The comparative effects of investments in human resources and physical investments on production and productivity in iron and steel. Application of production functions on an international cross-section (*).

by

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1. Introduction

It is, by now, a commonplace to say that growth of production or income is determined not only by physical capital and labour, but also by the qualifications and educational level of the work force or «human capital».

The question to what extent human capital formation contributes to increased production is, however, still a widely debated issue. It is an important issue too, since all nations spend a considerable part of their national income on education.

This paper presents the results of a research study in this area. The research was concentrated on an international sample of iron and steel industries and aimed to answer the following questions:

1) What is the quantitative impact of investments in education (human capital formation) on production and productivity?

2) How does the effect of investments in human resources compare with the effect of physical investments?

3) Do investments in human resources lead to a less than proportional, a proportional or a more than proportional decrease in physical capital?

4) What is the impact of increased factor inputs and scale on wages?

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Question 1 is concerned with the macro-economic returns on education. If we know how the money spent on education affects production and income, we have a yard-stick of their economic justification. Question 2 is intended to assess the comparative returns of human versus physical investments, while question 3 is connected with the problem of possible substitution between physical and human capital, \( \sigma_{K_1K_2} \).

In order to answer questions 1, 2 and 3 above we have fitted a number of production functions to the sample data collected. The simplest of these functions, the Cobb-Douglas function, can be expressed as follows:

\[
\log \frac{Y}{L} = a + b \log \frac{K_1}{L} + c \log \frac{K_2}{L} + d \log L
\]

\[
\log \frac{Y}{K_2} = a + b \log \frac{K_1}{K_2} + c \log \frac{L}{K_2} + d \log K_2
\]

where \( Y \) is production, \( K_1 \) is physical capital, \( K_2 \) is human capital, \( Y/L \) is human capital productivity and \( Y/L \) is labour productivity. The equations above are simple transformations of the Cobb-Douglas function

\[
Y = C K_1^b K_2^c L^f
\]

On this Cobb-Douglas function the following simple transformations are applied:

\[
Y = C \left( \frac{K_1}{L} \right)^b L^b \left( \frac{K_2}{L} \right)^c L^c L^f
\]

\[
\frac{Y}{L} = C \left( \frac{K_1}{L} \right)^b \left( \frac{K_2}{L} \right)^c L^{b+c+f-1}
\]

Taking logs and putting \( \log C = a \) and \( d = b + c + f - 1 \), leads to the first of the equations in log-linear form, presented above.

The second equation is derived in a similar manner. The coefficient \( d \), in the equations above, merely plays the role of scale factor, indicating whether there are constant returns to scale, in which case \( b + c + f = 1 \), increasing returns to scale, in which
case \( b + c + f > 1 \), or decreasing returns to scale, whereby \( b + c + f < 1 \). Those Cobb-Douglas functions are tested out in section 2.1.

The Cobb-Douglas function implicitly assumes that the elasticity of substitution between all the factors is one unit:

\[
\sigma_{K_1 K_2} = \sigma_{K_1 L} = \sigma_{K_2 L} = \sigma_{K_2 L} = 1
\]

To test whether this assumption of unit elasticity is valid, we fitted some other production functions, whereby the elasticity can be constant but different from one (the well-known CES-function) or can be variable (functions with Variable Elasticity of Substitution called VES-functions).

There are two ways of estimating the parameters of the CES-function. The first method, called the direct method, is based on a direct estimate of the parameters of the function. Estimation of the CES-function by this method is tried out in section 2.2.

Section 2.3. analyses the relationship between wages on the one hand, and productivity and capital intensities, \( K_1/L \), \( K_2/L \) and \( K_1/K_2 \) on the other hand.

Comparative wage levels are often used as an indirect measurement of the parameters of the CES-function and various forms of VES-functions. The final sections 3 and 4 of this study are devoted to some similar statistical estimates. At the same time they highlight some of the problems connected with the implicit assumptions underlying this estimation procedure.

