BRUSSELS ECONOMIC REVIEW – CAHIERS ECONOMIQUES DE BRUXELLES
VOL. 55(1) SPRING 2012

BANKS CONNECTIVITY, CREDIT RISK TRANSFER, AND STABILITY OF THE BANKING SYSTEM

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ABSTRACT:

Our dynamic model captures the network relations generated by credit risk transfer and securitization. Each bank determines its own level of risk according to fundamentals and the level of risk of its environment, given the possibilities opened by credit risk transfer. The dynamics of the model is generated by the network structure of the interbank relations. A highly connected network generates forces able to make the long term equilibrium of the bank industry dependant on initial conditions. Irregularity in the network can also explain that a final heterogeneity appear in the final situation of banks, even when their fundamentals were originally similar.

JEL CODES: G01, G2.

KEYWORDS: Banking networks, securitization, financial crisis contagion.

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INTRODUCTION

This paper contributes to analyze the stabilizing / destabilizing role of interbank network relations. It focuses on network relations generated by the effect of risk management practices, and the level of risk made admissible for each financial agent and for the banking industry. We consider a theoretical setting in which identical risk-averse financial agents can adapt their own level of risk, return, and level of activity to their environment. These adaptations are motivated by the depressing or stimulating influence of the level of activity, risk, and return of their business partners which we assimilate to other banks. From the view point of a given financial agent, increases in activity, risk, and return of the business partners can give out opposite messages. First, they may promise an increase of its own activity induced by the relations with these more active partners. Second, they may signal increased fragility in these same partners. The first message stimulates the level of activity in the bank that observes them, while the second tends to depress this level of activity. When the same messages are associated with the development of risk management practices and securitization, the depressive effects of increased activity, risk, and return in the environment tend to vanish and the positive influences dominate. In the case of securitization, these positive influences are further fueled by the additional financial activity generated by trading in structured products and derivatives.

In this paper we analyze the dynamics of the banking industry associated with the stimulating effect of securitization practices, according to the extent and the form of the interbank network relations. We do not analyze the choice of securitization which is assumed to be an efficient way to manage the credit risk of each bank individually; we focus on its consequences for individual and systemic risk. We address the following issues: what sort of dynamics are associated with such interbank influences when all banks are basically the same, have the same risk aversion and the same fundamentals except but are inserted in different interbank relations? Are the external effects on the individual and systemic risk transitory or permanent, and under what conditions? Are these effects on the banking system homogeneous and under what conditions? What might happen if the effects of credit risk transfer are not homogeneous across the banking system?

Credit risk transfer devices developed rapidly prior to the events of 2007-2008. They matched protection and return within the financial system, and apparently dissipated the risk across numerous institutions and portfolios, rendering individual risks safe for their originators. However, the subprime crisis has revealed that misjudgments about the effect of securitization encouraged banks to increase their exposure and did not provide the necessary protection (Morrison, 2005; Dodd, 2007; Muromachi, 2007; Shao and Yeager, 2007; Rey, 2008; Eichengreen, 2008; Mah-Hui Lim, 2008; Gerardi, Lehnert, Sitslund and Willen, 2008, Minton, Slutz and Williamson, 2009). Since this crisis, much attention has been devoted to understanding how contagion among banks operates. Attempts have been made to understand how interbank network connections can generate the conditions of a financial crisis, i.e. increases of financial activity, risk, and return to disproportionate levels given the fundamentals of the economy.

The model proposed in this paper contributes to this research. It builds on classical behavioral assumptions and examines the capacity of interbank networks to generate individual and systemic financial fragility. Our results support the view that the influence generated by the levels of activity, risk, and return of a financial agent's partners can generate stationary equilibria that are different from the homogeneous equilibrium obtained without links among banks. We find also that when network relations are moderate, the stimulating effects of a given shock on activity, risk, and return are transitory; in the presence of strong network relations these effects are permanent. The model predicts also that, when an increase in the level of risk of partner-banks is perceived in the same way by all the banks within a strong network of relations, small inequalities in the initial distribution of the risk among banks can generate large disparities in the final level of risk for those banks, even if all the components of the industry are similar. In this case, the long run dynamics of each bank is determined by the average level of risk of its partners and not by its own initial level of risk. Some properties of the banking network can generate localized conditions for a financial crisis, without increasing the risk for other parts of the financial industry.

The paper is organized as follows. *Section 1* provides an overview of the recent literature devoted to interbank relations. It underlines the links between hedging and risk management practices generated by the banks. *Section 2* presents the model and the results of our analysis. *Last section* comments on these results and the conclusion of the paper.

1. The Literature and The Problem

Physicists and biologists would define networks as specific forms of the organization of life or matter that can be represented by mathematical graphs, reduced to edges and vertices, or to a collection of interconnected components. Economists have only recently developed methodologies adapted to this form of organization of the industry and of the market. One of the major problems was to conciliate the structural form and the meso-analytical content of network analysis with the micro-founded framework used in economic theory. However, a bridging has been achieved between strategic and network analysis approaches which reconciles - at least partly - the analytical tools of graph theory with traditional microeconomics methods (see e.g. Bala and Goyal, 2000; Jackson, 2005; Goyal, 2007). In addition, the renewed attention to network theory in the 1990s gave birth to new strands of studies in several fields, including social sciences, finance and economic analysis (see e.g. Dorogovtsev and Mendes, 2003; Gallegati and Kirman, 2004).

From this perspective, financial and banking issues, along with the Internet and the World Wide Web, are particularly suited to network analysis. In standard financial theory, the usual assumption is to consider that correlation matrixes of stock returns or prices are dominated by positive components. Network analysis can be very useful to validate (or not) the market models thereby elaborated. Bonanno et al. (2004) suggest using the minimal spanning trees method to confirm or falsify these models in different time periods.

