A PERSPECTIVE OF SOUTH AFRICAN UNEMPLOYMENT

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ABSTRACT:
Based on stylized facts on South Africa, this article provides a simple theoretical framework to analyze the relations between reducing working time, unemployment, education and growth. It is flexible enough to provide some main macro-economic implications of work sharing, especially in terms of education and growth.

JEL CLASSIFICATION: E24, I29, J22.

KEYWORDS: reducing working time, unemployment, education, growth.

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INTRODUCTION

South Africa is a unique case in the developing world where transition has occurred in a context of high inequalities and relatively strong growth potential. The issue of inequalities in South Africa is of particular importance as it was reinforced by Apartheid. Thus, most of the poor have not had normal access to education, which in turn makes them less employable. In this context there is little hope that growth alone will help reduce unemployment: it will mostly be driven by investment that will be relatively more focused towards high-skilled labor than traditional activities (mining, agriculture). There is a trade-off between long-term solutions (that require further investments in human capital and technology) and short-term social needs: the poor need to benefit from the new democratic regime (not only in terms of basic rights but also in terms of access to economic resources). If the poor don't see their situation improving they will be reluctant to support structural reforms that are needed for the development of the country. This process could eventually damage the long-run growth potential of the economy.

The HIV/AIDS epidemic and the volatility of the Rand combined with perceived regional instability are also important factors that contribute to the economy's fragility. In such a context, it is particularly important to seek means to reduce unemployment more quickly than what human capital investment will achieve over a long period of time, not only to address the issue of inequalities but also to maintain a strong political and social consensus on reforms.

A common problem facing developing countries is the link between unemployment and the informal sector. This naturally has strong policy implications, as the issue may turn out to be more of attracting people to the formal sector rather than reducing unemployment. South Africa is, in this respect, an unusual developing country. Unemployment is high, equaling 41.8% in 2002 based on the broad definition and 30.5% on the narrow definition from the statistical releases of Statistics South Africa, and has increased over the last years. At the same time, the size of the informal sector remains rather low (16.0% according to the official statistics office), especially compared to other developing or emerging countries as shown in the table left:

<table>
<thead>
<tr>
<th>Countries (years)</th>
<th>Informal sector as % of total GDP</th>
<th>Informal sector as % of non-agricultural GDP</th>
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<tbody>
<tr>
<td>Tunisia (1995)</td>
<td>20.3</td>
<td>22.9</td>
</tr>
<tr>
<td>Morocco (1988)</td>
<td>24.9</td>
<td>30.7</td>
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<tr>
<td><strong>North Africa</strong></td>
<td><strong>22.6</strong></td>
<td><strong>26.8</strong></td>
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<tr>
<td>Tanzania (1991)</td>
<td>21.5</td>
<td>43.1</td>
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<tr>
<td>South Africa (1995)</td>
<td>6.9</td>
<td>7.2</td>
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<tr>
<td><strong>Sub-Saharan Africa</strong></td>
<td><strong>27.0</strong></td>
<td><strong>40.9</strong></td>
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<tr>
<td>Philippines (1995)</td>
<td>25.4</td>
<td>32.5</td>
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TABLE 1. CONTINUED

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<tr>
<td>Thailand (1994)</td>
<td>15.9</td>
<td>16.9</td>
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<tr>
<td>South Korea (1995)</td>
<td>32.4</td>
<td>48.1</td>
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<tr>
<td>India (1990-91)</td>
<td>27.7</td>
<td>37.3</td>
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<tr>
<td>Asia *</td>
<td>12.7</td>
<td>13.4</td>
</tr>
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These results point out a crucial problem further analyzed in Kingdom and Knight (2001) on the nature of the informal sector in South Africa. Strong barriers for entry into this sector exist, which explains the unusual situation where the informal sector remains low while unemployment is high. These barriers are explained by differences in access to networks that are necessary to develop activity in the informal sector.

Thus, it can't be advocated that unemployment in South Africa conceals as a cover for an activity in the informal sector. Furthermore, Kingdom and Knight also suggest that unemployed people are worse off then those informally employed. This highlights that South African unemployment should not be considered as voluntary, and as such is neither an ‘equilibrium’ rate nor a side effect of the development of the informal sector. A solution that could be considered is to increase labor market flexibility. Nevertheless the problem seems to largely depend on skill mismatches and thus requires further investigating the incentive for investing in human capital. Given the fragility of the situation of the poor, it might be difficult to secure a political and social consensus on reform based on more flexible wages that would likely be interpreted as downward pressure on real wages. A socially acceptable alternative is to promote a measure of reducing working time, which has the merit of emphasizing to the public its social positive impact through work sharing.

Other countries have experienced similar problems in reducing unemployment and have sought to share existing jobs, which has proven to be an appropriate measure in some cases (in France the measure of reducing working time in the Nineties could arguably be put forward to explain the decrease in unemployment). The measure could be interesting in the short run but may exhibit drawbacks in the long run (see OECD (1998) for a literature review):

- In the short run not only unskilled workers could gain access to employment, which could benefit those people but also society as a whole, as they would contribute to the fiscal system.