Before presenting the statistical results obtained, some explanation will be given on the definition of variables and on the composition of the sample.

a) Productivity was defined as the relation between man-hours required and man-hours actually used. The European Communities provided us with manpower requirements: man-hours required to produce various iron and steel products. Multiplying these standard requirements with actual production data determined total man-hours required. Productivity indices were obtained by dividing the man-hours required by actual man-hours used in production.

b) Two alternative measures were used to evaluate physical capital: present value of existing capital stock and accumulated investments over a 14-year period. Both series of capital data are highly
correlated. Since we disposed of investment data for all 22 cases, whereas capital stock data were lacking for some of them, the final regressions were carried out with accumulated investments.

c) Human capital refers to costs of formal education embodied in employed labour. The evaluation of these costs was effected by means of the following successive steps:

— subdivision of the labour force into occupational groups;

— determination of the educational structure within each of the occupational groups on the basis of a sample, the sample size varying from 10 to 20 % in the low level occupational groups to 40 — 60 % in the high level occupational groups;

— multiplication of diploma levels by corresponding average years of education in the primary, secondary (general, on the one hand, and technical on the other) and higher levels;

— multiplication of the years of education by the corresponding unit costs per student-year.

The basic part of our sample consisted of 12 European enterprises (3 French, 3 German, 3 Belgian, 2 Italian and 1 Dutch), for which we had detailed information, either collected in the firms themselves or (with the permission of the firms) in the European Economic Communities. The sample size within the firm varied from 10 to 20 % for the unskilled groups, to 40 — 60 % in the higher occupational groups. Employment by the firms included in the sample comprised 17 % of the German iron and steel workers, 96 % of the Dutch, 19 % of the Italian, 10 % of the French. The Belgian sample represented 44 % of the iron and steel workers of Belgium and 32 % of the iron and steel workers of Belgium and Luxembourg.

The European sample was subsequently enlarged to include Japan, the United Kingdom, the United States, Argentina, Brazil, Formosa and India. The data for Argentina, Formosa and India refer to individual firms. The data for Brazil refer to the large iron and steel companies of the State of Sao Paulo. The data for the U.K., U.S.A. and Japan are «national » averages of the large firms.

2. Presentation of the statistical results

2.1. The relative impact of physical and human capital on productivity and production

The results of the simple log-linear regression equations are given in Table 1. These regressions express labour productivity
TABLE I

Simple log-linear or Cobb-Douglas relationships between productivity, capital intensities and scale

<table>
<thead>
<tr>
<th>Equation number</th>
<th>Independent variable</th>
<th>Explanatory variables</th>
<th>Scale factor</th>
<th>Coefficient of determination $R^2$</th>
<th>Underlying functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>log $K_1/L$</td>
<td>log $K_2/L$</td>
<td>log L</td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td></td>
<td>0.872 (0.152)</td>
<td></td>
<td>0.261 (0.203)</td>
<td>1.261</td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td>0.708 (0.180)</td>
<td>0.031 (0.238)</td>
<td></td>
<td>0.456 Two-variable Cobb-Douglas with $K_2$ and $L$</td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td>0.738 (0.094)</td>
<td>0.536 (0.090)</td>
<td>0.337 (0.122)</td>
<td>1.337</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>log $K_1/K_2$</td>
<td>log $L/K_2$</td>
<td>log $K_2$</td>
<td>Scale factor</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td>0.642 (0.072)</td>
<td>0.272 (0.072)</td>
<td></td>
<td>1.272</td>
</tr>
<tr>
<td>(5)</td>
<td></td>
<td>0.625 (0.105)</td>
<td>0.127 (0.110)</td>
<td></td>
<td>1.127</td>
</tr>
</tbody>
</table>
and human capital productivity respectively as a function of capital intensities and scale. They are of the form

\[
\frac{Y}{L} = a + b \frac{K_1}{L} + c \log L \tag{1}
\]

\[
\frac{Y}{L} = a + b \frac{K_2}{L} + c \log L \tag{2}
\]

\[
\frac{Y}{L} = a + b \frac{K_1}{L} + c \frac{K_2}{L} + d \log L \tag{3}
\]

\[
\frac{Y}{K_2} = a + b \frac{K_1}{K_2} + c \log \frac{K_2}{L} \tag{4}
\]

\[
\frac{Y}{K_2} = a + b \frac{K_1}{K_2} + c \log \frac{L}{K_2} + d \log K_2 \tag{5}
\]

The underlying production functions are of the classical Cobb-Douglas type.

From the statistical point of view three things are worth underlining.

1) All the results obtained are highly significant: coefficients of determination are good, standard errors are low (except for the coefficient of log L in equation (2)).

