 & 0 & 0 \\ F_{31} & F_{32} & 1 & 0 & 0 \\ F_{41} & F_{42} & F_{43} & 1 & 0 \\ F_{51} & F_{52} & F_{53} & F_{54} & 1 \end{pmatrix} \text{ and } F^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \phi_2 F_{21} & \phi_2 & 0 & 0 & 0 \\ \phi_3 F_{31} & \phi_3 F_{32} & \phi_3 & 0 & 0 \\ \phi_4 F_{41} & \phi_4 F_{42} & \phi_4 F_{43} & \phi_4 & 0 \\ \phi_5 F_{51} & \phi_5 F_{52} & \phi_5 F_{53} & \phi_5 F_{54} & \phi_5 \end{pmatrix} \quad (6b)$$

---

[4] To check the restriction, the cycle j contribution to variable i $F_{ij}\psi_{jt} + F^*_{ij}\psi^*_{jt}$, can be written as

$$\delta_{ij}\left(\cos(p_{ij}\lambda)\psi_{jt} + \sin(p_{ij}\lambda)\psi^*_{jt}\right) \text{ with } p_{ij} = \frac{1}{\lambda} arctg\left(\frac{F^*_{ij}}{F_{ij}}\right) \text{ and } \delta_{ij} = sign(F_{ij}) \sqrt{F^2_{ij} + F^{*2}_{ij}}$$

The trigonometric form is similar to (4). It implies that the cycle in variable i is leading the Ψ_{jt} structural cycle by p_{ij} periods (p_{ij} times the one-period rotation λ). δ_{ij} is a scaling factor. Imposing a common timing on each structural cycle implies $p_{ij} = p_{ik} \Leftrightarrow \frac{F^*_{ij}}{F_{ij}} = \frac{F^*_{ik}}{F_{ik}} = \phi_i$.

To fix a reference point, no phase shift is introduced on the cycle present in the first variable ($\phi_1 = 0$). The system (4) to (6b) is equivalent to the cycle model of Koopman-Azevedo (2004) where additivity also holds.

Additivity introduces $(N - 1)(N - 2) / 2$ constraints on F*. Coming from $N (N - 1) / 2$ bilateral leads (or lags), we are left with only $(N - 1)$ leads (or lags) with respect to the first variable, which allows to derive all the bilateral leads. If the number of structural cycles is lower than N, the number of coefficients present in F* without additivity will drop and the number of constraints will fall accordingly. With one common cycle, additivity implies no further constraint.

2. ESTIMATION RESULTS

The models are estimated over the period 1983Q1-2004Q4 using the SSFPack algorithms for Ox 3.0 (Doornik, 1998; Koopman et al., 1998).[5] To initialize the Kalman filter, cycle and slope components are set to zero; trends are set to the value of the first observation. The initial variance-covariance matrix of the state vector is the unconditional variance for stationary variables. For non stationary variables, it is set to a very large value (diffuse prior). To derive reasonable starting values for the coefficients, extensive grid searches were conducted on univariate models -one per variable- preliminary to their estimation. The univariate coefficients were then used to initialize the multivariate model, assuming diagonal variance-covariance matrices. Some judgement is still necessary as λ and ρ may differ between variables.[6]

In the first subsection, phase shifts are left aside (F* = 0) and only similar "contemporaneous" cycles are considered. In the second subsection, F* is set free and leads and lags may appear between cycles.

2.1. SIMILAR CYCLES

The results for similar cycles without phase shifts can be found in the two first columns of table 1, respectively for the trends modelled as integrated random walks (IRW) and for the trends following the damped slope (DS) specification in (3b).

As usual, the goodness of fit statistics are based on the one-step ahead prediction errors. RD2 is the coefficient of determination R^2, computed on the first difference of the variables.[7] The Normality test statistic is the Doornik - Hansen statistic (1994), distributed

---

[5] More information is available on the website http://www.ssfpack.com/ where the package can be downloaded.

[6] To check the sensitivity of the results to the procedure, an extensive grid search experiment was run at the multivariate level (damped slope case) respecting the constraint of single λ and ρ coefficients. A bit less than 15 millions points were evaluated in the parameter space. Using the best point as starting value, the procedure converged to the optimum found under the "univariate" approach.

[7] More precisely, RD2 is equal to 1 minus the ratio of the variance of the one-step ahead prediction errors to the variance of the first differences of the variable.

as $X^2(2)$. The heteroscedasticity test is the non-parametric test found in Koopman et al. (1995). It is distributed as $F(29,29)$. The Ljung-Box statistic is based on the 12 first autocorrelations.

Normality is never a problem.[8] There is some evidence of heteroscedasticity in the LDUC errors and there is autocorrelation in the LDUC and URmin errors. The problem is less pronounced on LHOURS and LPART, especially for the DS model.

TABLE 1. ESTIMATION RESULTS

	similar cycles		phase shifts	
	IRW	DS	IRW	DS
cycle				
λ (frequency)	0.45	0.44	0.46	0.43
periodicity	3.5 years	3.5 years	3.4 years	3.7 years
ρ (damping factor)	0.90	0.86	0.91	0.88
2*SD (in p.c.)				
LDUC	2.51	2.40	2.73	2.44
LHOURS	0.48	0.33	0.52	0.43
LPART	0.39	0.09	0.42	0.11
LTFP	1.26	1.11	1.33	1.25
URmin	0.05	0.03	0.09	0.07
trend				
slope θ				
LDUC	1.00	0.75	1.00	0.79
LHOURS	1.00	0.54	1.00	0.43
LPART	1.00	0.68	1.00	0.65
LTFP	1.00	0.83	1.00	0.90
URmin	1.00	0.98	1.00	0.99
fit				
Log-Likelihood	1934.13	1992.99	1945.28	2001.31
# coefficients	53	63	63	73
RD2				
LDUC	0.300	0.459	0.436	0.489
LHOURS	0.313	0.412	0.345	0.496
LPART	0.484	0.530	0.501	0.529
LTFP	0.385	0.400	0.346	0.425
URmin	0.819	0.824	0.820	0.822
Normality				
LDUC	3.12	0.85	3.52	0.40
LHOURS	2.70	0.47	0.60	2.41
LPART	3.27	1.26	3.81	1.38
LTFP	2.26	1.47	0.56	2.58
URmin	3.96	4.19	3.73	4.36

[8] We computed the "auxiliary" residuals - the smoothed estimates of the shocks - to detect outliers. Applying a benchmark of 3.5 times the standard deviation, the following outliers were removed: LDUC(1984q3, 1999q1), LHOURS(1994q4), LPART(1987q4, 1988q1), LTFP(1992q1), URmin(1986q2).

TABLE 1. (CONTINUED)

	similar cycles		phase shifts	
	IRW	DS	IRW	DS
Heteroscedasticity				
LDUC	1.92	1.97	1.32	1.89
LHOURS	0.99	1.39	0.94	1.15
LPART	1.28	1.50	1.35	1.53
LTFP	1.49	1.40	1.40	1.58
URmin	1.24	1.28	1.11	1.21
Ljung-Box Q(12)				
LDUC	19.10	18.49	22.69	23.74
LHOURS	18.48	6.31	12.42	8.25
LPART	15.64	11.59	15.81	11.23
LTFP	6.18	7.91	6.92	7.15
URmin	26.82	29.33	28.73	31.73

The coefficients of determination are much better on LDUC and LHOURS in the DS case. Estimated freely, the θ slope coefficients are clearly below unity on 4 out of 5 variables. URmin is the exception: with θ at 0.98, the damped slope specification comes close to an IRW trend. But for URmin, the DS specification better matches the data. Our comments will focus on this specification.

The smoothed cycles can be seen on chart 2. The length of the cycle is estimated at 3.5 years.