The banking industry offers other potentialities for applications of network analysis. Financial intermediation and the bank industry historically were organized as an interconnected system of decentralized posts technically associated by compensatory devices. Banks and other financial agents have worked continuously to maintain and develop the structure of this network which they considered a shared input to the production of their financial services. For centuries, financial innovations did not challenge the traditional organization of the banking industry as a technical and informative network. The network externalities accessible to each bank from the banking industry are generic components of financial intermediation activity. When many banks evolve in the same economic area or compete to provide the same financial services, they tend to conciliate competition in price and other antagonistic attitudes, with cooperative actions able to develop efficient interconnections and to manage their common interest. Joint management of the interbank market (Bos, Elsinger, Summer and Thurner, 2004; Soramäki, Bech, Arnold, Glass and Beyeler, 2010) and the foreign exchange market are representative examples of this tendency of banks to manage the network architecture of their activity in collaboration.

Since the Asian financial crisis, theoretical and empirical works have focused mainly on the capacity of banking networks to propagate systemic risk. Allen and Gale (2000), Freixas, Parigi and Rochet (2000), Thurner, Hanel and Pichler (2003), and Babus (2007) compare different network topologies and their respective capacities to diffuse systemic risk. Risk management practices devoted to controlling the level of systemic risk in the economy have been the motivation for the creation and maintenance of medium term links between banks. Vivier Lirimont (2006) studies the capacity of banks to use their network structure to manage the liquidity risk. However, the actions that increase their protection against financial risk can also simultaneously increase their risk, and the instability of the whole economy. Leitner (2006) considers situations where, within a network structure, liquid banks are forced to bail out illiquid banks. In this case, “a network in which agents are closely interlinked may be optimal both because of and despite the potential for contagion”. Boissay (2006) presents the conditions of financial contagion as the consequence of the development of credit chains between banks and/or firms. In this case, the structure of the network seems also to matter. Pröpper, van Lelyveld, and Heijmans (2008) analyze the impact of the removal of a node on the properties of a banking network. They use European data to show that the removal of a small number of crucial nodes in a network structure strongly modifies the structure of the network and increases the risk of illiquidity crises.

Analyses of the consequences of the development of credit risk transfer start from before the 2007-2008 meltdown and develop with it. Wagner and Marsh (2006, p. 173) find that “the incentive to transfer credit risk transfer is aligned with the regulatory objective of improving stability” while Chiesa (2008, but written in 2004) uses contract theory to provide a more balanced appreciation. Credit risk transfer “enhances loan monitoring and expands financial intermediation” but also potentially generates the risk of financial fragility: “the extent of credit enhancement needs to be precisely delimited. Above that exact level, monitoring incentives are undermined (loan quality deteriorates) and wealth is transferred from the bank’s financiers to the bank” (Chiesa, 2008, p. 464). Other premonitory views point to the destabilizing nature of securitization and its potential negative effects on welfare (Morrison, 2005). Nicoló and Pelizon (2005/2008) address how credit derivatives affect the design of the contracts that buyers use to signal their type. Allen and

Carletti (2006) find that when banks hedge this risk in an interbank market, credit risk transfer is detrimental to welfare and can generate contagion. In the context of the 2007-2008 financial crisis, credit risk transfer is generally perceived as destabilizing (Duport, 2008, Heyde and Neyer, 2010). Hakenes and Schnabel (2010) find that with imperfect information on credit risk, credit risk transfer decreases welfare due to the excessive number of unprofitable loans. In a recent paper, Nijskens and Wagner (2011) try to reconcile the pros and cons of credit risk transfer through observation of the β of the banks. Before the crisis, banks increased their protection via securitization which reduced the risk for the banks. In the same time, they also increased their β, a sign that the market integrated the systemic risk which their increased correlation then generated.

Securitization and credit risk transfer devices create new links between banks and other financial intermediaries. “Securitization increases the dimensionality, and thus the complexity of the financial network. Nodes grow in size and interconnections between them multiply” (Haldane, 2009, p. 7). When the 2007-2008 crisis appeared, the most urgent challenge was to understand how securitization and the new links it had created modified exposure of the system to systemic crisis. Allen and Carletti’s (2006) seminal work distinguishes between good links created by securitization among banks and insurance, and bad links that connect banks. Allen, Babus and Carletti (2009) then analyze the role of securitization in the network propagation of financial risk. Credit derivative products diversify portfolios (and reduce the risk of bankruptcy) but introduce due diligence costs for participation in additional projects selected by other banks. A trade-off between these costs and benefits determines the optimal level of interaction for each bank. Babus and Carletti assume also that financial institutions use short term deposits to finance long term assets. This transformation involves the possibility of interim signals about the possibility of lenders defaulting. In this case, and for intermediate levels of bankrupcy costs, clustered networks more frequently lead to liquidations than unclustered ones. One of the most interesting papers on the network relations associated with securitization is Shin (2009) which proposes an accounting model for the banking system that captures the “interlocking claims and obligations” in relation to one “unleveraged sector” aggregating the balance sheets of the other financial agents, household investors and an “end-user” who is the ultimate borrower (Shin, 2009, p. 314). The model analyzes the link between the leverage of the financial intermediary sector, the profile of leverage of the individual banks, and the proportion of funding obtained outside the financial intermediary sector. Shin’s interpretation of the subprime crisis is that the “greater risk-taking capacity of the shadow banking system leads to an increased demand for new assets to fill the expanding balance sheets and an increase in leverage… However, once they have exhausted all the good borrowers, they need to scour for other borrowers - even subprime ones. The seeds of the subsequent downturn in the credit cycle are thus sown” (Shin, 2009, p. 310).

Nier et al. (2007), following Eboli (2007), suggest applying the physics of flow networks to analyze default dynamics of interbank links. Nier and co-authors identify external assets as the source of shocks and consider that depositors are the final bearers of the losses. They use a simulation model to analyze their effect on the financial stability of diverse structures of connections among banks. They find that the effect of the degree of connectivity is non-monotonic: small increases of connectivity tend to accelerate the contagion effect but when connectivity improves more, the good effects begin to dominate and reduce the contagion. They explore the role of concentration and asymmetry in the banking system on the risk of systemic instability. Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz (2007) depict the dynamics of the production networks associated with the “supplier-customer relationships involving extension of trade-credit” and test the comparative robustness of diverse network structures against the domino effect (diffusion of bankruptcies) among the industry. In a more recent paper (Battiston et al., 2009), they propose a model that integrates both the good and bad effects of networks connections. The good effect is that a complete network tends to improve the efficiency of the diversification mechanism. The bad effect is that all increases in the number of counterparts increase the default risk of each component in the system: all things being equal, individual and systemic risks also increase with the number of links in the network. With a feed-back effect that Battiston and colleagues assimilate to the financial accelerator, they find that a complete network more easily generates a systemic crisis than an incomplete one.