2) Of all the two-variable combinations, the function with physical and human capital as inputs gives by far the best results: the coefficient of determination is 0.805 (as against 0.639 for the second best function with the traditional inputs K_1 and L).

3) The three-variable combinations with K_1, K_2 and L give better results than any of the two-variable combinations.

The equation

\[
\frac{Y}{L} = a + 0.738 \log \frac{K_1}{L} + 0.536 \log \frac{K_2}{L} + 0.337 \log L \tag{3}
\]

has a R^2 of 0.877 which is slightly higher than the corresponding R^2 of

\[
\frac{Y}{K_2} = a + 0.625 \log \frac{K_1}{K_2} - 0.284 \log \frac{L}{K_2} + 0.127 \log K_2 \tag{5}
\]
which has a $R^2$ of 0.819. The latter difference in $R^2$ has probably to be ascribed to the fact that $L$ is a better indicator of scale than $K_2$.

As far as the economic significance of the results is concerned, it should be noted that economies of scale, as revealed by the scale factor, are substantial.

Furthermore, the contribution of human capital to growth is considerable, almost as high as the contribution of physical capital. This can best be shown with the help of equation (3) which converted into growth rates, is equal to

$$y = 0.738 \, k_1 + 0.536 \, k_2 + 0.065 \, l$$

The coefficients of $k_1$ and $k_2$ add up to 1.274 and account for a very large part of the scale factor 1.377.

It is worth noticing that the results do not change significantly if we restrict the sample to the developed countries, as can be seen from the data in Table 2.

### TABLE 2

Results of the regression equations (3bis) and (4bis) for the restricted sample of developed countries

<table>
<thead>
<tr>
<th>Regression equations</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation (3bis) $\frac{Y}{L} = a + 0.569 \log \frac{K_1}{L} + 0.427 \log \frac{K_2}{L} + 0.269 \log L$</td>
<td>0.723</td>
</tr>
<tr>
<td>Equation (4bis) $\frac{Y}{K_2} = a + 0.625 \log \frac{K_1}{K_2} + 0.172 \log K_2$</td>
<td>0.705</td>
</tr>
</tbody>
</table>

For the sake of completeness, it should be mentioned that various indicators were tried out for the technological factor (such as percentage of steel produced by the LD-process, percentage of sheets in the production of rolled products, percentage of pig iron produced by the process of continuous casting, etc.). None of these explanatory variables related to technological progress proved significant. Nor did the factor capacity utilization yield significant results.
2.2. Test on the elasticity of substitution by means of the CES-function (Direct estimation of parameters)

When the Cobb-Douglas function is chosen as the underlying structural form of the regression equation, all elasticities of substitution between the factors involved equal unity:

$$\sigma_{K_1L} = \sigma_{K_2L} = \sigma_{K_1K_2} = 1.$$ 

Substantial research has been carried out in recent years to test the hypothesis of unit elasticity and the direct or indirect measurement of the elasticity $\sigma_{K_1L}$.

This is done by using other production functions than the Cobb-Douglas function, more particularly: the CES-function (Constant Elasticity of Substitution) and one or other of the many VES-functions (Variable Elasticity of Substitution).

Little has been done to estimate the elasticity of substitution $\sigma_{K_1K_2}$. To this end, we tried out various forms of CES- and VES-functions. The simplest of these is the two-variable CES-function with $K_1$ and $K_2$ as inputs:

$$Y = A \left( \psi_1 K_1^\theta + \psi_2 K_2^\theta \right)^{\mu/\theta}$$

whereby $\mu$ is the scale factor and $\rho = \frac{1 - \sigma}{\sigma}$ or $\sigma = \frac{1}{1 + \rho}$.

The Kmenta-approximation [1] (1) of this function is equal to

$$\log Y = \log A + \mu \psi_1 \log \frac{K_1}{K_2} + (\mu - 1) \log K_2 - \frac{\rho}{2} \mu \psi_1 \psi_2 \left( \log \frac{K_1}{K_2} \right)^2$$

$$= a + b \log \frac{K_1}{K_2} + c \log K_2 + d \left( \log \frac{K_1}{K_2} \right)^2$$

whereby $\mu - 1 = c$, $\mu \psi_1 = b$ and $-\frac{\rho}{2} \mu \psi_1 \psi_2 = d$, implying that

$$\mu = 1 + c, \ \psi_1 = b/\mu \text{ and } \rho = -\frac{2d}{\mu \psi_1 \psi_2}.$$
The foregoing equation differs from (4) only by the quadratic term
\[ d \left( \log \frac{K_1}{K_2} \right)^2 \]