CHART 2. SMOOTHED SIMILAR CYCLES - (DS TRENDS)
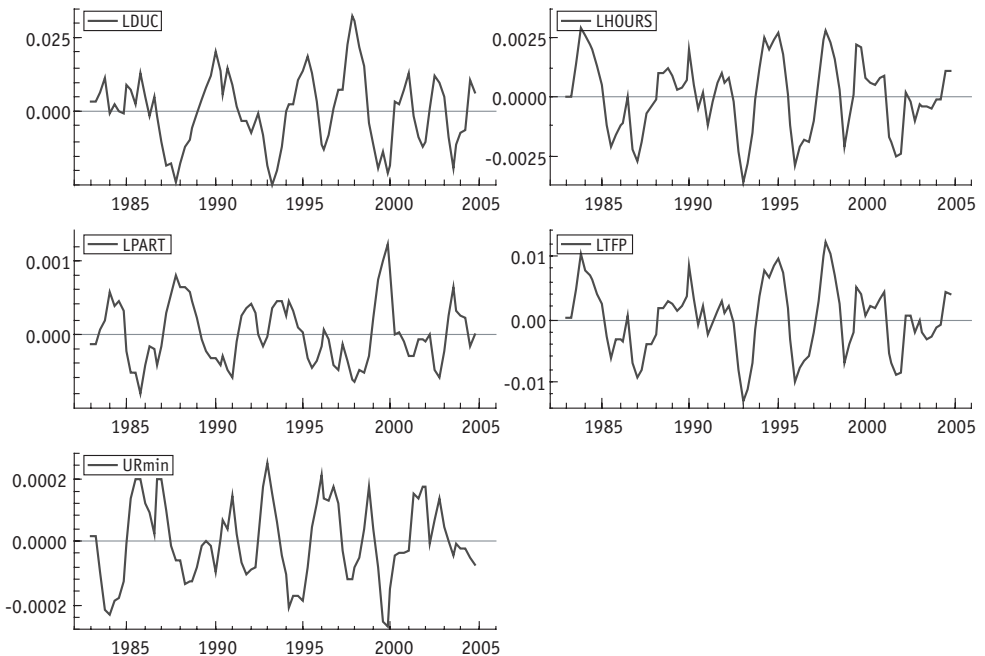
The cycle found on URmin is tiny and does not reflect the long waves in unemployment that are identified as pertaining to the trend. The cycle on LPART has very limited amplitude as well. In table 1, the standard deviations of the cycles (times 2) are computed to give an idea of the cycles amplitude. The cycle found on LDUC is markedly different from raw capacity utilization. Its amplitude is also much lower. Finally, the cycles on LTFP and LHOURS look quite similar but have different amplitude.

To assess the number of structural cycles that hide behind the cycles of the five variables, we decompose the variance of the cycles between the five "structural" cycles Ψ. We get the following results:

TABLE 2. STRUCTURAL CYCLES SHARES IN CYCLE VARIANCE - (DS TRENDS)

	$\Psi 1$	$\Psi 2$	$\Psi 3$	$\Psi 4$	$\Psi 5$
LDUC	1.000	0.000	0.000	0.000	0.000
LHOURS	0.130	0.870	0.000	0.000	0.000
LPART	0.514	0.486	0.000	0.000	0.000
LTFP	0.317	0.683	0.000	0.000	0.000
URmin	0.001	0.999	0.000	0.000	0.000

The matrix is lower triangular by construction, like the F matrix in (6a). Only the first two structural cycles matter to generate the cycles on all the variables. As a consequence, there is a link between the resulting cycles and the multivariate approach may benefit from it. The cycle in LTFP is the result of the 2 structural cycles $\Psi 1$ and $\Psi 2$ that explain 100 p.c. of the LDUC and LHOURS cycle variances. It is the same for the cycles on LPART and URmin but these cycles are negligible. We are left with 2 structural cycles for 3 variables with a significant cycle. To give an economic interpretation based on the Cholesky decomposition is dangerous as such decomposition is not unique. For the order LDUC - LHOURS - LPART - LTFP - URmin, the first two structural cycles are each responsible for a very high percentage of the variance of LDUC and LHOURS. So, for the lack of a better denomination and with the previous caveat in mind, the two cycles $\Psi 1$ and $\Psi 2$ could be named "LDUC" and "LHOURS" cycles.

In terms of correlations, we get the following results:

TABLE 3. CORRELATIONS OF CYCLES - (DS TRENDS)

	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	1	0.361	-0.717	0.563	-0.034
LHOURS	0.361	1	0.391	0.974	-0.944
LPART	-0.717	0.391	1	0.172	-0.673
LTFP	0.563	0.974	0.172	1	-0.845
URmin	-0.034	-0.944	-0.673	-0.845	1

As the LPART and URmin cycles are negligible, the correlations of the corresponding rows and columns do not have much sense and will not be discussed. As expected, there is a very high correlation between the cycles in LTFP and LHOURS. The LDUC cycle is correlated with both of them. For a part, the link between LDUC and LTFP cycles could be the result of a measurement error in LTFP since LDUC is absent from the production function when TFP is computed.

In the IRW case, the major difference is the presence of a significant cycle in LPART (cycles and tables are given in appendix 2). Three structural cycles are now present but the third one only matters for LPART. It turns out that the LPART cycle is completely independent of the others. Moreover, the LHOURS cycle is not correlated with the cycle in LDUC anymore and less correlated with the LTFP cycle.

Coming back to the DS case, we can also compute correlations between the slopes of the trends, β, since they are stationary:

TABLE 4. CORRELATIONS OF TREND SLOPES – (DS TRENDS)

	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	1	-0.106	0.242	0.774	0.334
LHOURS	-0.106	1	-0.415	-0.029	0.172
LPART	0.242	-0.415	1	-0.338	-0.108
LTFP	0.774	-0.029	-0.338	1	0.420
URmin	0.334	0.172	-0.108	0.420	1

There is a strong correlation between the LTFP and LDUC trend slopes, suggesting that permanent changes in LDUC could be linked to changing productivity paces. It could also result from a measurement problem. There is also some correlation between the trend slopes of the two variables and the trend slope of URmin. It is negatively affected by the high θ coefficient found on URmin: shocks that affect the slope of LTFP and LDUC for some time seem to affect the slope of URmin for a protracted period, levelling down the high correlation present between shocks. It is worth noting that in providing some information on the trend affecting LTFP and LDUC, URmin also contributes to the determination of the cycles present in LTFP and LDUC. So although it has only a tiny cycle, URmin contributes indirectly to the estimation of the other cycles.

For the sake of completeness, we can also mention that the trend slope of LHOURS is independent of the trend slope in the previous variables and that the slope of LPART appears to be negatively correlated with the slopes of LTFP and LHOURS.

A drawback of similar cycles is the assumption that cycles reach their highest correlation at the current period, making no allowance for leads and lags between cycles. The next subsection will introduce phase shifts to test this restriction.

2.2. CYCLES WITH PHASE SHIFTS

Thanks to the introduction of F* in (6a), the phase shifts are estimated freely, together with the other parameters. The estimation results are given in the right part of table 1. In the IRW model, the phase shifts are significant at the 95 p.c. level. The likelihood ratio test gives a value of 22.30 against 18.31 for the $X^2(10)$. In the DS model, the phase shifts are significant at the 90 p.c. level.[9] The absence of shifts is rejected. Heteroscedasticity is now less of a problem on LDUC but there is no improvement in terms of autocorrelation. Allowing a damped slope specification for trends produces results equivalent to those found for the model without phase shifts. Except for URmin, all the θ slope coefficients move away from 1, with strong improvements on the coefficients of determination for LHOURS and LTFP. Again, the DS specification better matches the data.

The smoothed cycles can be seen on chart 3 (DS case). The cycle period is barely affected. The amplitude of cycles is a bit larger but the cycles of URmin and LPART are still negligible. Phase shifts are not the solution to the lack of cycle and we will not comment further on the two cycles.

