EXPERIMENTAL ANALYSIS AND MODELING OF MANGOES DRYING KINETICS

C. Heilporn¹, C. Wylock¹, L. Spreutels¹, B. Haut¹

¹Service TIPs, Université Libre de Bruxelles
50, avenue F.D. Roosevelt CP 165/67, 1050 Bruxelles, Belgium
Tel.: +32 2 650 29 18, E-mail: bhaut@ulb.ac.be

Abstract: In this work, the drying rate of mango slices is experimentally studied and modeled. The developed model takes into account the physical transformations occurring during the drying process. The simulated drying rates, computed with the proposed model are compared with a set of experimental drying rates obtained by analyzing several drying trials. The model is also used to analyze the sensitivity of the drying rate with respect to its physico-chemical parameters. The results show a good fit between the model and the experimental data.

Keywords: drying, mango, modeling, food preservation, drying rate

INTRODUCTION

Fruits drying is a widely used process in the food industry. This process enables increasing significantly the products shelf life and reducing their volume and their mass (Aghbashlo et al., 2009). Moreover, undesirable changes of the products can be greatly avoided by the elimination of most microorganisms during fruits drying (Ochoa-Martínez et al., 2012). Nevertheless, it is still a major engineering challenge because fruits are highly sensitive to heat changes.

Several studies have been realized in order to improve the drying process, regarding to the energy consumption and the quality of the dried products (Koua et al., 2009). Especially, one of the most important issue in food drying is the analysis and the modeling of the drying rate. Indeed, an accurate and robust (working for a wide range of operating conditions) modeling enables highly improving the design and the optimization of the dryers.

A literature review shows that the mango drying have been widely studied these last years. The studies realized by (Desmorieux et al., 2008; Dissa et al., 2009) led to several drying rates expressions. Gbaha et al. developed an empirical model for a solar mango dryer (Gbaha et al., 2007). Goyal et al. compared several empirical drying models for mangoes. They showed that a good agreement can be reached using an exponential Arrhenius type model (Goyal et al., 2006). Janjay et al. realized numerical simulations with the finite element method for the drying of a mango slice (Janjai et al., 2008).

Most of these models are based on empirical or semi-empirical approaches, which limit their use to specific ranges of drying conditions. These studies aim generally to improve the drying process in terms of energy consumption, process design, products quality and energy consumption (Mujumdar, 2007). Nevertheless, the use of such empirical models to design the dryer suffers from poor control of the drying operating conditions that can lead to poor product quality. In particular, the food is often overheated, resulting in changes of nutritional properties, taste and color.

The development of a mathematical model taking into account simultaneously the heat and mass transfer phenomena during the drying process is therefore of a fundamental interest for the design of efficient dryers (Nonclerq et al., 2009).

This study is devoted to the drying process of mangoes. Mangoes are one of the most exported dried fruit in West Africa (Rivier et al., 2009). Actually, without this transformation process, a large part of the mango production is wasted due to a lack of local, national and international trade opportunities.

This work has three main objectives:

1. To carry out several drying trials and to analyze their drying rates.
2. To develop an original mathematical model, based on heat and mass balances and on the expression of transfer coefficients for a complex porous media. This model leads to a mathematical expression to simulate the drying rate.
3. To discuss the influence of the different physico-chemical parameters of the model, by comparing the analysis of the drying rates trial with the drying rate simulated by the developed model.

MATERIALS AND METHODS

Experimental dryer

The dryer used for the experiments is schematically presented in Figure 1. It is made up of a circular fixed bed (diameter of 0.14 m) equipped with a heat exchanger. Ambient air is pumped and conveyed through a heat exchanger, in order to generate a hot air flow for the fixed bed. A heating coil is placed just before the fixed bed inlet, is used to regulate the air temperature in the fixed bed.

Fresh mango slices are placed on three trays in the fixed bed. The hot air flows perpendicularly to the trays and it leaves the device by the top of the fixed bed. The air exiting the device is charged with moisture, resulting from the evaporation of the water contained in the mango slices.