The regression result of this equation is
\begin{align*}
\log \frac{Y}{K_2} &= a + 0.869 \log \frac{K_1}{K_2} + 0.288 \log K_2 - 0.166 \left( \log \frac{K_1}{K_2} \right)^2 \\
& \quad (0.284) \quad (0.074) \quad (0.140) \quad (0.140) \quad (0.284)
\end{align*}

(6)
giving a scale factor \( \mu = 1.288 \) and an elasticity of substitution \( \sigma_{K_1 K_2} = 0.459 \).

The elasticity of substitution is very low, which would seem to indicate that there is little scope for choice between «physical capital intensive» techniques and «human capital intensive» techniques. In other words, human capital would to a large extent be complementary to physical capital and embodied technology, the relative inputs of both types of capital being fixed fairly rigidly.

However, the standard error of the quadratic form is high.

Moreover, addition of the quadratic term involves but a small increase in the coefficient of determination. In other words, the shift from Cobb-Douglas to CES-function does not lead to a significant increase in explanatory power and the hypothesis of unit elasticity of substitution cannot be rejected on the basis of the foregoing statistical results.

It should be noted that experiments with other quadratic terms, such as \( \left( \log \frac{K_1}{L} \right)^2 \) and \( \left( \log \frac{K_2}{L} \right)^2 \) yielded wholly insignificant results.

The above equation (6) neglects the factor \( L \) altogether.

This factor can be dealt with in two ways. The simplest course is to extend the two-variable CES-function to a Uzawa-function. Alternatively the simple CES-function can be converted into a double stage CES-function, using the Kmenta-approximation in each of the two stages.
Simply adding the term \( \log L \) to the foregoing Kmenta-approximation, leads to

\[
Y = a + b \log \frac{K_1}{K_2} + c \log K_2 + d \left( \log \frac{K_1}{K_2} \right)_2 + e \log L \tag{7}
\]

This is the Kmenta-approximation of the Uzawa-function [2]

\[
Y = A \left[ \psi_1 K_1 - e' + \psi_2 K_2 - e' \right]^{-\mu' / \theta'}
\]

whereby the elasticity of substitution of the variables between brackets, \( K_1 \) and \( K_2 \), can take a (constant) value different from one, while the elasticities of substitution between each of these variables and the third variable is equal to one: \( \sigma_{K_1L} = \sigma_{K_2L} = 1 \).

Fitting the above equation to the sample data gives

\[
\frac{Y}{K_2} = 0.972 \log \frac{K_1}{K_2} + 0.267 \log K_2 - 0.215 \left( \log \frac{K_1}{K_2} \right)_2 + 0.122 \log L \tag{8}
\]

with a \( R^2 \) of 0.830 and \( \sigma_{K_2L} \) being 0.344. The \( R^2 \) of the corresponding Cobb-Douglas was 0.805.

By using the Uzawa-function one imposes \( \sigma_{K_1L} \) and \( \sigma_{K_2L} \) as one unit. This hypothesis is relaxed in applying a double stage CES-function (Sato [3]) of the form

\[
Y = B \left[ \xi_1 Z - e' + \xi_2 L - e' \right]^{-\mu' / \theta'}
\]

with

\[
Z = A \left[ \psi_1 K_1 - e' + \psi_2 K_2 - e' \right]^{\mu / \theta'}
\]

In this case \( \sigma_{K_1L} = \sigma_{K_2L} = \text{constant} \times \sigma_{K_1L_2} \).

The second stage regression gives us the function

\[
\log \frac{Y}{L} = a + 0.257 \log \frac{Z}{L} + 0.064 \left( \log \frac{Z}{L} \right)_2 + 0.330 \log L \tag{9}
\]

with a high standard error for the quadratic term and a \( R^2 \) which is lower than for the Uzawa function.
We will come back to the problem of the measurement of elasticity, after we have introduced wages in the model, but the preliminary conclusion of this section is that shifting from Cobb-Douglas to CES does not give better statistical results.