CHART 3. SMOOTHED CYCLES WITH PHASE SHIFTS - (DS TRENDS)

---
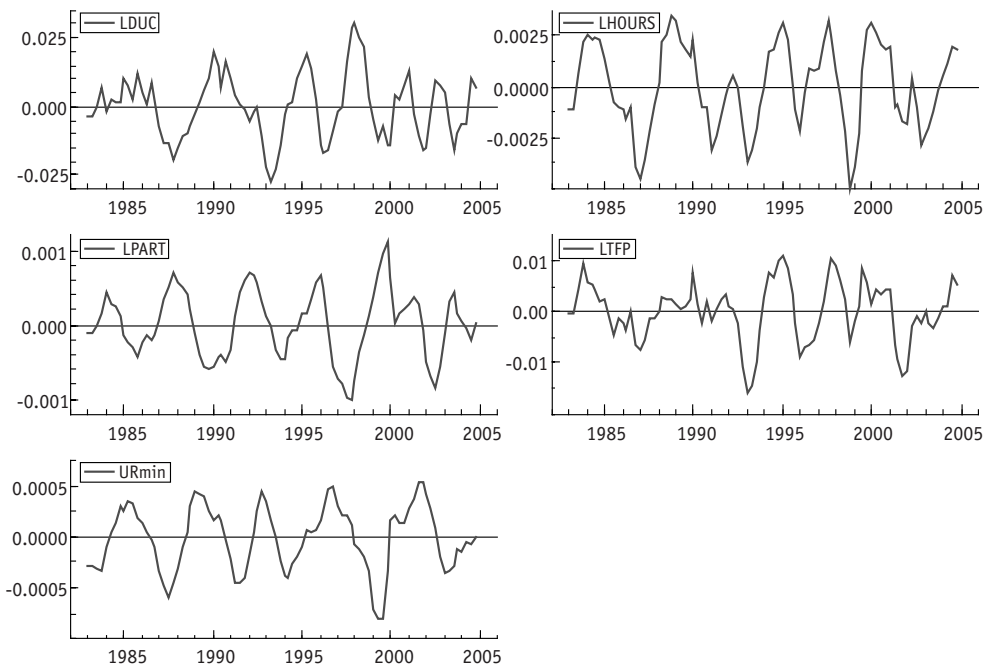
[9] If one takes into account the limited number of structural cycles, the dimension of F* goes down and the phase shifts are significant at much higher levels.

The cycle on LHOURS is quite different from the cycle found without phase shifts. Now it differs from the LTFP cycle. If we decompose the variance of the cycles between structural cycles, we get the following results:

TABLE 5. STRUCTURAL CYCLES SHARES IN CYCLE VARIANCE - (DS TRENDS)

	$\Psi 1$	$\Psi 2$	$\Psi 3$	$\Psi 4$	$\Psi 5$	$\Psi 1^*$	$\Psi 2^*$	$\Psi 3^*$	$\Psi 4^*$
LDUC	1.000								
LHOURS	0.070	0.480				0.450			
LPART	0.131	0.449	0.000			0.306	0.114		
LTFP	0.425	0.145	0.000	0.000		0.116	0.314	0.000	
URmin	0.001	0.053	0.000	0.000	0.000	0.033	0.914	0.000	0.000

There are still 2 structural cycles Ψ (and their companion cycles Ψ^*) behind the cycles. Companion cycles responsible for phase shifts matter and represent a large share of the cycle variance on many variables. With a share of 0.450 in LHOURS coming from its companion cycle, the first structural cycle will generate a closer link between the LDUC and LHOURS cycles once leads (or lags) are taken into account.

The phase shifts between cycles appear in the next table.[10] By normalization, the phase shifts are always within bounds equal to ± 25 p.c. of the cycle period. With a period of 3.7 years (14.6 quarters), the shifts will be lower than 3.7 quarters (in absolute value).

TABLE 6. PHASE SHIFTS BETWEEN CYCLES (LAG OF I WITH RESPECT TO J) IN QUARTERS
        - (DS TRENDS)

i \ j	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	0	2.8	2.3	1.1	-3.3
LHOURS	-2.8	0	-3.6	0.1	-2.7
LPART	-2.3	3.6	0	3.6	2.9
LTFP	-1.1	-0.1	-3.6	0	1.7
URmin	3.3	2.7	-2.9	-1.7	0

Leaving URmin aside, LDUC is lagging by several quarters, from 1.1 to 2.8. Part of the lag on LDUC may be attributed to a publication lag. LDUC is published in January, April, July and October, making reference to the capacity in the month before. As a consequence, a decay of about 2 months or 0.7 quarter will result in our quarterly setting. LHOURS and LTFP are generally leading.

---

[10] Their computation follows Rünstler (2004).

Although LHOURS is leading LDUC by more than LTFP, LHOURS is almost contemporary with LTFP from a bilateral point of view. The relative position of variables over the cycle apparently depends on the reference variable considered. The validity of additivity restrictions will be further investigated in section 3.

Until now, the procyclical or countercyclical character of cycles has not been checked. Rünstler (2004) shows how to compute levels of "association" between cycles. Associations give a measure of correlations after adjusting for phase shifts.

TABLE 7. ASSOCIATION BETWEEN CYCLES - (DS TRENDS)

	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	1	0.721	-0.661	0.735	-0.183
LHOURS	0.721	1	-0.331	0.666	0.657
LPART	-0.661	-0.331	1	0.484	-0.839
LTFP	0.735	0.666	0.484	1	-0.545
URmin	-0.183	0.657	-0.839	-0.545	1

All the non trivial cycles are positively correlated with more or less the same degree of association, 0.70. With the introduction of shifts, the cycles in LDUC and LHOURS are more strongly correlated. The three quarters lead of LHOURS explains the difference with table 3. The correlation of LDUC with LTFP improved as well.

In the IRW case, a cycle is also present on LPART (chart and tables can be found in appendix 3). Three structural cycles are enough to generate the cycles on LDUC, LHOURS, LPART and LTFP. LDUC is still lagging with respect to LHOURS and LTFP. Additivity is again not verified. Without phase shifts, the cycle in LPART was independent from the other cycles. Now there is some *negative* correlation with LDUC and LHOURS meaning that LPART is weakly countercyclical. There is a positive association between the three other cycles.

Coming back to the DS case, the correlation between trend slopes may again be derived. Phase shifts do not change the picture and the previous comments still apply. A robust result is the correlation between the trend slopes of LDUC and LTFP (and URmin to a lesser extent). This is due to the high correlation between their slope shocks.

TABLE 8. CORRELATIONS OF TREND SLOPES - (DS TRENDS)

|	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	1	-0.131	0.060	0.741	0.383
LHOURS	-0.131	1	-0.256	0.024	0.082
LPART	0.060	-0.256	1	-0.425	-0.098
LTFP	0.741	0.024	-0.425	1	0.571
URmin	0.383	0.082	-0.098	0.571	1

2.3. THE OUTPUT GAP

From the previous analysis, we know that phase shifts are present and significantly different from zero. Moreover, the damped slope specification for the trend can degenerate into the IRW specification, which proved useful for the unemployment rate. As a consequence, the damped slope model with phase shifts will be the "preferred" model, used to discuss the Belgian output gap and to test restrictions on the cycles in the next section.

The output gap is computed with formula (2). Given the negligible cycles found on LPART and URmin in the preferred model, only the cycles on LTFP and LHOURS matter. Among them, the fluctuations in LTFP are dominant. The correlation between the output gap and the LTFP cycle amounts to 0.99 (0.81 with LHOURS).

The output gap is noticeably different from the cycle on LDUC, for which the correlation drops to 0.59 (see chart 4). Previous results showed that a second structural cycle is present in LTFP, on top of the structural cycle in LDUC. In the preferred model, it is responsible for 46 p.c. of the variance in the LTFP cycle. If the LDUC cycle is shifted backward with one quarter, the correlation with the output gap improves somewhat to 0.64.