Empirical works have observed the form of the banking network around the world, and particularly the bad links that are able to generate contagion. They usually consider networks as a collection of moving market interactions and also as more permanent links determined by bilateral stable relationships. They use an adapted descriptive methodology borrowed from physics and biology to evaluate the properties of the links, the nodes, and the network. Müller (2006) simulates the potentialities of contagion in the Swiss interbank market using new data on bilateral bank exposure and credit lines and finds that the possibilities are substantial. Hattori and Suda (2007) and the Bank of Japan employ network analysis to investigate cross-border bank exposure. It is interesting that since 2010, IMF publications and internal documentation have been using network methodology to test the systemic linkages among banks and financial institutions. Bech, Chapman and Garratt (2008, 2010) consider the positions of Canadian banks within the Large Value Transfer System. They rank the banks belonging to this network to provide an empirical measure of which banks are likely to hold the most liquidity at any given time. Following the 2007-2008 meltdown, these works provide new tools to identify the weaknesses in world financial relations and to prevent a future crisis. Espinosa-Vega and Solé (2010) use network analysis to propose a simulation algorithm to locate the channels of risk transfer in the system. The IMF (2010) Monetary and Capital Markets Department has elaborated a staff paper adopting a network based methodology defined in 2009 by the BIS and the Financial Stability Board, which identifies the jurisdictions with systemically important financial sectors. This document uses four measures of centrality previously proposed by von Peter (2007) and Kubelec and Sá (2010), to capture the interconnectedness of financial jurisdictions and the resulting risks that financial crisis will be propagated. All these papers conclude that financial networks - which are supposed to optimize protection against risk - can serve also to propagate financial instability.

The model that we present and analyze in this paper is micro-founded following the main works reviewed in this section. Each financial agent maximizes, at each moment, a utility function whose properties are usual in banking and financial economics. The connections among banks are given and denote the partnership relations between agents. The form taken by credit risk transfer is given by the type of influence that an increase in the risk to a given bank exerts on the level of utility of its partners (formally, its neighbors). As a counterpart to these simplifications, the model analyzes the dynamics of the banking industry, according to the importance and the weight of the banking interconnections.

2. THE MODEL

The first subsection discusses the interconnected decisions of n individual banks in the banking industry using credit risk transfer. We explore the dynamics of the banking industry in different circumstances related to the weight and the form of the network connections between banks.

2.1. THE GENERAL FRAMEWORK

Let us consider a set of n banks i with $i = 1,\ldots,n$. We suppose that all banks have the same payoff function $W(y_i, \bar{y}_i)$ where the first argument y_i is the individual level of risk associated with the bank's investment choices and the second argument $\bar{y}_i$ is the level of systemic risk computed by the bank[1]. We assume that W is continuous, two times derivable in its two arguments, and satisfies for all $\bar{y}_i$ the following properties: $W(0, \bar{y}_i) = 0$, $\lim_{y_i \to +\infty} W(y_i, \bar{y}_i) < 0$, $\frac{\partial W_{y_i=0}}{\partial y_i} > 0$, $\frac{\partial^2 W}{\partial^2 y_i} < 0$. These assumptions mean simply that each bank has to take a strictly positive but finite level of risk to obtain a positive payoff. In addition, due notably to regulation policy and to capital requirements, W is concave in y_i.

The level of systemic risk y_i is computed by each bank i as an average of its neighbors on the interaction graph. In formula:

$$\bar{y}_i = \frac{1}{k_i + 1} \sum_{j=1}^{n} g_{ij} y_j \tag{1}$$

where k_i is the connectivity of bank i (its number of neighbors) with $k_i = \sum_{j \neq i} g_{ij}$ and g is the adjacency matrix of the interaction graph ($g_{ij} = 1$ if i and j are neighbors, $g_{ij} = 0$ otherwise). We use the convention $g_{ii} = 1$, assuming that each bank considers its own risk as one index component - among others - of the level of risk of the system. The definition of $\bar{y}$ captures the fact that each bank is embedded in a financial system characterized by the development of a

[1] If ρ_i is the return of the bank *i*, this return is a differentiable and concave relation $\rho_i = h(y_i)$ between return and risk; then the payoff function W can be written as $W(y_i, \bar{y}_i) = W(h(y_i), y_i, \bar{y}_i)$.

network of management and dispersion of risk by the way of securitization devices, frequently considered as having a positive effect on the capacity of each element in the system to tolerate a given level of activity and risk.
We suppose that at any given time t, each bank i adapts its control variable y_i according to the gradient dynamics

$$\dot{y}_i = \frac{1}{\tau_y} \partial_1 W(y_i, \overline{y}_i) \tag{2}$$

where $\partial_1 W = \frac{\partial W}{\partial y_i}$, and τ_y is a characteristic time for the adjustment of the variables y_i by the banks. For any given value of its second argument $\overline{y}_i$, the payoff function $W(y_i, \overline{y}_i)$ has a unique maximum $y_i(\overline{y}_i)$. Accordingly, if the terms $\overline{y}_i$ were considered as exogenous and fixed by each bank, this dynamics would tend to $y_i(\overline{y}_i)$. That is, if the coupling induced by the variables $\overline{y}_i$'s is discarded, each bank independently chooses the same level of risk, the unique maximum $y_i(\overline{y}_i) = \tilde{y}(\overline{y})$. It is easy to check that in this benchmark case, this equilibrium is linearly stable for the dynamics (2).

More interesting in our issue is the coupling where each bank computes the global level of risk $\overline{y}_i$ as an average over its neighbors on the interaction graph, since coupling and interactions may induce distortions in risk appreciation and instability. The rest of the paper investigates the existence and stability properties of homogeneous equilibriums with coupling.