2.3. Wage determination

In the usual analysis of production functions, there are only two variables, $K_1$ and $L$, and wages are attributed wholly and exclusively to labour. When human capital is added as production factor, one should, from a theoretical point of view, break wages down into two parts:

— the minimum or subsistence wage, being a remuneration of gross labour;

— the difference between actual and subsistence wage (surplus wage or wage differential), being a return on human capital invested.

We were unable to break the wages down into these two components.

It is worth mentioning however, that if one considers subsistence wage as being determined biologically, then theoretical subsistence wage can be considered as equal in all countries, and wage differentials can be wholly attributed to skill differentials ($^2$).

The first set of regressions in Table 3 refers to the wage level $W$ in its traditional sense: $\omega = \frac{W}{L}$, total wage bill divided by the number of workers. The second set of regressions have as independent $W$ variable: $\frac{W}{K_2}$, wages per year of education received (or human capital invested).

Let us first indicate the major results with respect to determination of wages in the traditional sense. The correlation with productivity is high: as equation (10) indicates, 70% of wage differences can be attributed to differences in productivity.

($^2$) The wage data have been collected from various sources, mainly: the I.L.O Yearbook of Statistics (1971) and E.E.C. — Social Statistics (1969).
TABLE 3

Wages as a function of productivities, capital intensities and scale

<table>
<thead>
<tr>
<th>Equation number</th>
<th>Independent variable</th>
<th>Explanatory variables</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega = \frac{W}{L}$</td>
<td>$\frac{Y}{L}$ $\frac{K_1}{L}$ $\frac{K_2}{L}$</td>
<td></td>
</tr>
<tr>
<td>(10)</td>
<td></td>
<td>0.692 (0.098)</td>
<td>0.714</td>
</tr>
<tr>
<td>(11)</td>
<td></td>
<td>0.293 (0.112) 0.570 (0.115)</td>
<td>0.694</td>
</tr>
<tr>
<td>(12)</td>
<td></td>
<td>0.283 (0.121) 0.566 (0.118) -0.043 (0.155)</td>
<td>0.695</td>
</tr>
<tr>
<td>(13)</td>
<td></td>
<td>0.497 (0.110) ... 0.459 (0.187) -0.186 (0.116)</td>
<td>0.815</td>
</tr>
<tr>
<td>(14)</td>
<td></td>
<td>0.497 (0.114) ... 0.499 (0.192)</td>
<td>0.800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation number</th>
<th>Independent variable</th>
<th>Explanatory variables</th>
<th>$\bar{R^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega = \frac{W}{K_2}$</td>
<td>$\frac{Y}{K_2}$ $\frac{K_1}{K_2}$ $\frac{L}{K_2}$</td>
<td></td>
</tr>
<tr>
<td>(15)</td>
<td></td>
<td>0.619 (0.144)</td>
<td>0.480</td>
</tr>
<tr>
<td>(16)</td>
<td></td>
<td>0.461 (0.113)</td>
<td>0.480</td>
</tr>
<tr>
<td>(17)</td>
<td></td>
<td>0.410 (0.107) ... 0.263 (0.080)</td>
<td>0.620</td>
</tr>
</tbody>
</table>

Note: ... stands for insignificant.

Since the correlation with $Y = F(K_1, K_2, L)$ is high, one would expect a high correlation with $\frac{K_1}{L}$, $\frac{K_2}{L}$, $L$. 
This turns out to be the case: relating $\omega$ directly to the factors $K_1$ and $K_2$ provides a $R^2$ of 0.694 (See equation (11)). The scale factor $L$ is, however, insignificant (See equation (12)). Moreover, the sum of the coefficients $\frac{K_1}{L}$ and $\frac{K_2}{L}$ is close to one. This would indicate that differences in employment as such, have little effect on wage levels and that wage differentials can be ascribed to a large extent to differences in human and physical capital stock.

If we put productivity together with capital intensities and scale in one regression equation, the combination leading to a significant increase in $R^2$ is $\frac{Y}{K_2}$ and $\frac{W}{L}$, (see equation (14)).

The regressions between $\frac{Y}{K_2}$ and the corresponding variables $\frac{K_1}{K_2}$ and $K_2$, yield somewhat less satisfactory results. The combination giving the best results is

$$\log \frac{W}{K_2} = a + 0.410 \log \frac{K_1}{K_2} + 0.263 \log K_2,$$

(17)

with a coefficient of determination of 0.620.