Fig. 1. Block-diagram of the experimental dryer used to dry the mango slices

Measurements

The initial mass of the fresh mango slices, \( M_{\text{in}} \) (kg of mango), is weighted in a Sartorius balance CP12001S (d=0.1 g), before to be placed on the three trays of the fixed bed. For each drying trial, the mangoes are sliced with chosen thickness, \( \Delta_0 \) (m).

A rotameter Rota is used to set the airflow rate to a chosen value \( Q \) (m\(^3\)s\(^{-1}\)) (Heilporn et al., 2012). The superficial velocity of the air flowing in the fixed bed, \( u \) (m s\(^{-1}\)), is calculated by:

\[
  u = \frac{Q}{\Omega}
\]

where \( \Omega \) (m\(^2\)) is the fixed bed section.

The air temperature is measured at several positions (see Figure 1): at the fixed bed inlet, inside the fixed bed and at the fixed bed outlet. They are noted \( T_{\text{in}} \) (K), \( T_{\text{r}} \) (K) and \( T_{\text{out}} \) (K), respectively.

\( T_{\text{in}} \) and \( T_{\text{out}} \) are measured using a data logger Testo 400 connected to thermo-hygrometric probes. \( T_{\text{r}} \) is measured with a type K thermocouple connected to a data logger Agilent.

An additional type K thermocouple is used for the temperature regulation in the fixed bed. The measured temperature is noted \( T_{\text{b}} \).

The air moisture content is measured using a data logger Testo 400 connected to thermo-hygrometric probes at the fixed bed inlet and outlet (see Figure 1). These measures are noted \( Y_{\text{in}} \) and \( Y_{\text{out}} \), respectively.

Data processing

The experimental drying rate at the time \( t \) can be expressed by:

\[
  \dot{M}_3(t) = \frac{Q (Y_{\text{out}}(t) - Y_{\text{in}}(t))}{M_s}
\]

where \( J_{\text{exp}}(t) \) is the mango slices drying rate at the time \( t \) (kg water kg dry mass\(^{-1}\) h\(^{-1}\)), \( Q \) is the air flow rate in the dryer (m\(^3\) h\(^{-1}\)), \( M_s \) is the dry mass of the mango slices in the dryer (kg dry mass), \( Y_{\text{in}}(t) \) is the air moisture content at the fixed bed inlet at the time \( t \) (kg water kg dry air\(^{-1}\)) and \( Y_{\text{out}}(t) \) is the air moisture content at the fixed bed outlet at the time \( t \) (kg water kg dry air\(^{-1}\)).

At the same time as each experiment with the fixed bed dryer, a homogeneous sample of mango slices is placed in a stove at 70°C during 24 hours, according to the European standard EN 12145 (Disa et al., 2009; Disa et al., 2008). After 24 hours, the mango slices are completely dried and the final dried mass of the sample is measured. This enables estimating the initial moisture content of the mangoes, \( X_0 \), using Equation 3:

\[
  X_0 = \frac{M_{f,\text{stove}} - M_{0,\text{stove}}}{M_{f,\text{stove}}}
\]

where \( M_{0,\text{stove}} \) is the initial mass of fresh mango slices placed in the stove (kg mango), \( M_{f,\text{stove}} \) is the final dried mass of mango slices after 24 hours drying (kg dry mass) and \( X_0 \) is the initial moisture content of mango slices.

The dry mass of the mango slices in the dryer, \( M_f \) (kg dry mass), can then be deduced by the following equation:

\[
  M_f = \frac{M_{0,\text{stove}}}{X_0}
\]
\[ M_f = \frac{M_{ma}}{1 + X_0} \] (4)

Where \( M_{ma} \) is the initial mass of mango slices introduced in the dryer (kg mango) and \( M_f \) is the final mass (kg).

**Experimental conditions**

The influences of several operating conditions on the drying rate are investigated: the superficial velocity of the air, \( u \), the thickness of the mango slices, \( \Delta_0 \), and the temperature in contact with the food product, \( T \) (K).