CHART 4. OUTPUT GAP, LTFP AND LDUC CYCLES (SMOOTHED CYCLES WITH PHASE SHIFTS - DS TRENDS) IN PERCENTAGE POINTS
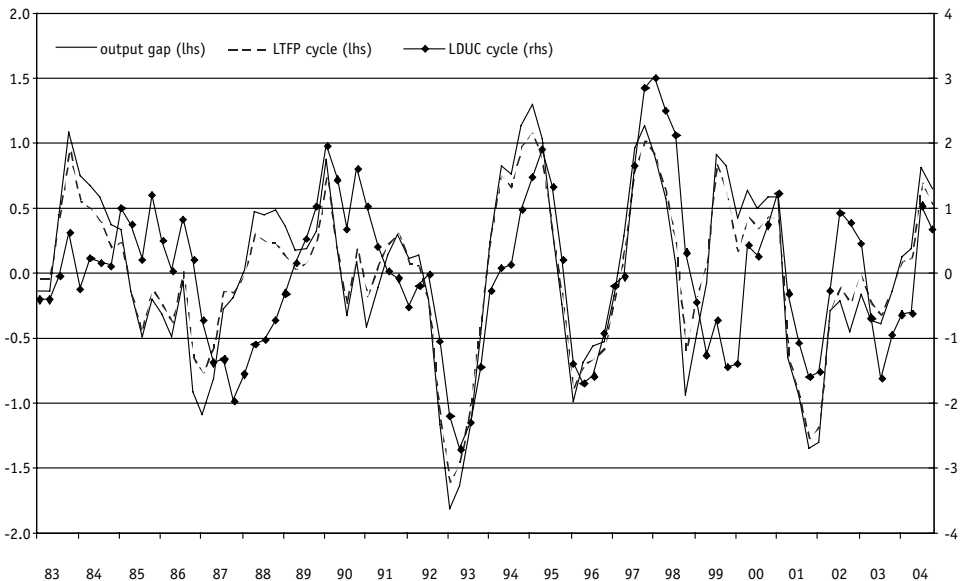
It is interesting to compare on chart 5 the output gap with the business survey indicator of the National Bank of Belgium.[11] The indicator was not included in the estimations but since 1988, a close link exists between the two variables. The correlation is equal to 0.76 (0.63 over the whole sample). Like the output gap, the confidence indicator is affected by short cycles.

CHART 5. OUTPUT GAP AND NBB BUSINESS SURVEY INDICATOR (STANDARDIZED VALUES)
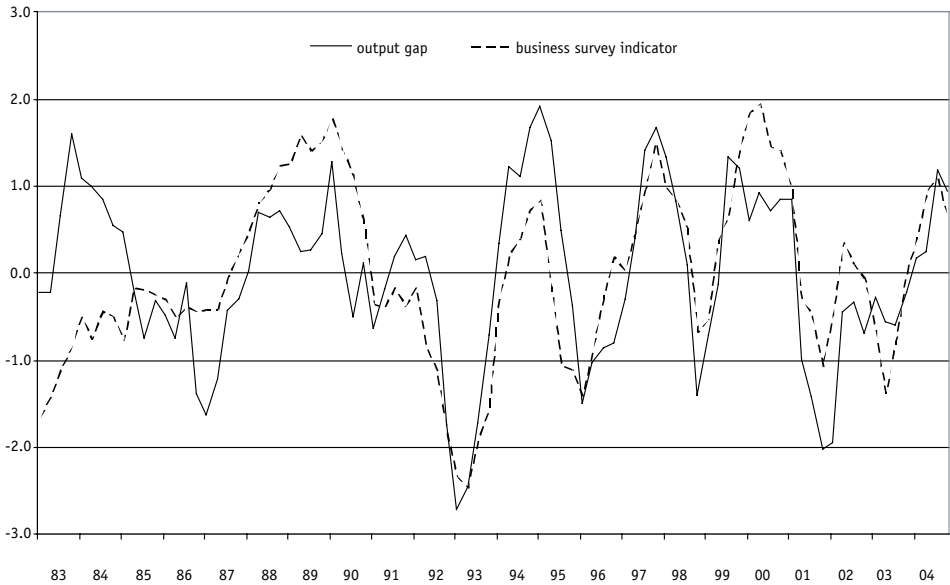
The correlation of the output gap with raw capacity utilisation is lower, at 0.58 (0.41), although it is higher with the cycle estimated on LDUC.[12] The NBB business survey indicator gives a better idea of the output gap notwithstanding the inclusion of capacity utilization at the estimation level. But capacity utilisation was treated in a symmetric way, without constraining the output gap to replicate its movements *by construction.*

In the next section, we will check whether one can improve on the present model, by imposing a single common cycle on the whole set of variables or by imposing additivity on the phase shifts present in the cycles.

---

[11] Quarterly averages of the gross overall synthetic curve are used.

[12] A lead of one quarter is used on raw capacity utilisation to account for the publication lag. Otherwise the correlation drops even further, to 0.47 (0.33).

3. CYCLE RESTRICTIONS

Some polar representations of cycles can be found in the literature on multivariate models. Often, a single cycle is assumed to govern the cycles of all the variables in the system. On N-1 variables, idiosyncratic shocks may be added to the common cycle. This kind of model can be found in Scott (2000), Proietti et al. (2002) or Rünstler (2002). If the restriction is warranted, a better estimate of the cycle will result. Here, we will check whether a common cycle is a realistic assumption. The irregular components introduced in the models may be interpreted as the idiosyncratic cycle shocks from the literature, but on N-1 variables we will allow idiosyncratic cycles to be present as well. This specification is less restrictive than the common cycle restriction and has another advantage: it can be tested with log-likelihood ratios. It does not change the essence of the common cycle restriction: as the other cycles are idiosyncratic, the commonality between cycles will still go through the common cycle. A common cycle with N-1 idiosyncratic cycles can also be found in Koopman-Azevedo (2004). They call the resulting model, "common similar cycles" model.

Not independent from the common cycle restriction, additivity is often present in multivariate models. But with more than one common cycle, additivity does not necessarily hold. In Koopman-Azevedo (2004), additivity is imposed by construction although several structural cycles are present. We will test for additivity in a second subsection, without constraining the number of common cycles.

3.1. COMMON CYCLE RESTRICTIONS

With N-1 idiosyncratic cycles, the model is asymmetric: there is one variable without idiosyncratic cycle. The cycle on this variable will give the reference cycle, common to all variables (phase shifts are still allowed). The corresponding F and F* matrices are:

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ F_{21} & 1 & 0 & 0 & 0 \\ F_{31} & 0 & 1 & 0 & 0 \\ F_{41} & 0 & 0 & 1 & 0 \\ F_{51} & 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } F^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ F_{21}^* & 0 & 0 & 0 & 0 \\ F_{31}^* & 0 & 0 & 0 & 0 \\ F_{41}^* & 0 & 0 & 0 & 0 \\ F_{51}^* & 0 & 0 & 0 & 0 \end{pmatrix} \quad (6c)$$

The first structural cycle corresponds to the first column of F (and F*). As it is shared by all the variables, possibly with a phase shift, it is the common cycle. The N-1 other structural cycles are the idiosyncratic cycles, one for each variable but the first one (the reference variable). Phase shifts are unnecessary as the idiosyncratic cycles are present in a single variable. Given the results from the preferred model, only LDUC, LTFP and LHOURS will be selected as reference variable for the common cycle. Results may change depending on the reference variable.

Compared to (6a), the common cycle model with idiosyncratic cycles (IC) introduces 12 zero restrictions on the F and F* matrices. However, the preferred model favoured a specification with two common cycles.[13] "Imposing" two common cycles with IC will leave the likelihood unchanged. The F and F* matrices are in this case:

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ F_{21} & 1 & 0 & 0 & 0 \\ F_{31} & F_{32} & 1 & 0 & 0 \\ F_{41} & F_{42} & 0 & 1 & 0 \\ F_{51} & F_{52} & 0 & 0 & 1 \end{pmatrix} \text{ and } F^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ F_{21}^* & 0 & 0 & 0 & 0 \\ F_{31}^* & F_{32}^* & 0 & 0 & 0 \\ F_{41}^* & F_{42}^* & 0 & 0 & 0 \\ F_{51}^* & F_{52}^* & 0 & 0 & 0 \end{pmatrix} \quad (6d)$$

Moving now to the single common cycle specification in (6c) only implies 6 restrictions.