3. **Estimation of the CES by means of comparative wages**

The literature on the subject shows that, if there are only two factors of production, $K_1$ and $L$, and wages equal marginal productivity of labour, the following relationship can be derived from a CES-function:

$$\log \frac{Y}{L} = a + b \log \omega + c \log L$$

Hereby, $\omega = \frac{W}{L}$, equals total wage bill $W$, divided by the number of workers $L$, $b = \mu/(\mu + \rho)$, $c = (1 - b) (\mu - 1)$, $\mu - 1 = c/(1 - b)$ and $\sigma_{K_1 L} = b/(1 + c)$. By analogy, if the
only factors of production are $K_1$ and $K_2$, and wages equal marginal productivity of human capital, the following relationship must hold:

$$\log \frac{Y}{K_2} = a + \frac{\mu'}{\mu' + \rho} \log \frac{W}{K_2} + (1 - b) (\mu' - 1) \log K_2$$

In the foregoing regression equations, $K_2$ usually produced better results than $L$. Therefore we first tried out the latter of the two above equations, obtaining the following results:

$$\log \frac{Y}{K^2} = a + 0.613 \log \frac{W}{K^2} + 0.195 \log K_2 \quad (18)$$

with an $R^2$ of 0.536 and $\sigma_{K_1K_2}$ being 0.513. The coefficient of determination is low but the results are consistent with the corresponding results obtained in the Kmenta-approximation as given by equation (6). This equation, which had a $R^2$ of 0.819, gave an elasticity of substitution $\sigma_{K_1K_2}$ of 0.459.

The results for the well known and traditional expression

$$\log \frac{Y}{L} = a + b \log \omega + c \log L$$

are as follows:

$$\log \frac{Y}{L} = a + 1.088 \log \omega + 0.217 \log L \quad (19)$$

with $R^2 = 0.738$ and a $\sigma_{K_1L}$ close to one unit (0.894). It may be recalled that the same combination, $K_1$ and $L$, did not give any result in a CES-function of the Kmenta-form. The $R^2$ of the two-variable Cobb-Douglas with $K_1$ and $L$ was 0.639. Statistically the results obtained are not inconsistent with the foregoing, unless one would consider a $\sigma_{K_1L}$ of 0.894 to be significantly different from one.

There is a problem, however, in the fact that in equation (18) wages are assumed to be equal to marginal productivity of human capital, while in (19) the same wages are considered to be equal to marginal productivity of labour.

Both assumptions cannot be maintained simultaneously, unless subsistence (or minimum) wages are proportional to surplus wages. If observed wage levels are closer to surplus wages, which is pro-
bably the case, then equation (19) is a hybrid and ambiguous form, where the L refers to gross unweighted labour, while the coefficient of log $\omega$ refers to human capital. In other words, if wage differentials are wholly due to differences in qualificational levels of the work force, the true relation is (18). If wage differentials reflect varying subsistence levels, the true relationship is (19). If wages are a mixture of subsistence and surplus wages, then neither (18) or (19) reflect the true relationship.

4. Testing the hypothesis of a possible VES-relationship between $K_1$ and $K_2$ and between $K_1$ and L by means of regressions involving wages

In recent years various generalized production functions have been modeled with either the CES or the Cobb-Douglas as a starting point. The more important of these forms are the Mukerji-function [4] and generalized Mukerji- or CRESH-functions (Hano<ref>no [5]), the VES-functions involving linear relationships between factor prices and productivity or capital intensities (Revankar [6]), and the VES-functions involving log-linear relations between factor prices and productivities or capital intensities (Lu, Fletcher [7]).

Most of these intricate production functions involve numerous parameters. The estimation of such parameters is a difficult matter and can usually be done only by greatly simplifying the hypothesis in the estimation procedure (more particularly in the side-relations used). They mostly end up being applied as a test of hypothesis about the constancy of the elasticity of substitution rather than an estimation of the elasticity as such. We have in this paper eliminated the Mukerji- and generalized Mukerji-function as they can be regarded as special cases of the two-stage CES-function, which has already been dealt with in an earlier paragraph. Moreover, the estimation of the parameters of these functions requires information on profits, $r$, and we have no reliable data on profits of the enterprises included in the sample. We carried out some tests with proxy-variables of profits (interest rates and capital charges), but no significant correlation was found between Y and $K_1$ on this basis. (Later on, we present some correlations between $K_1$, $K_2$, $W/K_2$, $\omega/r$, and between $K_2$ and $r$ in an attempt to test the hypothesis of possible VES-relations between $K_1$, $K_2$, and L).