- As more opportunities are created for low-skilled workers there could be some incentive not to invest further in education, as enough opportunities would exist for the lowest-skill intensive jobs.
This last point is of particular importance as the future prospects of the South African economy heavily depend on the promotion of education. It is therefore crucial to analyze the interactions between reducing working time, unemployment, education and growth. The approach adopted here proposes a general and theoretical framework within which it is possible to discuss those issues.

The rest of the paper is organized as follows: section 1 investigates the impact of reducing working time on labor demand, section 2 introduces labor supply and analyzes the general equilibrium and then, section 3 considers the impact on growth.

1. Labor Demand and Partial Equilibrium

This section presents the behavior of the representative enterprise, and then studies the partial equilibrium, i.e. when labor supply is exogenous. The first sub-section presents assumptions regarding the production function. The second one examines assumptions regarding wage determination. An essential feature of a reduction in working time is the partial compensation of the workforce for forgone remuneration due to reducing working time. The third sub-section studies how labor demand is expected to respond to different possible levels of compensation for their sacrifice of total remuneration.

1.1. The enterprise

All enterprises are considered to be homogeneous, operating in a purely competitive market environment. We also assume that technology exhibits constant returns to scale. These assumptions enable us to study only the behavior of the representative firm.

1.1.1. A general framework

The first technical issue to address is the degree of substitution between workers and hours worked. This problem is usually addressed by the assumption that effective employment of an enterprise is the product of quantity of labor multiplied by an index of working time efficiency. This property is formally represented as:

\[ Y = F(\lambda(d)E) \]  

(1)

where \( E \) is the total number of workers engaged in productive activity, and \( \lambda(.) \) measures the total efficiency of \( d \), the amount of hours worked per worker. This formulation implicitly assumes that the utilization rate of equipment is constant. In a more general framework one should take into account that a reduction in working time may lead to a change in this utilization rate (see Hart and McGregor (1988) or Anxo et al. (1995)).
The impact of reducing working time on employment depends on:

- The rule of wage determination.
- The shape of the function $\lambda(.)$.

To understand how reducing working time may impact the overall levels of employment, the production function can be rewritten in the following form for the sake of convenience:

$$ Y = F \left( \frac{d}{\lambda(d)} dL \right) $$

(2)

This enables us to interpret this argument as the product of efficiency of hours worked per worker, $\lambda(d)/d$, and the total number of hours worked, $dE$. If we consider that $\lambda(.)$ is the identity function, it implies that $d$ (amount of hours worked per worker) and $E$ (size of the total workforce) are perfect substitutes in the production function. Under these circumstances, there would be a definite possibility for arbitrage between hours worked and employment. In such a case, where a perfect substitutability between workers and hours worked exists, the production function becomes:

$$ Y = F(dE) $$

(3)

Hence the demand of labor only depends on the total amount of hours worked. Assuming perfect competition, the demand for labor would be:

$$ F'(dE) = w \Leftrightarrow dE = F'^{-1}(w) $$

(4)

where $w$ is the real wage. Hence, for $w$ given, decreasing $d$ will automatically have a positive effect on the number of workers hired, as enterprises will seek to keep the quantity $dE$ constant.

If $\lambda(.)$ is not the identity function, the impact of reducing working time is less straightforward, as the demand for total hours worked, $dE$, will depend on the efficiency of working hours, $\lambda(d)$:

$$ dE = \frac{d}{\lambda(d)} F'^{-1}(w) $$

(5)

In such a case, the demand of total worked hours depends on the individual working time. A decrease in $d$ will change the total amount of hours demanded by enterprises, and hence the overall effect on the demand for workers will be more ambiguous. From a microeconomic point of view, it is common to assume that $\lambda(.)$ will be a decreasing function (as work is also usually tiring, thus reducing work efficiency). Therefore the quantity $d/\lambda(d)$ will be increasing in $d$, so that when $d$ is reduced there will also be a reduction in the demand of total hours worked.
1.1.2. **High-skilled and low-skilled jobs**

This analysis may be improved by distinguishing between skilled and unskilled workers. Most of the empirical studies (see Feldstein (1967), Craine (1973), Leslie and Wise (1980) or Anxo and Bigsten (1989)) consider the substitutability between hours and a homogenous aggregate of workers. The assumption of homogeneity is too strong. As the quality of human capital influences the marginal productivity of labor, the number of hours worked by workers with different levels of quality of human capital will definitely have a differentiated effect on total output. The first original element of this article is then to provide an analysis by which we can distinguish between two types of workers:

- The influence of unskilled workers on production depends on the number of hours worked, and not their specific skills.

- Skilled workers influence production through their human capital stock, rather than the amount of hours worked.

Hence we can rewrite the production function as the following\(^1\):

\[
Y = F(dL, Q)
\]

(6)

where \(L\) is the number of unskilled workers and \(Q\) the human capital stock.

1.2. **Partial equilibrium**

Technology is supposed to exhibit constant returns to scale. It is therefore possible to limit the study to the representative enterprise. This section presents the partial equilibrium, i.e. when labor supply is given. We assume that:

- wage of skilled workers is competitive;

- wage of unskilled workers is sticky.