In table 9, the new shares of the structural cycles in the cycle variance of the different variables are given. Only two structural cycles emerge. One is the common cycle and the other is idiosyncratic, involving either LTFP or LDUC depending on the reference variable. The possibility to get more than two structural cycles by having more idiosyncratic cycles was not selected in the optimization process. Apparently, cycles are too intertwined to allow for a third or more idiosyncratic cycles. The models with LDUC and LHOURS used as reference variable turned out to be the same. The model with LTFP used for the reference cycle is different.

TABLE 9. STRUCTURAL CYCLES SHARES (ONE COMMON CYCLE MODEL WITH IC)

LDUC USED AS REFERENCE VARIABLE

	$\Psi 1 + \Psi 1^*$	$\Psi 2 + \Psi 2^*$	$\Psi 3 + \Psi 3^*$	$\Psi 41 + \Psi 4^*$	$\Psi 5 + \Psi 5^*$
LDUC	1.000				
LHOURS	1.000	0.000			
LPART	1.000		0.000		
LTFP	0.660			0.340	
URmin	1.000				0.000

LHOURS USED AS REFERENCE VARIABLE

	$\Psi 1 + \Psi 1^*$	$\Psi 2 + \Psi 2^*$	$\Psi 3 + \Psi 3^*$	$\Psi 41 + \Psi 4^*$	$\Psi 5 + \Psi 5^*$
LHOURS	1.000				
LTFP	0.660	0.340			
LPART	1.000		0.000		
LDUC	1.000			0.000	
URmin	1.000				0.000

---

[13] For identification, one of the common cycles is not entirely common. It must be absent from one variable, the first variable in the system.

LTFP USED AS REFERENCE VARIABLE

	$\Psi1+ \Psi1^*$	$\Psi2+ \Psi2^*$	$\Psi3+ \Psi3^*$	$\Psi4+ \Psi4^*$	$\Psi5+ \Psi5^*$
LTFP	1.000				
LDUC	0.569	0.431			
LHOURS	1.000		0.000		
LPART	0.998			0.002	
URmin	1.000				0.000

The use of LDUC as reference variable for the cycle is common in the literature. This model delivers a marginally higher likelihood compared to the "LTFP model" (see column (2) of table 10). But the model should not be confused with a single cycle model. A strong idiosyncratic cycle is present in LTFP.

The fit of LDUC is unaffected when imposing the single common cycle (with IC), no matter what the reference variable is. It is also the case for LPART and URmin but they did not have much cycle in the preferred model and it is still the case. To the contrary, the fit for LTFP and LHOURS deteriorates. LTFP is more affected when LDUC or LHOURS gives the reference cycle, LHOURS more affected when LTFP is used.

**TABLE 10. FIT OF COMMON CYCLE MODELS[a]**

	2 common cycles with IC (≡ no restrictions) (1)	1 common cycle with IC (2)	2 common cycles with IC + restrictions on LPART and URmin (3)
order: **LDUC - LHOURS** - LPART - LTFP - URmin			
Log-Likelihood	2001.31	1995.79	1999.59
RD2 LDUC	0.489	0.485	0.480
RD2 LHOURS	0.496	0.462	0.497
RD2 LPART	0.529	0.527	0.530
RD2 LTFP	0.425	0.365	0.415
RD2 URmin	0.822	0.822	0.820
order: **LHOURS - LTFP** - LPART - LDUC - URmin			
Log-Likelihood	2001.31	1995.79	1999.65
RD2 LDUC	0.489	0.485	0.458
RD2 LHOURS	0.496	0.462	0.499
RD2 LPART	0.529	0.527	0.528
RD2 LTFP	0.425	0.365	0.411
RD2 URmin	0.822	0.822	0.822
order: **LTFP - LDUC** - LHOURS - LPART - URmin			
Log-Likelihood	2001.31	1995.07	1999.64
RD2 LDUC	0.489	0.491	0.473
RD2 LHOURS	0.496	0.433	0.497
RD2 LPART	0.529	0.527	0.529
RD2 LTFP	0.425	0.379	0.422
RD2 URmin	0.822	0.823	0.823

[a] In columns (1) and (3), two variables selected among three candidate variables (LDUC, LTFP and LHOURS) are necessary to identify the two common cycles. As the order between the two variables does not matter, three different orderings need to be tested.

Do these differences in the fit of LTFP and LHOURS matter? Looking at likelihood ratios between column 1 and column 2, the 6 restrictions are not rejected, no matter what the reference variable is. But LPART and URmin have only tiny cycles and many of the restrictions for the one common cycle model with IC in (6c) are zero restrictions concerning these two variables. Once the zero restrictions corresponding to tiny cycles in LPART and URmin are set on the models (results are presented in the third column of table 10), the 2 restrictions implied by the single common cycle are systematically rejected. Turning the second common cycle into an idiosyncratic cycle is not allowed. A second common cycle is necessary in (two of) the three variables LDUC, LTFP and LHOURS.

From a practical point of view, does the single common cycle restriction make a difference? If we compare the output gap of the preferred model to output gaps generated under single common cycle restrictions, the difference amounts to 0.1 percentage point on average (see chart 6). What matters for the output gap is the LTFP cycle and if LTFP is not the reference, an idiosyncratic cycle will try to preserve the LTFP cycle. All patterns are similar and almost nowhere does the gap change sign.

CHART 6. OUTPUT GAPS (PERCENTAGE POINTS)
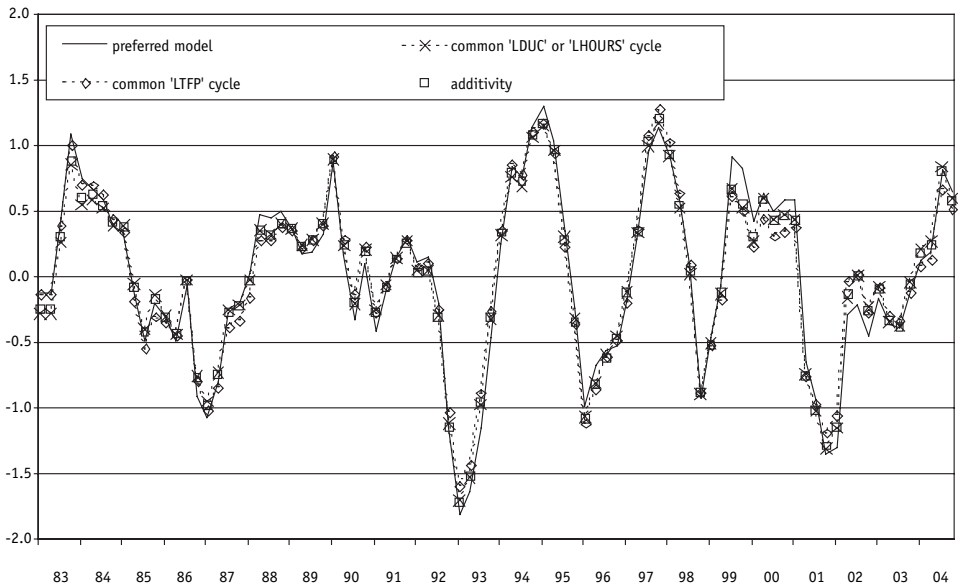
3.2. ADDITIVITY RESTRICTIONS

The additivity property is automatically verified when there is a single common cycle. The phase shifts on the common cycle will give the timing for the different variables. If present, idiosyncratic cycles are completely neutral. With more than one common cycle, additivity need not hold. To test for additivity, the preferred model is reestimated with the specification for matrices F and F* given in (6b).