A VES-function easily applicable to our available data, is the so-called Lu-Fletcher function, which, in turn, is an adapted version of the Liu-Hildebrand function [8].
It allows a test of hypothesis on the constancy of elasticities without involving profits. It is however restricted to two variables. Furthermore, it implies again that wages equal marginal productivity of either labour or human capital. For the two variable case $K_1$ and assuming constant returns to scale, the function has the form

$$Y = A \psi_1 K_1^{\epsilon} + \psi_2 \eta \left( \frac{K_1}{L} \right)^c (1+\epsilon) L^{-\epsilon} \left( \frac{1}{\epsilon} \right)$$

This expression is reduced to a CES when $c = 0$.

Taking the partial derivative of $Y$ with respect to $L$, taking logs and putting wages equal to marginal productivity of labour, leads to

$$\log \frac{Y}{L} = \log a + b \log \omega + c \log \frac{K}{L}$$

where, $c$, the coefficient of $\log \frac{K}{L}$, is the $c$ appearing as a power in the general form above and $\eta = \frac{1-b}{1-b-c}$. Thus $c \neq 0$ in the log-linear regression above would imply the rejection of the hypothesis of a constant elasticity of substitution between $K_1$ and $L$.

The tests have been performed for

$$\frac{Y}{L} = F (\omega, \frac{K_1}{L}, L) \quad \text{and} \quad \frac{Y}{K_2} = F (\frac{W}{K_2}, \frac{K_1}{K_2}, K_2).$$

The findings are shown in Table 4.

The results for $\frac{Y}{K_2} = F (\frac{W}{K_2}, \frac{K_1}{K_2}, K_2)$ give a $c$, significantly different from 0, but the $R^2$ is lower than for the corresponding Cobb-Douglas and CES.

The results for $\frac{Y}{L} = F (\omega, \frac{K_1}{L}, L)$, however, are as good, or even superior to any of the results hitherto obtained.
For the regression
\[
\log \frac{Y}{L} = a + 0.539 \frac{K_1}{L} + 0.776 \log \omega + 0.362 \log L
\]
(23)
which is the derivative of a Lu-Fletcher function with increasing returns to scale, the \( R^2 \) is 0.920.

Note that this equation is close to the three-variable Cobb-Douglas equation (5)
\[
\log \frac{Y}{L} = a + 0.738 \log \frac{K_1}{L} + 0.536 \log \frac{K_2}{L} + 0.337 \log L
\]
(5)
the only difference being in the replacement of \( \log \frac{K_2}{L} \) by \( \log \omega \).

The \( R^2 \) of that latter equation was 0.877.

In any case the coefficient of \( \log \frac{K_1}{L} \) is significantly different from zero. According to the Lu-Fletcher test, this would imply a variable \( \sigma_{1L} \). Implicit in this test however is the assumption that wages equal marginal productivity of labour and that human capital does not intervene in the picture. As equation (24) of Table 4 shows, the addition of \( \frac{\log K_2}{L} \) to equation (23) leads to a further slight increase in \( R^2 \). Probably the true function underlying equation (23) and/or (24), is some, even more intricate, three variable function involving minimum wages and surplus wages.

***

We now come to the last set of tests involving regressions of \( \frac{K_1}{K_2} \) with \( \frac{W}{K_2} \), and of \( \frac{K_1}{L} \) and \( \frac{K_2}{L} \) with \( \omega \). These regressions are shown in Table 5.

Since we have no data on profits, we used discount rates of Central Banks (*) as proxy-variable for profits.