2.2. HOMOGENEOUS EQUILIBRIUMS WITH COUPLING

We define a homogeneous equilibrium as a stationary solution of Eqs. (2) such that each bank chooses the same risk level. This reads $y_1 = \ldots = y_n = y^*$, where y^* is solution of

$$\partial_1 W(y^*, y^*) = 0 \tag{3}$$

since in this case $\overline{y}_i = y^*$, according to Eq. (1). Equivalently, Eq. (3) may be written as a fixed point of the continuous function $\tilde{y}$

$$\tilde{y}(y^*) = y^* \tag{4}$$

Without coupling, a bank is not at all influenced by its neighbors and $\tilde{y}$ is a constant function. In this case, given that we assume that W is the same for all banks, y^* is also the same for all banks. Then, in this case, the levels of activity, risk, and return of partner-banks or neighbors have no effect on the decisions of bank

i . With coupling, a bank is influenced by its neighbors. If this influence is sufficiently weak, the derivative of $\tilde{y}$ remains smaller than 1 and the unique fixed point property still holds. This implies that there is a unique homogeneous equilibrium, similar to the equilibrium of the system without coupling: the perturbations of this equilibrium generated by external influences are only transitory. If the neighbors' influence is sufficiently strong, the derivative of $\tilde{y}$ may become larger than 1 , and several homogeneous equilibriums may exist. If this same influence of the actions of banks on their neighbors is moderate, this stimulation is not permanent, that is, the initial equilibrium is stable.

2.3. LINEAR STABILITY OF HOMOGENEOUS EQUILIBRIUMS WITH COUPLING

Let us now consider the linear stability properties of such a homogeneous stationary solution y^* . We linearize the dynamics around this equilibrium and study the evolution of a perturbation v_i :

$$\dot{v}_i = \partial_{11} W v_i + \partial_{12} W \sum_j \frac{g_{ij}}{k_i + 1} v_j \tag{5}$$

where all partial derivatives are taken at the point (y^*, y^*) . Using the notations

$$\alpha = \partial_{11} W \text{ , } \beta = \partial_{12} W \tag{6}$$

we rewrite in vectoral form the equation for the v_i 's:

$$\dot{\vec{v}} = \alpha I \vec{v} + \beta B \vec{v} \tag{7}$$

where I is the $n \times n$ identity matrix, and $B = (b_{ij})_{i,j=1}^n$ is the graph connectivity matrix normalized to 1 line by line: $b_{ij} = g_{ij}/(k_i + 1)$. We now need to study the eigenvalues and eigenvectors of the matrix $\alpha I + \beta B$. We first consider a complete graph configuration in which each agent is connected to all others; then we discuss the case of an arbitrary connection graph.

2.3.1. THE COMPLETE GRAPH

Proposition 1: Consider a complete interaction graph, and let $y_1 = \ldots = y_n = y^*$ be a homogeneous stationary solution of Eqs. (2). This solution is linearly unstable if and only if $\alpha + \beta > 0$.

Proof of Proposition 1: If each agent is connected to all others, we simply have $b_{ij} = 1/n$. In this case, the eigenvalues of B are 1 (multiplicity 1), and 0 (multiplicity $n-1$). The eigenvalues of the matrix $\alpha I + \beta B$ are then $\alpha + \beta$ (multiplicity 1) and α (multiplicity $n-1$), and the corresponding eigenvectors are those for the matrix B. Thus, a homogeneous stationary solution is linearly unstable if and only if $\alpha+\beta>0$.

Note that since y^* is a maximum of W at fixed $\bar{y}=y^*$, then $\alpha<0$. Thus, α has a stabilizing effect and the potential destabilizing element comes from β which contains the coupling between the agents.

The most unstable eigenvector is the vector $\vec{w}_1$ corresponding to the eighenvalue of highest modulus

$$\vec{w}_1=\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix} \tag{8}$$

all other eigenvectors being stable. Consequently, starting close to an unstable homogeneous solution, we can expect a collective dynamics of all agents in the same direction.

The intuition behind this result is the following. In this complete graph configuration, all agents correctly compute the same and actual levels of systemic risk $\bar{y}_i=\bar{y}$ where $\bar{y}_i=\frac{1}{n}\sum_{j=1}^{n}y_j$. But at the same time, the generalization of credit risk transfer technologies may induce agents to tolerate higher levels of risk.

2.3.2. An arbitrary graph

We consider an arbitrary bank interaction graph. The result of the previous paragraph essentially extends to this case.

Proposition 2: Consider an arbitrary interaction graph, assume $\beta\geq 0$, and let $y_1=\ldots=y_n=y^*$ be a homogeneous stationary solution of Eqs. (2). This solution is linearly unstable if and only if $\alpha+\beta>0$.

Proof of Proposition 2: B is a stochastic matrix (all its entries are non-negative, and each of its rows sums to 1). Thus, $\vec{w}_1$ is an eigenvector of B, associated with eigenvalue 1, and we know that all other eigenvalues of B have modulus at most 1 (for a reference, see for instance Serre, 2002). Thus, the eigenvalue of $\alpha I+\beta B$ with the largest real part is again $\alpha+\beta$, and $\alpha+\beta>0$ is again a necessary and sufficient condition for linear instability in this case.

In this case, the most unstable eigenvector is still $\vec{W}_1$. However, other eigenvectors, corresponding to different evolutions for different agents, may also be unstable. For instance, this result is obtained if the graph possesses strongly connected clusters of agents, with loose connections between clusters. This configuration is depicted in Figs. 1 and 2 below. In this case, different forms of instability can be observed for the components of the network. The eigenvector associated with the second eigenvalue (red points) defines identical dynamics for agents 2 to 5 and 7 to 10. Due to their position on the graph, agents 1 and 6 display a different path.

FIGURE 1. AN EXAMPLE OF A GRAPH WITH TWO WELL CONNECTED CLUSTERS, LOOSELY CONNECTED ONE WITH THE OTHER.