With two structural cycles (the number did not change), (6b) implies a fall in the number of shift coefficients from seven to four. The likelihood ratio test gives a value of 6.67. Additivity is not rejected and a unique timing appears. But the two acyclical variables LPART and URmin are again responsible for the result: timing is irrelevant when there is almost no cycle and every timing will be fine.[14] This favours additivity. To get rid of the problem, the preferred model is reestimated putting the cycles in LPART and URmin to zero. Then (6b) is imposed on the model. Additivity amounts to a single restriction, imposing a coherent timing between the cycles of LDUC, LTFP and LHOURS. Additivity is strongly rejected between these three variables, with a likelihood ratio test equal to 7.20.

The lags of variable i with respect to j are the followings (lags under additivity in parentheses):

i ⟍ j	LDUC	LHOURS	LTFP
LDUC	0.0	3.0 (2.6)	1.0 (1.7)
LHOURS	-3.0 (-2.6)	0.0	0.2 (-0.9)
LTFP	-1.0 (-1.7)	-0.2 (0.9)	0.0

LHOURS leads LDUC by three quarters whereas LTFP leads LDUC by one quarter. But taken together, the cycles of LHOURS and LTFP are contemporaneous. Looking at the lags in terms of structural cycles may help to understand the result:

	1st cycle	2nd cycle
LDUC	0.0	
LHOURS	-3.0	0.0
LTFP	-1.0	-2.1

Although LHOURS is leading LTFP by two quarters on the first structural cycle, LTFP is leading LHOURS by 2 quarters on the second cycle. All in all, the two series are almost contemporaneous.

From a practical point of view, the output gap computed under the additivity constraint is almost indistinguishable from the output gap computed with one common cycle imposed and LDUC (or LHOURS) used as reference variable (see chart 6).

---

[14] Signs of this instability can be seen in the phase shifts derived for LPART and URmin under the two different trend specifications, IRW and DS.

4. Unemployment, the Missing Cycle

If we compare on chart 7 the output gap of the preferred model to the output gaps computed by the OECD and the European Commission, their estimates show much greater variability. In fact, if the gap computed by the Commission is split up into factor inputs, our estimate becomes closer to the TFP cycle component (including also the cycle in hours). The EC "TFP" cycle has larger amplitude since it is the result of a standard HP filter whereas our estimated model discards low frequencies.[15] But the major difference between output gap estimates comes from the unemployment cycle.

Chart 7. Comparison of output gaps (percentage points)
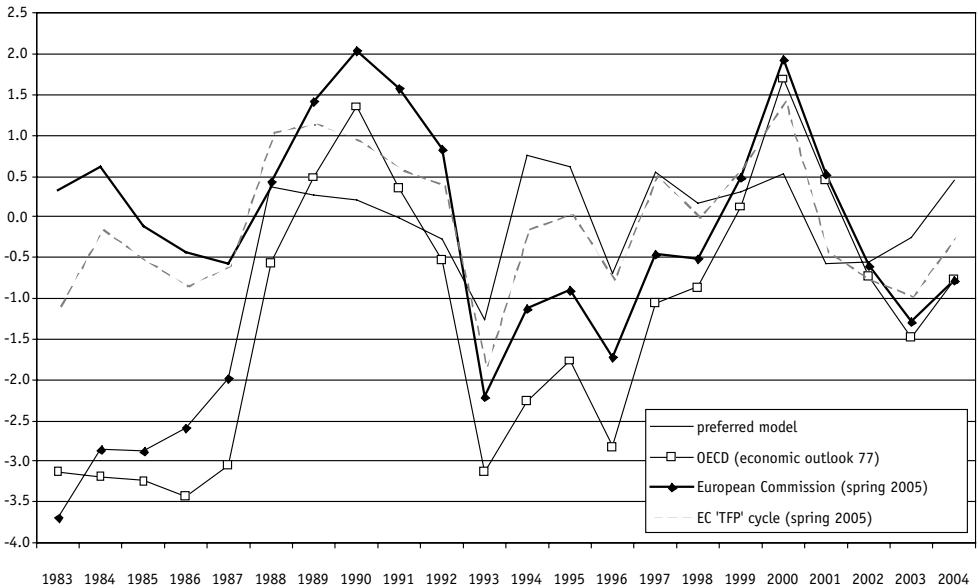
A striking result in our empirical section is the negligible cycle found on URmin. The long wave present in unemployment is attributed to the trend component, which would suggest that it is of a permanent nature. Even the changes in unemployment are long-lasting contrary to what happens to other variables, as the trend slope is close to a random walk (DS case). In sharp contrast, the unemployment cycles on chart 8, computed by applying a standard HP filter or using a bivariate STS model to estimate the NAWRU as the European Commission does, deliver a long cycle close to the raw unemployment rate over the period 1983-2004. When included, it has a strong (negative) impact on the output gap.

---

[15] By construction, the annual HP filter tries to keep periodicities lower than 19.8 years in the cycle.

**Chart 8. Long cycles in the unemployment rate (percentage points)**
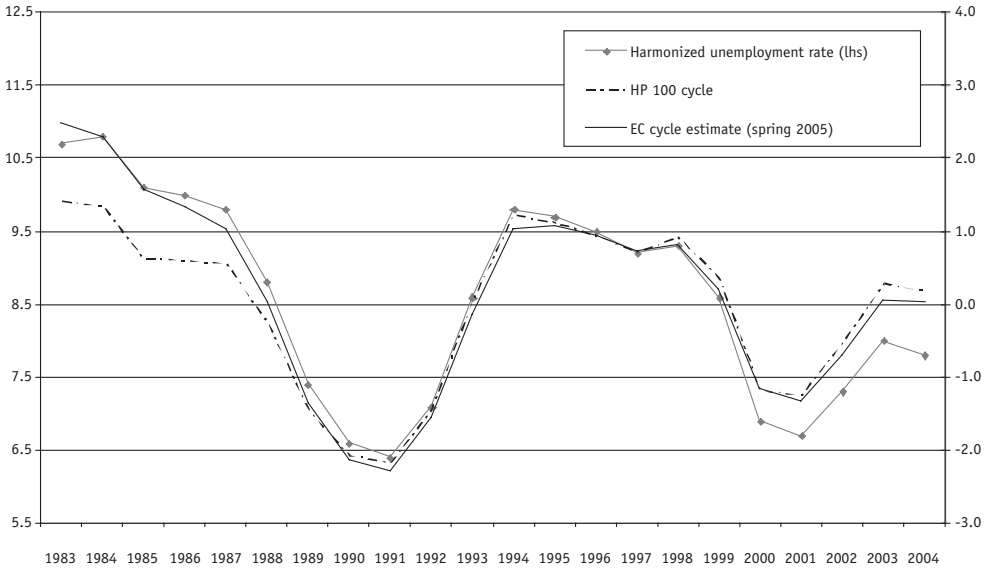
A long cycle could be missing in the specification of the model. Does this imply that the model is flawed? Not really. In the estimation of this kind of model, there is often a hesitation in the assignment of a long wave to the trend component or to the cycle component. In our estimates, when it was necessary, the long wave was included in the trend component of the variable.

On chart 9, "long cycles" computed from the trend components of LDUC and URmin are shown for the preferred model. The time series for LDUC is the centered trend without drift while for URmin, it is the centered trend slope.[16] The two time series have much in common, which is the result of the high correlation between trend slope shocks.

---

[16] The asymmetric treatment reflects the near unit root found in the unemployment trend slope.