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Explanatory variables</th>
<th>( R^2 )</th>
<th>Underlying assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{Y}{K_2} \log \frac{W}{K_2} )</td>
<td>( \log K_1/K_2 )</td>
<td>( \log K_2 )</td>
<td>0.671</td>
</tr>
<tr>
<td>(20) 0.331 (0.199)</td>
<td>0.453 (0.136)</td>
<td></td>
<td>0.740</td>
</tr>
<tr>
<td>(21) -0.041 (0.188)</td>
<td>0.562 (0.140)</td>
<td>0.317 (0.089)</td>
<td></td>
</tr>
<tr>
<td>( \frac{Y}{L} \log W/L = \log \omega )</td>
<td>( \log K_1/L )</td>
<td>( \log K_2/L )</td>
<td></td>
</tr>
<tr>
<td>(22) 0.732 (0.126)</td>
<td>0.466 (0.106)</td>
<td></td>
<td>0.856</td>
</tr>
<tr>
<td>(23) 0.776 (0.098)</td>
<td>0.539 (0.085)</td>
<td>0.362 (0.098)</td>
<td>0.920</td>
</tr>
<tr>
<td>(24) 0.545 (0.133)</td>
<td>0.577 (0.078)</td>
<td>0.233 (0.100)</td>
<td>0.366 (0.088)</td>
</tr>
</tbody>
</table>
TABLE 5

Simple regressions between $K_1/K_2$ and $\frac{W/K_2}{r}$ and between $K_1/L$ and $\omega/r$

<table>
<thead>
<tr>
<th>Equation number</th>
<th>Independent variable</th>
<th>Explanatory variable</th>
<th>Regression coefficient and standard error</th>
<th>Constant</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(25)</td>
<td>log $K_1/K_2$</td>
<td>log $\frac{W/K_2}{r}$</td>
<td>0.982 $(0.241)$</td>
<td></td>
<td>0.452</td>
</tr>
<tr>
<td>(26)</td>
<td>$K_1/K_2$</td>
<td>$\frac{W/K_2}{r}$</td>
<td>...</td>
<td></td>
<td>0.108</td>
</tr>
<tr>
<td>(27)</td>
<td>log $K_1/L$</td>
<td>log $\frac{\omega}{r}$</td>
<td>...</td>
<td></td>
<td>0.208</td>
</tr>
<tr>
<td>(28)</td>
<td>$K_1/L$</td>
<td>$\frac{\omega}{r}$</td>
<td>0.185 $(0.090)$</td>
<td>0.815</td>
<td>0.377</td>
</tr>
</tbody>
</table>

Note: ... stand for insignificant.

As Sato [3] has shown

$$\log \frac{K_1}{K_2} = a + \sigma \log \frac{W/K_2}{r}$$

is a test equivalent to

$$\log \frac{Y}{K_2} = a + \sigma \log \frac{W}{K_2}$$

However, the test involved assumes not only equality between wages and marginal productivity of human capital, but also complete independance between the factors $K_1$ and $K_2$ on the one hand, and the factor $L$, on the other.

The test is significant for $log K_1/K_2$, although the coefficient of determination is low. According to the results obtained $\sigma \frac{K_1 K_2}{K_1 K_2}$ would not be significantly different from one, which would indicate a Cobb-Douglas relation between $K_1$ and $K_2$. 
The test for the equation
\[
\frac{K_1}{L} = a + \sigma \log \frac{\omega}{r}
\]
does not yield significant results.

On the other hand, the linear relationship
\[
\frac{K_1}{L} = a + b \frac{\omega}{r}
\]
is of some significance, even though the coefficient of determination is still very low.

This equation (28) and the corresponding equation (26) are tests set out by Revankar [6] for another family of VES-functions. Any coefficient significantly different from zero, would indicate a variable elasticity of substitution. As in the Sato-test, it is implicitly assumed, not only that wage differentials solely reflect differences in productivity of labour, but also that the factors \(K_1\) and \(L\) are fully independent of the factor \(K_2\). The latter assumption is even more questionable than the former.

The low correlations in Table 4 are undoubtedly due in part to the fact that the discount rates of Central Banks do not adequately reflect general profit levels \(^{(4)}\).

What conclusions can be drawn from to the findings of this last section?

With respect to \(\sigma_{K_1K_2}\) the additional tests do not justify rejection of the Cobb-Douglas hypothesis of unit elasticity, a conclusion which confirms the findings of section 2.

The partial derivative of the Lu-Fletcher function with respect to labour, gives very good results. We do not think, however, that it can therefore be inferred that the underlying relation between \(K_1\) and \(L\) involves a VES-function, inter alia because the validity of the Lu-Fletcher test is based on the implicit assumption that wages \(K_1\) are determined exclusively by \(\frac{L}{K_1}\)

\(^{(4)}\) Using capital charges did not give any better results.
Bibliography