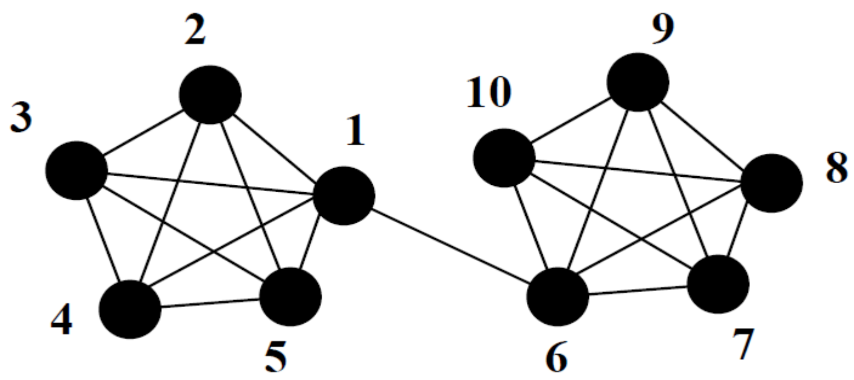
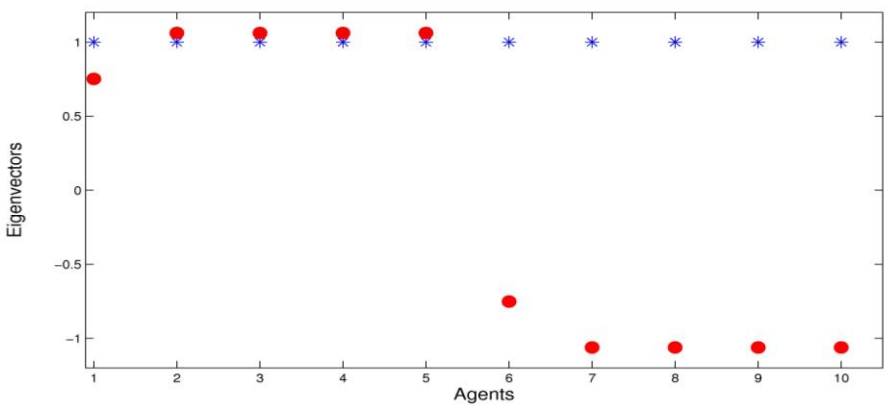


FIGURE 2. THE FIRST EIGENVECTOR OF THE CONNECTIVITY MATRIX B OF THE GRAPH ON THE LEFT (STARS), CORRESPONDING TO EIGENVALUE $\lambda_1 = 1$ AND THE SECOND EIGENVECTOR (CIRCLES), CORRESPONDING TO EIGENVALUE $\lambda_2 = 0.942$



We can illustrate these analytical properties with an example.

2.4. An example

The utility function is specified as follows:

$$W(y_i, \overline{y}_i) = y_i\left(2 - y_i + \frac{y_i}{2} tanh[\sigma(\overline{y}_i - 1)]\right), \;\; i = 1, \ldots n \quad (9)$$

where $\sigma \geq 0$ is a parameter which encapsulates the coupling between agents. The function $\tanh$ encapsulates the influence of the environment of a bank, captured by $\overline{y}_i$ the average level risk, on his individual decisions. This function is increasing in $\overline{y}_i$, thus a bank is likely to take higher risks when its neighbors over the interaction graph make the same type of decision. However, the shape of the function implies that this influence is bounded. The coupling strength σ is a parameter measuring the weight given by the individual agents to this collective mechanism, in their decision process.

For $\sigma = 0$ there is no coupling and the dynamics is simply given by:

$$\partial_1 W(y_i, \overline{y}_i) = 2(1 - y_i) \quad (10)$$

In this case the homogenous stationary solution $y^* = 1$ is asymptotically stable.

For $\sigma > 0$ there is coupling and the dynamics is given by:

$$\partial_1 W(y_i, \overline{y}_i) = 2(1 - y_i) + y_i \tanh[\sigma(\overline{y}_i - 1)], \;\; i = 1, \ldots n \quad (11)$$

In this case a homogenous stationary solution y_i^* verifies $y_i = \frac{2}{2 - tanh[\sigma(\overline{y}_i - 1)]}$.