To this end, a setpoint \( T_d(t) = T_{ref} \) is imposed to the temperature controller, which regulates the power of the heating coil in order to meet this setpoint. The experimental conditions used for the drying trials are presented in Table 1.

**Table 1. Experimental conditions used for the drying trials**

<table>
<thead>
<tr>
<th>Drying trial</th>
<th>( u ) (m/s)</th>
<th>( T_{ref} ) (°C)</th>
<th>( \Delta_0 ) (mm)</th>
<th>( M_{ma} ) (kg mango)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \sim 0.15 )</td>
<td>60</td>
<td>4</td>
<td>0.196</td>
</tr>
<tr>
<td>2</td>
<td>( \sim 0.15 )</td>
<td>50</td>
<td>4</td>
<td>0.284</td>
</tr>
<tr>
<td>3</td>
<td>( \sim 0.4 )</td>
<td>60</td>
<td>4</td>
<td>0.272</td>
</tr>
<tr>
<td>4</td>
<td>( \sim 0.4 )</td>
<td>60</td>
<td>7</td>
<td>0.225</td>
</tr>
<tr>
<td>5</td>
<td>( \sim 0.4 )</td>
<td>50</td>
<td>4</td>
<td>0.278</td>
</tr>
</tbody>
</table>

Each experiment is realized in approximately 4 hours of drying.

**Drying rate**

A mathematical expression is proposed to model the drying rate of mango slices. This expression is based on the hypothesis that the drying rate \( J_{th} \) (expressed in kg of water per kg of dry mass and per hours) is proportional to the deviation from equilibrium between the water vapor concentration in the gas in contact with the evaporation surface and the water vapor concentration in the bulk of the gas phase. Therefore, this expression writes:

\[ J_{th}(t) = aM_wk(X(t)) \left( \frac{p_{sat}(T(t))}{R_gT(t)} - \frac{Y(t)}{M_w} \right) \] (5)

where \( a \) is the mango slices specific surface area (m\(^2\) external surface kg dry mass\(^{-1}\)), \( k(X(t)) \) is the mass transfer coefficient depending on the moisture content at the time \( t \) (m\(^2\)s\(^{-1}\)), \( M_w \) is the molar mass of water (kg mol\(^{-1}\)), \( R_g \) is the perfect gas constant (J mol\(^{-1}\)K\(^{-1}\)), \( T(t) \) is the temperature in contact with the mango slices at the time \( t \) (K), \( \lambda(X(t)) \) is the moisture content of the mango slices (kg water kg dry mass\(^{-1}\)) and \( Y(t) \) is the air moisture content in contact with the mango slices at the time \( t \) (kg water kg dry air\(^{-1}\)).

It is assumed that the air is a perfect gas and that the mango porosity does not influence the specific surface area of the mango slices, can then be expressed using Equation 6:

\[ a = \frac{2(1 + X_0)}{\Delta_0\rho_{ma}} \] (6)

where \( \Delta_0 \) is the initial thickness of the mango slices (m) and \( \rho_{ma} \) is the mango slice density (kg m\(^{-3}\)). Since mangoes contain about 85% of water, \( \rho_{ma} \) is approximated by the water density.

It is also assumed that the internal mass transfer limit the water evaporation, meaning that there is no significant constant drying rate period. The mass transfer coefficient is then expected to decrease the moisture content of the mango slices decreases. A linear correlation between \( k \) and \( X \) is proposed:

\[ k(X) = k_{max} \left( 1 - \beta \left( \frac{X(t) - X}{X_0} \right) \right) \] (7)

where \( k_{max} \) is the maximum mass transfer coefficient (ms\(^{-1}\)) and \( \beta \) is the decreasing rate of \( k \) with respect to \( X \).