CHART 9. TREND COMPONENT CYCLES
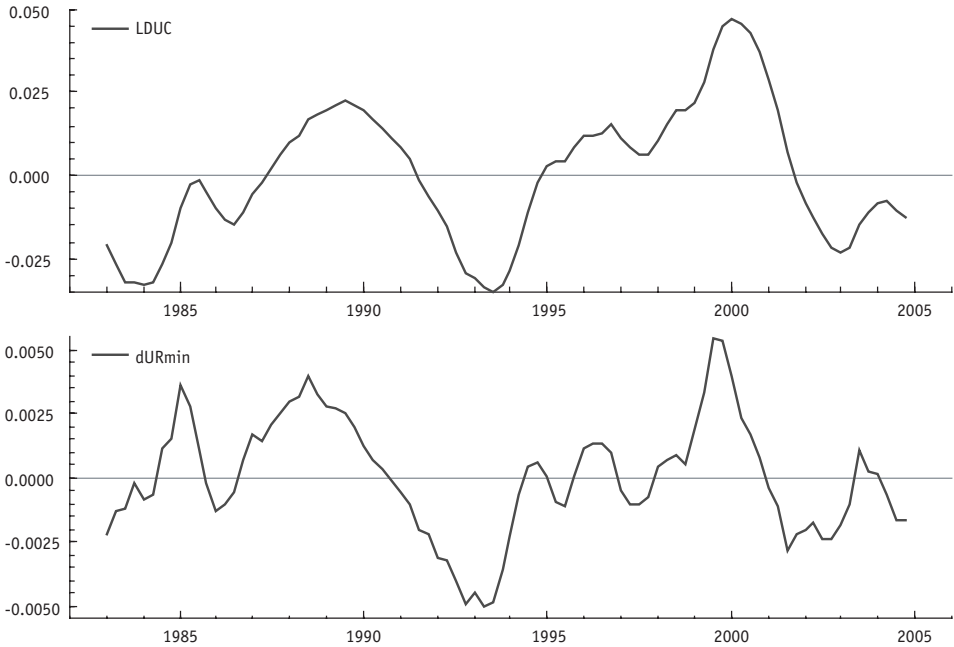
A model with LDUC and URmin (taken in first difference) but also LTFP - as suggested by the correlations of slope shocks - could be a good starting point to search for a long cycle in the data. URmin could possibly stay in level if hysteresis is introduced, linking the cycle and trend evolutions of the variables as in Proietti et al. (2002). If successful, this attempt would deliver another measure of the output gap, more in line with the computations done by the EC and the OECD. Further analysis would be necessary to assess the relative merit of the two measures in explaining inflation or budget cycles. Potential growth could also be fruitfully investigated once the long cycle is removed from the trend components.

CONCLUSION

The production function approach, with its richness of (sub)factor inputs, has become very popular to compute measures of output gaps and potential growth rates. Structural time series models with explicit cycle and trend components are well suited to deliver the corresponding estimates for each (sub)factor without a priori constraining cycle frequencies. In a multivariate setting, the models can exploit the commonalities existing between variables to provide better estimates of the cycles and trends.

In the paper, a common cycle is not imposed at the outset on the data. All variables are treated in a symmetric way and may have their own cycle. Commonalities are evaluated, not imposed. In particular, capacity utilization - that was introduced to provide extra information on the cycle - is not assumed to give the reference cycle and may have a trend. But the presence of a common cycle was investigated. Phase shifts were introduced along the lines of Rünstler (2004) to allow leads and lags between cycles. They were estimated simultaneously, without imposing a given timing between cycles and additivity of leads and lags does not necessarily hold. The objective is again to let the data speak, without a priori restrictions.

Two types of smooth trends were considered: integrated random walks and the damped slope specification of Proietti et al. (2002). The latter better matches the data, with the notable exception of the unemployment rate whose trend is close to an IRW.

In the damped slope case, "short" cycles of about 3.5 years are found in hours, TFP and capacity utilization. They are tightly correlated. The output gap implied by these short cycles turned out to be very close to the NBB business survey indicator. The indicator could probably add useful information to the estimation of the output gap.

The cycles cannot be reduced to capacity utilization as is often postulated. Capacity utilization delivers useful information but two independent structural cycles are at the root of the different cycles, with different weightings between variables. If capacity utilization is wrongly imposed as the reference variable for a common cycle, an idiosyncratic cycle emerges in TFP, explaining 34 p.c. of the TFP cycle variance. Other reference variables give the same result: the one common cycle model is rejected (even after allowing for idiosyncratic cycles).

Phase shifts are present. Hours lead capacity utilization by 2.8 quarters. TFP and hours are contemporaneous but TFP leads capacity utilization by 1.1 quarters only, showing that additivity of leads and lags does not hold. Put to the test, additivity is actually rejected.

More work is needed to get a clear picture of the Belgian output gap. The short cycles identified in this paper may not be the end of the story. The correlations found between slopes in the damped slope specification suggest that a long cycle could hide in the trend components, involving TFP, capacity utilization and the unemployment rate. Adding an extra set of cycles to the model at another frequency could prove a tricky task, the more so if phase shifts are considered. Bayesian techniques could make the estimation more tractable. Other extensions are possible involving nominal information. The link between price (wage) inflation and the two structural cycles could then be investigated.

REFERENCES

Apel, M. and P. Jansson, 1999. "A theory-consistent system approach for estimating potential output and the NAIRU", *Economics Letters*, 64, 271–275.

Azevedo, J., S. Koopman and A. Rua, 2006. "Tracking the business cycle of the euro area: a multivariate model-based band-pass filter", *Journal of Business and Economic Statistics*, 24, 278–290.

Clark, P., 1989. "Trend reversion in real output and unemployment", *Journal of Econometrics*, 40, 15–32.

Cotis, J-Ph., J. Elmeskov and A. Mourougane, 2005. "Estimates of potential output: benefits and pitfalls from a policy perspective" in Reichlin L. (ed.), *The Euro Area Business Cycle: Stylized Facts and Measurement Issues*, London, CEPR.

Denis, C., D. Grenouilleau, K. Mc Morrow and W. Röger, 2006. *Calculating potential growth rates and output gaps - a revised production function approach*, ECFIN, Economic paper 247.

Doornik, J., 1998. *Object-oriented matrix programming using Ox 2.0*, London, Timberlake Consultants Press.

Doornik, J. and H. Hansen, 1994. *An Omnibus Test for Univariate and Multivariate Normality, discussion paper*, Nuffield College, Oxford.

Fabiani, S. and R. Mestre, 2004. "A system approach for measuring the euro area NAIRU", *Empirical Economics*, 29, 311–341.

Giorno, C., P. Richardson, D. Roseveare and P. van den Noord, 1995. *Estimating potential output, output gaps and structural budget balances*, OECD, Economic Department Working Paper 152.

Harvey, A., 1989. *Forecasting, structural time series models and the Kalman filter*, Cambridge, Cambridge University Press.

Harvey, A. and S. Koopman, 1997. "Multivariate structural time series models" in Heij C., J. Schumacher, B. Hanzon and C. Praagman (eds), *System dynamics and financial models*, John Wiley & Sons Ltd.

Harvey, A. and T. Trimbur, 2003. "General model-based filters for extracting trends and cycles in economic time series", *Review of Economics and Statistics*, 85, 244–255.

Koopman, S., A. Harvey, J. Doornik and N. Shephard, 1995. *Stamp, structural time series analyser, modeller and predictor*, London, Timberlake Consultants.

Koopman, S. and J. Azevedo, 2004. *Measuring synchronization and convergence of business cycles*, Tinbergen Institute, Discussion Paper 052/4 (forthcoming in Oxford Bulletin of Economics and Statistics).

Koopman, S., N. Shephard and J. Doornik, 1998. "Statistical algorithms for models in state space using SsfPack 2.2", *Econometrics Journal*, 1–55.

Orphanides, A. and S. van Norden, 2002. "The Unreliability of Output Gap Estimates in Real Time", *Review of Economics and Statistics*, 84(4), 569-583.

Proietti, T., A. Musso and T. Westermann, 2002. *Estimating potential output and the output gap for the euro area: a model-based production function approach*, European University Institute, Working Paper ECO 2002/09 (forthcoming in *Empirical Economics*).

Rünstler, G., 2002. *The information content of real-time output gap estimates: an application to the euro area*, ECB, Working paper 182.

Rünstler, G., 2004. "Modelling phase shifts among stochastic cycles", *Econometrics Journal*, 7, 232–248.

Scott, A., 2000. *A multivariate unobserved components model of cyclical activity*, Reserve Bank of New Zealand, Discussion Paper 2000/04.