FIGURE 3. STABILITY OF THE HOMOGENOUS STATIONARY SOLUTION $y^* = 1$ WITHOUT COUPLING AND $n = 30$

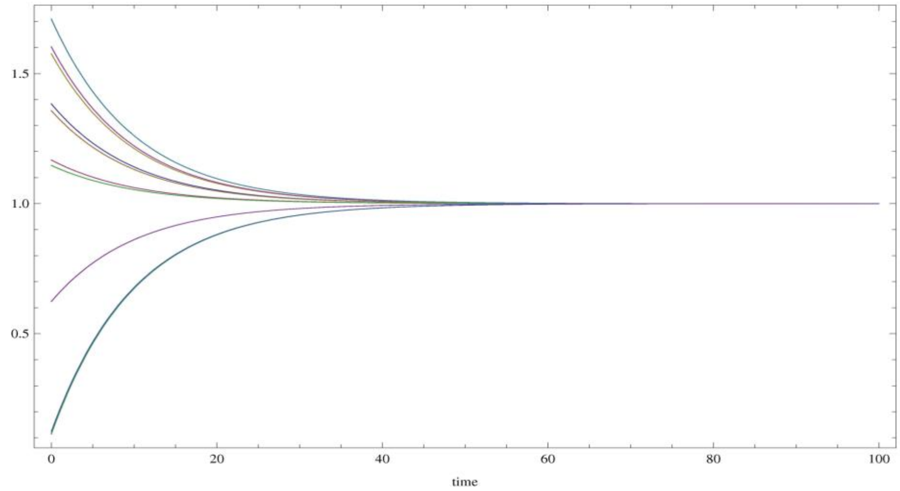
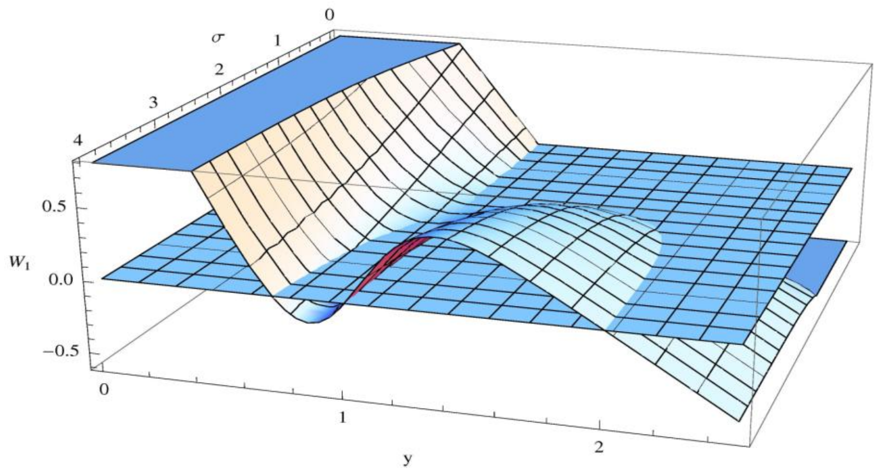
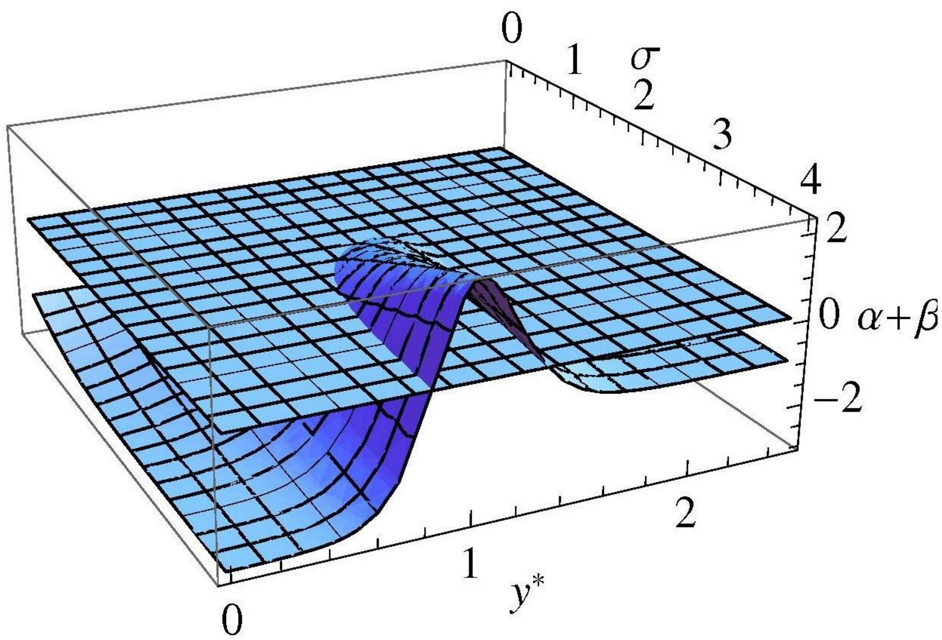


FIGURE 4. EXISTENCE OF HOMOGENEOUS STATIONARY SOLUTIONS y^* FOR σ WITH COMPLETE GRAPH



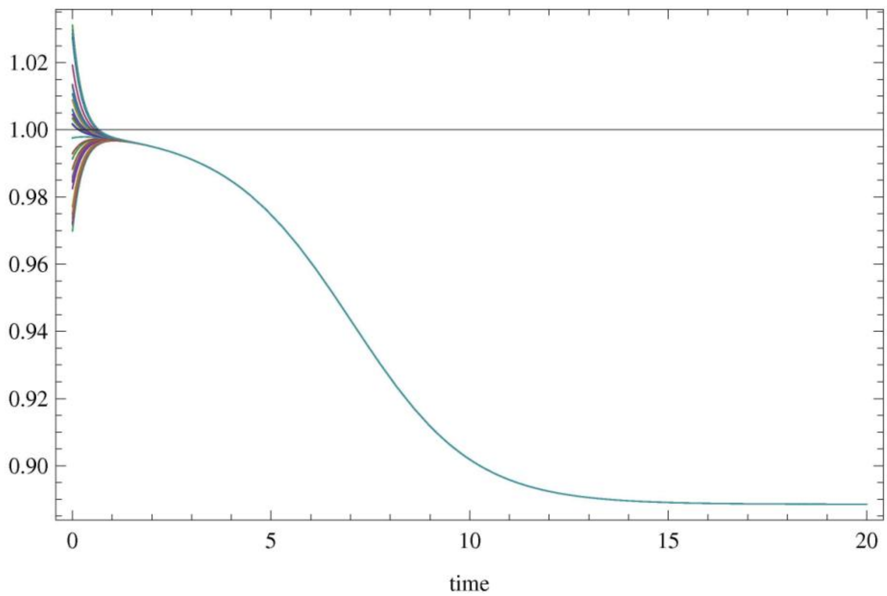
As shown in Fig. 4 below, for small values of the coupling strength the unique homogenous solution is still $y^* = 1$. For larger values of the parameter σ, multiple homogenous solutions with $\frac{2}{3} < y^* < 2$ exist. Using the notations defined in section 2.3 we have $\alpha = \partial_{11} W = -2 + \tanh[\sigma(\bar{y}_i^* - 1)] < 0$ and $\beta = \partial_{12} W = \sigma y_i^* \text{sech}[\sigma(\bar{y}_i^* - 1)]^2 > 0$. Then the stability criterion of these solutions, $\alpha + \beta < 0$ depends nonlinearly on σ.