Thanks to a water mass balance on a mango slice, the mango drying rate is correlated to the time derivative of the mango water content by Equation 8:

\[ J_{th}(t) = -\frac{dX(t)}{dt} \] (8)

Thanks to the experimental drying rate results, numerical values of the model parameters \( \beta \) and \( k_{max} \) can be estimated. These values are estimated using an iterative fitting procedure, implemented with the Mathematica 5.1 software.

Within this procedure, for each drying trial, the experimental time evolution of the drying rate \( J_{exp}(t) \) is first evaluated using Equation 2. In a second time, the time evolution of the theoretical drying rate \( J_{th}(t) \) is simulated by injecting Equation 5 in Equation 7 and by solving numerically Equation 8. In Equation 5, the temperature of the air in contact with the mango slices is assumed equal to \( T_d(t) \) and the air moisture content in contact with the mango slices is assumed equal to \( Y_{air}(t) \). The numerical resolution of Equation 8 is realized using the “NDsolve” function of Mathematica.

The numerical values of \( \beta \) and \( k_{max} \) are then fitted by minimizing the quadratic difference between \( J_{th}(t) \) and \( J_{exp}(t) \), using the “FindMinimum” function of Mathematica.

**RESULTS**
The experimental time evolution of the drying rate (obtained using Equation 2) for the set of drying trials are presented in Figure 2.

Figure 2: Time evolution of the experimental drying rate for the set of drying trials.

It is observed that there is no constant drying period, as it is assumed in the proposed model and that the drying rate becomes very small after three hours.

The fitted values of the proposed model parameters, $\beta$ and $k_{\text{max}}$, are presented in Table 2, according to the corresponding trial number. The values of the initial moisture content of the mango $X_0$ and the maximum drying rate $J_{\text{max}}$ are also presented in Table 2. $J_{\text{max}}$ is the maximum drying rate observed for each drying trials performed.

Table 2: Measured and fitted results of the experimental drying trials

<table>
<thead>
<tr>
<th>Drying trials</th>
<th>$X_0$ (kg water/kg dry mass)</th>
<th>$J_{\text{max}}$ (kg water/kg dry mass/h)</th>
<th>$k_{\text{max}}$ (ms$^{-1}$)</th>
<th>$\beta$ (–)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.93</td>
<td>2.70</td>
<td>$2.54 \times 10^{-3}$</td>
<td>0.87</td>
</tr>
<tr>
<td>2</td>
<td>6.06</td>
<td>2.14</td>
<td>$1.43 \times 10^{-3}$</td>
<td>0.74</td>
</tr>
<tr>
<td>3</td>
<td>4.32</td>
<td>2.06</td>
<td>$2.12 \times 10^{-3}$</td>
<td>1.15</td>
</tr>
<tr>
<td>4</td>
<td>6.42</td>
<td>3.32</td>
<td>$1.65 \times 10^{-3}$</td>
<td>0.93</td>
</tr>
<tr>
<td>5</td>
<td>8.37</td>
<td>3.42</td>
<td>$2.13 \times 10^{-3}$</td>
<td>0.77</td>
</tr>
</tbody>
</table>

No clear trends are observed for $\beta$ and $k_{\text{max}}$ with respect to the experimental conditions. Therefore, it assumed that these parameters are constant. A statistical inference is then realized using the numerical values presented in Table 2. The average value of $k_{\text{max}}$ within a 95 % confidence interval is $2.0 \times 10^{-3} \pm 5 \times 10^{-4}$ ms$^{-1}$. The average value of $\beta$ within the same confidence interval is $0.9 \pm 0.2$.

A direct validation of the proposed model is realized by comparing the time evolution of the experimental drying rate and the simulated theoretical drying rate. This latter is calculated considering the corresponding operating conditions and using the corresponding fitted values of $\beta$ and $k_{\text{max}}$. These comparisons are presented for all drying trials in Figures 3a and 3b. The experimental and simulated results are compared for the trial 1 and 4 (realized using $T_{\text{ref}}=60^\circ\text{C}$) in Figure 3a and for the trial 2, 3 and 5 (realized using $T_{\text{ref}}=50^\circ\text{C}$) in Figure 3b.