APPENDIX 1. STATE SPACE FORM OF THE MODEL, DAMPED SLOPE SPECIFICATION

For $z_t = \begin{pmatrix} LDUC_t \\ LHOURS_t \\ LPART_t \\ LTFP_t \\ UR\min_t \end{pmatrix}$ the measurement equation is given by:

$$z_t = \begin{pmatrix} I_N & 0_N & 0_N & 0_N & I_N \end{pmatrix} \begin{pmatrix} \mu_t \\ \beta_t \\ \psi_t \\ \psi_t^* \\ C_t \end{pmatrix} + \varepsilon_t$$

The transition equation for the state vector is:

$$\begin{pmatrix} \mu_{t+1} \\ \beta_{t+1} \\ \psi_{t+1} \\ \psi_{t+1}^* \\ C_{t+1} \end{pmatrix} = \begin{pmatrix} m \\ 0_n \\ 0_n \\ 0_n \\ 0_n \end{pmatrix} + \begin{pmatrix} I_N & I_N & 0_N & 0_N & 0_N \\ 0_N & \theta & 0_N & 0_N & 0_N \\ 0_N & 0_N & \rho\,I_N\cos\lambda & \rho\,I_N\sin\lambda & 0_N \\ 0_N & 0_N & -\rho\,I_N\sin\lambda & \rho\,I_N\cos\lambda & 0_N \\ 0_N & 0_N & F & F^* & 0_N \end{pmatrix} \begin{pmatrix} \mu_t \\ \beta_t \\ \psi_t \\ \psi_t^* \\ C_t \end{pmatrix} + \begin{pmatrix} 0_n \\ \xi_t \\ \kappa_t \\ \kappa_t^* \\ 0_n \end{pmatrix}$$

with 0_n, the (N x 1) zero vector, m, a vector of drifts and θ, a diagonal matrix of damped slope coefficients. F and F* are lower triangular matrices, their precise shapes depending on the presence of additivity restrictions. Without restrictions,

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ F_{21} & 1 & 0 & 0 & 0 \\ F_{31} & F_{32} & 1 & 0 & 0 \\ F_{41} & F_{42} & F_{43} & 1 & 0 \\ F_{51} & F_{52} & F_{53} & F_{54} & 1 \end{pmatrix} \text{ and } F^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ F_{21}^* & 0 & 0 & 0 & 0 \\ F_{31}^* & F_{32}^* & 0 & 0 & 0 \\ F_{41}^* & F_{42}^* & F_{43}^* & 0 & 0 \\ F_{51}^* & F_{52}^* & F_{53}^* & F_{54}^* & 0 \end{pmatrix}$$

The variance-covariance matrix of shocks from the measurement and transition equations is block diagonal with identical diagonal variance-covariance matrices for κ_t and κ_t^*.

APPENDIX 2. SMOOTHED SIMILAR CYCLES - (IRW TRENDS)
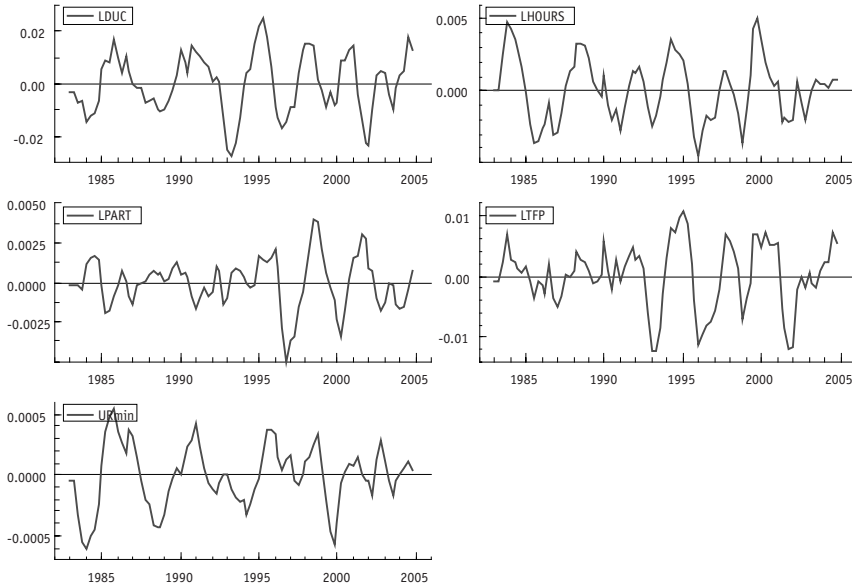
Structural cycles shares in cycle variance

	Ψ1	Ψ2	Ψ3	Ψ4	Ψ5
LDUC	**1.000**				
LHOURS	0.011	**0.989**			
LPART	0.002	0.009	**0.989**		
LTFP	0.531	0.465	0.004	0.000	
URmin	0.148	0.840	0.010	0.000	0.001

Correlations of cycles

	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	1	0.105	-0.049	0.729	0.385
LHOURS	0.105	1	0.088	0.755	-0.871
LPART	-0.049	0.088	1	-0.031	-0.206
LTFP	0.729	0.755	-0.031	1	-0.339
URmin	0.385	-0.871	-0.206	-0.339	1

APPENDIX 3. SMOOTHED CYCLES WITH PHASE SHIFTS - (IRW TRENDS)
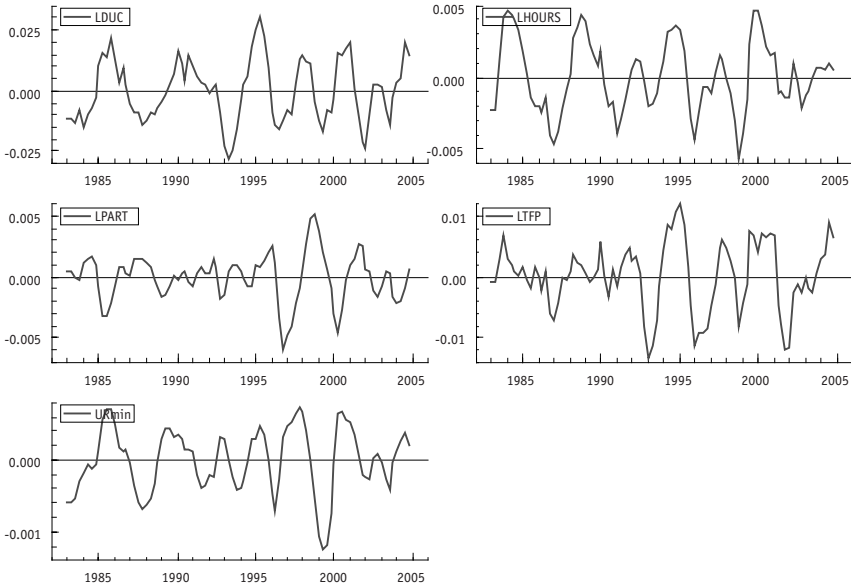
Structural cycles shares in cycle variance

	Ψ1	Ψ2	Ψ3	Ψ4	Ψ5	Ψ1*	Ψ2*	Ψ3*	Ψ4*
LDUC	1.000								
LHOURS	0.037	0.669				0.295			
LPART	0.024	0.003	0.887			0.021	0.065		
LTFP	0.432	0.099	0.007	0.000		0.189	0.264	0.008	
URmin	0.150	0.001	0.135	0.000	0.000	0.018	0.415	0.282	0.000

Phase shifts between cycles (lag of i with respect to j) in quarters

i \ j	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	0	2.7	1.7	1.3	-0.7
LHOURS	-2.7	0	-2.9	0.5	3.3
LPART	-1.7	2.9	0	1.7	-1.7
LTFP	-1.3	-0.5	-1.7	0	2.7
URmin	0.7	-3.3	1.7	-2.7	0

Association between cycles

	LDUC	LHOURS	LPART	LTFP	URmin
LDUC	1	0.576	-0.213	0.788	0.409
LHOURS	0.576	1	-0.271	0.637	-0.763
LPART	-0.213	-0.271	1	0.088	-0.779
LTFP	0.788	0.637	0.088	1	-0.386
URmin	0.409	-0.763	-0.779	-0.386	1