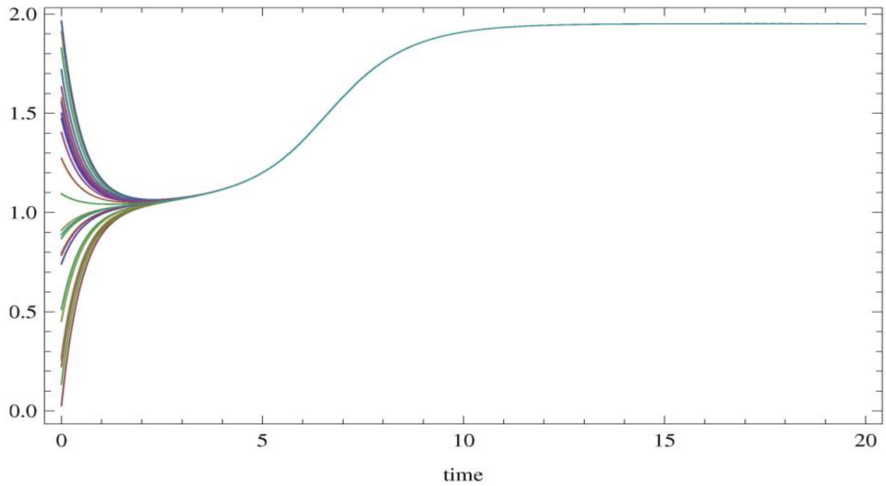
FIGURE 5. STABILITY OF HOMOGENEOUS SOLUTIONS FOR σ WITH COMPLETE GRAPH



The initial homogeneous solution $y^* = 1$ becomes unstable for $\sigma \geq 2$. Then, with a complete graph configuration and larger values of the coupling strength, the n levels of risk of the n banks eventually converge to a low homogeneous solution, $\frac{2}{3} < y_L^* < 1$ or to a high one, $1 < y_H^* < 2$. The low solution is obtained with $\bar{y}(t = 0) < 1$, that is, when the average initial distribution of risks is small enough. Conversely, the high equilibrium is reached when the average initial distribution of risks is large, that is, for $\bar{y}(t = 0) > 1$. Fig. 4 and Fig. 5 show that the convergence process toward the low or high solutions falls roughly into two phases: in the first phase, the different levels of risk tend to converge to a level of risk slightly lower or higher than the initial homogeneous solution. In this phase, the tendency of each bank to find its equilibrium level of risk is only moderately influenced by the level of risk of the other banks. In the second phase, a 'systemic' dynamics moves all the levels of risk, which are fairly equal, to a low or a high homogeneous equilibrium. In this second phase, coupling interacts strongly with the forces generated by the internal characteristics of each bank.

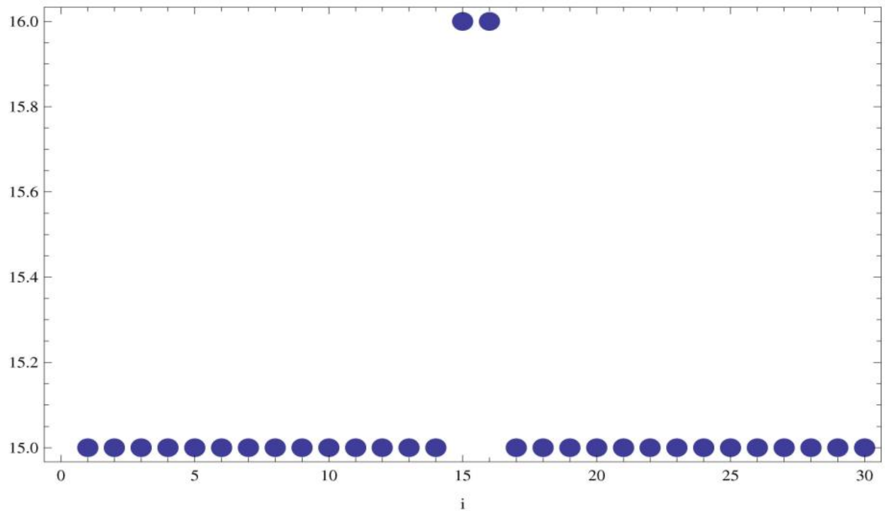
FIGURE 6. CONVERGENCE TO LOW (LEFT-HAND SIDE) AND HIGH (RIGHT-HAND SIDE) HOMOGENEOUS SOLUTIONS; COMPLETE GRAPH $n = 30$ AND $\sigma = 2.31$





Heterogeneous choices can be obtained with arbitrary incomplete connection graphs and sufficient coupling strength. From this standpoint, we can consider the connection topology described on Fig.1. Fig. 1 depicts two fully connected clusters of $\frac{n}{2}$ agents with only one connection between clusters. Then, with $n = 30$ connectivity is shaped as follows:

FIGURE 7. CONNECTIVITY k_i WITH $n = 30$



For small values of coupling strength, ($0<\sigma<2$), the initial homogeneous solution $y^*=1$ remains stable. While the parameter remains smaller than a critical value $\sigma \approx 2.31$, in this example the levels of risk converge to the high homogeneous solution $1< y_H^* <2$.

For larger values of the parameter, ($\sigma \geq 2.32$), heterogeneous choices, low and high, can be obtained for a large range of initial conditions, according to the location of the banks in the network.

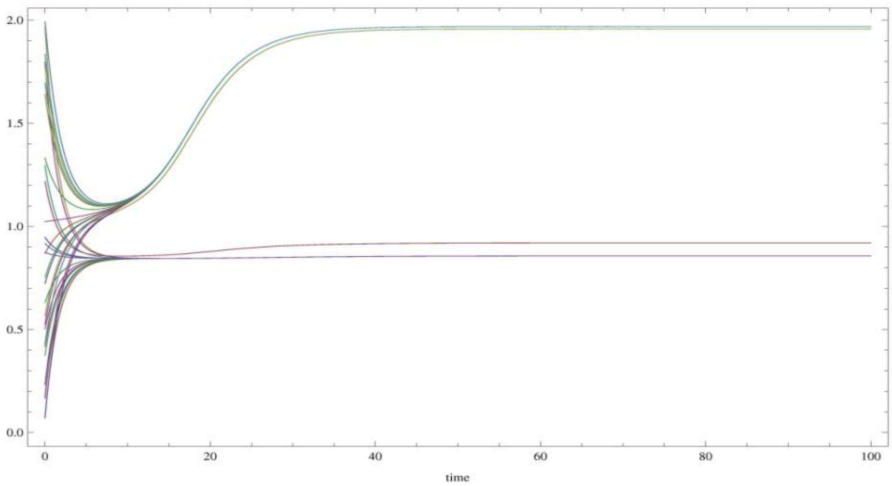


FIGURE 8. HETEROGENEITY WITH INCOMPLETE GRAPH; $n=30$, $\sigma=2.32$

THE RIGHT HAND SIDE FIGURE DISPLAYS THE DYNAMICS AND THE RIGHT HAND SIDE THE LEVELS OF RISK AT THE STATIONARY STATE.