Figure 3a: Direct validation of the model by comparing of the time evolution of the experimental drying rate and the simulated drying rate for the drying trials 1 and 4 ($T_{\text{ref}}=60^\circ\text{C}$).

Figure 3b: Direct validation of the model by comparing of the time evolution of the experimental drying rate and the simulated drying rate for the drying trials 2, 3 and 5 ($T_{\text{ref}}=50^\circ\text{C}$).

It is observed that there is a good fit between the experimental drying rates and the theoretical drying rates obtained with the model. The theoretical drying rates presented in Figures 3a and 3b is obtained with the specific values of $\beta$ and $k_{\text{max}}$ presented in Table 2.

A cross validation of the proposed model is also realized by comparing the time evolution of the experimental drying rates of two different drying
trials (1 and 5) with two simulated theoretical drying rates computed using the averaged values of $\beta$ and $k_{\text{max}}$. This comparison is presented in Figure 4.

It is observed that there is a good agreement between the experimental drying rates and the theoretical drying rates obtained by simulation with the average values of $\beta$ and $k_{\text{max}}$.

Nevertheless, as it could be expected, it is observed that the agreement is not as well as using the specific values of $\beta$ and $k_{\text{max}}$ corresponding at each drying trial.

![Figure 4: Cross validation of the model by comparing the time evolution of the experimental drying rate and the simulated drying rate for the drying trials 1 and 5, using the averaged values of $\beta$ and $k_{\text{max}}$.](image)

DISCUSSION

It is observed in Figure 2 that the initial stage of the experimental drying rate does not present a significant constant drying period. This phenomenon, observed here for mango slices, is often observed in foodstuff drying. This characteristic is implicitly included in the proposed drying rate model, which does not consider any constant drying period. It is worth to notice that the proposed model can simulate very well the observed experimental results during the initial drying stage. This tends to validate the assumption that there is no constant drying period in the case of mango drying.

The Figures 3a and 3b illustrate the excellent agreement between the experimental drying rates and the simulated ones, computed using the values of $k_{\text{max}}$ and $\beta$ presented in Table 2. It is then clearly shown that the developed drying rate model is able to predict with accuracy the overall time evolution of the drying rate. Moreover, it is observed in Figure 4 that, when the averaged values of $k_{\text{max}}$ and $\beta$ are used, the model predicts drying rates, which are very close to those observed experimentally.

In addition, this study highlights a high sensitivity of the drying rate with respect to the temperature in contact with the mango slices, which is not generally mentioned in literature. Indeed, on the one hand, the lowest values of $J_{\text{max}}$ are observed for the trials 2 and 3 (2.14 and 2.06 kg of water per kg of dry mass and per hour, respectively), which are performed at 50°C. On the other hand, the trials realized using a temperature of 60°C show a higher maximum drying rates (2.7, 3.32 and 3.42 kg of water per kg of dry mass and per hour, respectively). Such a sensitivity with respect to the temperature could be expected and it is worth to notice that the developed drying rate model can emulate it very well. Indeed, as it can be seen in Equation 5, the temperature acts on the driving force term, via the difference of water vapor concentrations in the bulk of the gas phase.

Concerning the influence of the airflow rate $Q$, it is observed that $Q$ has no significant influence on the drying rate. Indeed, the trials 1 and 4 show similar maximum drying rates whereas the gas superficial velocities are different (respectively 0.15 and 0.4 m s$^{-1}$). Actually, the airflow is used to prevent the gas phase to be saturated with water vapor. The airflow rate has only an indirect influence on the experimental drying rate computation, via the outlet air moisture content $Y_{\text{out}}$. Likewise, the airflow rate is not clearly included as a key parameter in the drying rate model, since it has an influence only via the air moisture content in contact with the mango slices $Y$ (which is set equal to $Y_{\text{out}}$).

The influence of the thickness and the initial moisture content of the mango slices on the drying rate is clearly highlighted by the experimental results. The drying rate increases as the thickness decreases and as the initial moisture content increases. In average, the drying rate seems to be directly proportional to the thickness. The sensitivity is even higher for very high initial moisture content, as shown in trial 5. The high sensitivity of the drying rate model with respect to these two parameters is included via the expression of the specific surface area (see Equation 6) and, for $X_0$, via the transfer coefficient (see Equation 7).

It is verified, by simulation of the drying rate model, that this sensitivity is well emulated.

CONCLUSIONS

In this work, the drying rate of mango slices is experimentally and theoretically studied. A set of drying trial is realized using a fixed bed dryer and an original drying rate model is developed.

From the experimental results, it appears that the drying rate of mango slices depends significantly on the air temperature used in the dryer but not on the air superficial velocity of the air. This result is very interesting as it enables determining with accuracy the optimum temperature to use in the dryer. Indeed, a higher temperature, within a range of temperature
that does not induce deterioration of the product, promotes a higher drying rate that lead to a good quality product in a shorter time than with a lower temperature.

Moreover, it is highlighted that the thickness and the initial moisture content (which is related to the mango maturity) of the mango slices also influence the drying rates. Therefore, the use of high initial moisture contents with low thickness of the mango slices should be promoted.

The original model presented in this paper is based on the physical changes that occur during the drying process. The excellent agreement between the experimental data and the simulated ones, computed using the proposed model, show that this drying rate model for mango slices can be used in a wide range of operating conditions. This is a real input in comparison with the previously developed models, which are mainly based on empirical or semi-empirical approaches, and which could lead to a poor quality product when varying the operating conditions.

NOMENCLATURE

\[ a \] mango slices specific surface area

\[ J \] drying rate general expression

\[ J_{exp}(t) \] mango slices drying rate at time \( t \)

\[ J_{th} \] theoretical drying rate evaluated using the proposed model

\[ k(X) \] mass transfer coefficient depending on the moisture content

\[ k_{max} \] maximum mass transfer coefficient

\[ M_a \] molar mass of air

\[ M_{s, stove} \] initial mass of mango slices placed in the stove

\[ M_{s, dried} \] dried mass of mango slices placed in the stove

\[ M_{ma} \] initial mass of mango slices introduced in the dryer

\[ M_{u} \] dry mass of the mango slices in the dryer

\[ M_w \] molar mass of water

\[ p_{sat}(T) \] saturated pressure at a chosen temperature

\[ Q \] Air flow rate in the dryer \( \text{m}^3 \text{s}^{-1} \)

\[ R_g \] perfect gas constant \( \text{J mol}^{-1} \text{K}^{-1} \)

\[ T \] temperature in contact with the K mango slices

\[ T_{in} \] air temperature at the fixed bed K inlet

\[ T_{out} \] air temperature inside the fixed K bed

\[ T_s \] air temperature at the fixed bed K outler

\[ t \] drying time \( \text{h} \)

\[ u \] superficial velocity of the air \( \text{ms}^{-1} \)

\[ X \] moisture content of the mango slices \( \text{kg water kg dry mass}^{-1} \)

\[ X_0 \] initial moisture content of mango slices \( \text{kg water kg dry mass}^{-1} \)

\[ Y \] air moisture content \( \text{kg water kg dry air}^{-1} \)

\[ Y_{in}(t) \] air moisture content at the fixed bed inlet at time \( t \) \( \text{kg water kg dry air}^{-1} \)

\[ Y_{out}(t) \] air moisture content at the fixed bed outlet at time \( t \) \( \text{kg water kg dry air}^{-1} \)

\[ \beta \] the decreasing rate of the mass transfer coefficient with respect to the moisture content of the mango slices

\[ \Delta_{0} \] initial thickness of the mango slices \( \text{m} \)

\[ \Omega \] fixed bed section \( \text{m}^2 \)

BIBLIOGRAPHY


Disa, A., Desmorieux, H. et al. (2008). Convective drying characteristics of Amelie mango (Mangifera Indica L. cv. "Amelie") with...