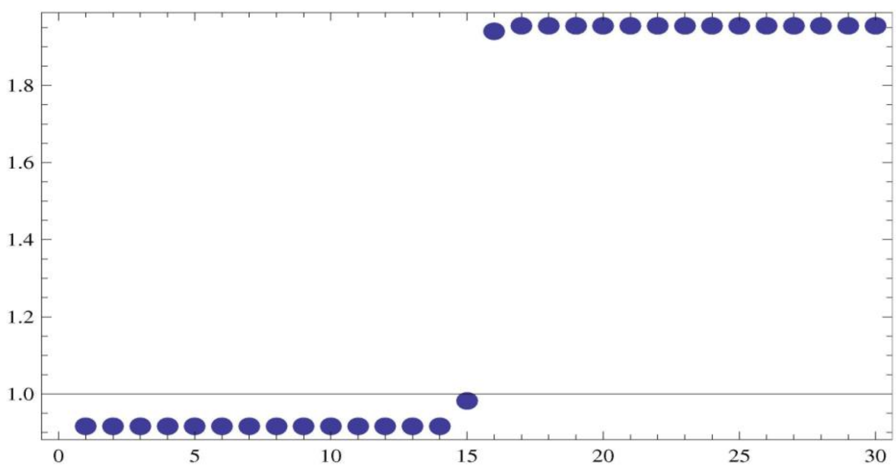
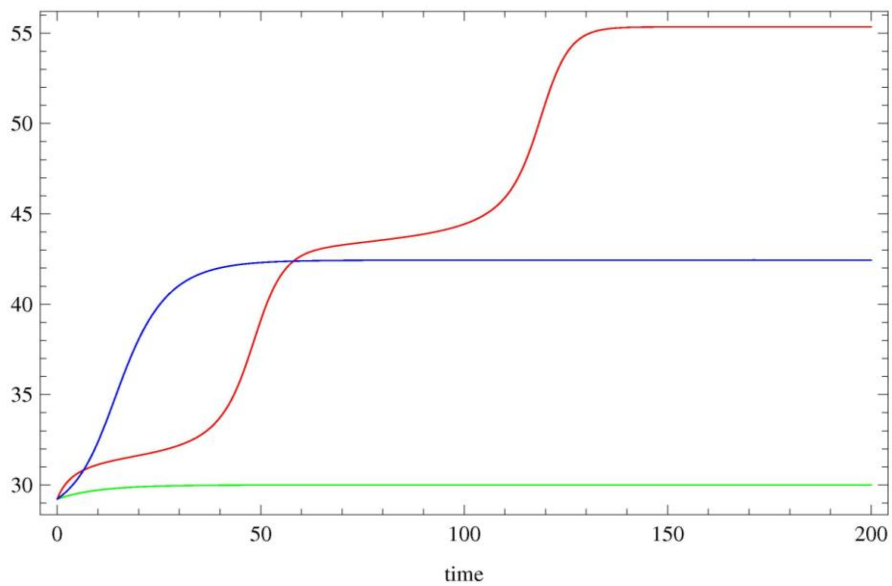


Fig. 8 shows that the banks located in the first cluster choose a low level of risk. We have for $i = 1,...14$ $y_L^* = .86$ and $y_L^* = .92$ for $i = 15$. In the second cluster, the banks take higher risks. For $i = 16$, $y_L^* = 1.96$ and for $i = 17,...30$, $y_H^* = 1.97$. Note from Fig. 8 that in this case also, we can distinguish two phases in the convergence process to the heterogeneous equilibrium. In the first phase, the risks tend to converge in each cluster toward average values close to the initial equilibrium. In the second phase, each cluster has an independent dynamics of convergence toward a low or high equilibrium.

Note also that, as a consequence of this heterogeneous equilibrium, the level of aggregate risk may decrease compared to the homogeneous case. In the following example we obtain a significantly lower systemic risk with higher coupling in the heterogeneous case than in the homogeneous case with lower levels of coupling.

FIGURE 9. SYSTEMIC RISK WITH INCOMPLETE GRAPH; $n = 30$. RED: HOMOGENEOUS CASE $\sigma = 2.31$;

BLUE: HETEROGENEOUS CASE $\sigma = 2.33$; GREEN: WITHOUT COUPLING $\sigma = 0$.



Finally, we note that these results obtained for arbitrary unweighted graphs can be extended to weighted graphs, which correspond to cases where the influence of each neighbor is given a weight.

CONCLUSION

The simple model developed in this paper analyzes the way that perception of change in the level risk of a bank temporarily or permanently affects the level of risk of its partners. We focus on situations where the use of credit risk transfer devices explains that the stimulating effects of an increase in the risk level environment of a given bank dominates over the depressing effects. The banks we consider try, at each moment, to adjust their indexes of utility, which depends on their risk, their return and the level of activity, risk, and return of their environment. Without any possibility to transfer the risk or any counterparts in the issuance or holding of credit derivatives, the regulation constraints on the amount of their net positions naturally bounds their exposure, their risk, and their utility index. The relationship with other financial agents provides each bank with possibilities to improve its management of the regulatory constraints.

We capture the mutual influence of the banks through a description of the network topology linking the banks. We find that the weight of the network interaction has consequences for the stationary equilibrium of each bank's risk. As the importance of the network effects increases, the 'autarky' equilibrium becomes unstable and banks move dynamically toward new equilibriums, with lower or higher levels of risk according to the form of the network and the initial risk (and return) conditions.
The role of the form of the network is interesting. We focus on the effects of local 'networks': connections among banks are not uniformly distributed across the global banking industry. We find that the network structure of the partnerships among similar banks can induce heterogeneous stationary equilibrium levels of risk and utility if the coupling is sufficiently strong. In the absence of links, the same banks would achieve homogeneous equilibrium. If the network effects are sufficiently strong and all banks manage credit risk in the same way, the environment can encourage some banks to take on too much risk and induce others to decrease their level of risk or to maintain it at reasonable levels. Our small size sample generated heterogeneous equilibrium involving two clusters and four different levels of risk. A higher level of heterogeneity can be generated with smaller samples. We can conjecture that in case of heterogeneous equilibriums, network effects have different influences on different clusters in the banking industry.

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