The assumption of wage stickiness for unskilled workers may be justified by implicit contract theory, models of negotiation with unions, or by using legal constraints that set a minimum wage.

Let \(w_L\) be the wage of unskilled workers. The wage of unskilled workers has two components: one, fixed, linked to the assumption of wage stickiness, and one variable, linked to working time. The wage of each unskilled worker is then:

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\(^1\) If the production function assumes perfectly complementary production factors, then for a given amount of skilled workers reducing working time should lead to a significant reduction of unemployment. This effect might be observed in the short run, as it takes longer for enterprises to adopt new labour saving technology. However, perfect substitutability in the long run can diminish the expected positive effects for labour demand.
\[ w_L = \bar{c} \bar{w} \] (7)

The coefficient \( c \) is the degree of compensation, and \( \bar{w} \) represents the fixed part of the wage. If \( c = 0 \), then whatever the working time is the salary is the same: in this case there is full compensation. If \( c = 1 \), then reducing working time leads to a decrease in income of unskilled workers. For intermediate values of \( c \), reducing working time implies a partial compensation of unskilled workers for reduced hours.

Let \( w_h \) be the wage of skilled workers endowed with an amount of human capital \( h \). \( w_h \) is the product of the quantity of human capital \( h \) multiplied by the salary per unit of human capital, \( \omega \).

\[ w_h = \omega h \] (8)

The profit function of enterprises is then given by:

\[ \Pi = F(dL, Q) - w_L L - \omega Q \] (9)

As the maximization of this function with respect to \( L \) and \( Q \) leads to the following labor demand functions:

\[ w_L = \frac{\partial Y}{\partial L} = dF'(dL, Q) \] (10)

\[ \omega = \frac{\partial \Pi}{\partial Q} = F'(dL, Q) \] (11)

Integrating equation (7) into the first relation gives the behavior of the enterprise (see annex 1):

\[ \left( \frac{L}{Q} \right)^D = \frac{1}{d} T(d^{c-1} \bar{w}) \] (12)

where \( T(\cdot) \) is a decreasing function, and \( \left( \frac{L}{Q} \right)^D \) refers to the relative labor demand.

At the general equilibrium the wage per unit of human capital will be defined by:

\[ \omega = F'_2(T(d^{c-1} \bar{w}), I) \equiv \Omega(d^{c-1} \bar{w}) \] (13)

where \( \Omega(\cdot) \) is an increasing function.

As returns to scale are constant, profits are equal to zero. Hence the enterprise is indifferent to the scale of production. Facing a supply of skilled labor \( Q \) and a non-competitive salary for unskilled workers \( w_L \), the enterprise adjusts its demand of unskilled labor according to the supply of skilled labor.
1.3. **Conditions for a positive effect on labor demand**

In this sub-section we analyze how the degree of wage compensation will influence the demand of labor. From equation (12) it is possible to determine the change in labor composition with respect to a change in legal working time (see annex 2):

\[
\frac{\partial(L/Q)}{L/Q} = \frac{c - l - \varepsilon}{\varepsilon} \frac{\partial d}{d}
\]

where \(\varepsilon = \frac{dL}{Q} \frac{F'(L/Q; I)}{F'(dL/Q; I)}\) is the elasticity of marginal productivity of labor with respect to \(dL/Q\). Hence the influence of a change in legal working time will depend on this elasticity and on the compensation degree.

To get a positive impact of reducing working time on labor demand, a first condition is: \(c \geq l + \varepsilon\). In other words the positive effect arises only if the influence on productivity is not overwhelmed by an increase in cost of labor. This can be hard to impose from a social point of view, as unskilled workers are also generally the poorest. Nevertheless in this simple version we do not take into account that the cost of labor can be reduced without diminishing wages. This can also be achieved through lower taxes paid by enterprises. In the short run, inequalities would increase among employees, but it would improve the access of "previously disadvantaged people" to employment. This logic can also be related to affirmative action. So far this law focuses on existing jobs, and the need to share jobs equally among races, even if applicants don't have equivalent qualifications. This drawback is in a way unavoidable because of the underlying idea, i.e. that there is a given amount of jobs that have to be shared. Here we want to distance the principle even further in terms of job sharing, and without the mentioned drawback. What this partial analysis shows is that by reducing working time it should be possible to create many new jobs, and thereby provide better possibilities to reducing exclusion, poverty, unemployment and eventually inequalities.

This first result explains the behavior of labor demands facing a change in working time. This analysis has to be completed by introducing supply. This one is going to change according to educational choices. An important assumption is the implicit segmentation of the labor market. Workers who are not hired on the unskilled labor market cannot enter the skilled labor market.

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1 If \(c \leq l + \varepsilon\) there is a partial compensation that induces a higher cost of labour, especially regarding the positive impact on labour productivity. The impact on cost is higher, and this can lead to an a priori non-desirable equilibrium. For a given amount of skilled workers the demand for unskilled workers is increasing, which imposes a higher pressure on the unemployment of such kind of labour. Due to the high level of unemployment and more generally of poverty, this effect is not necessarily the one that the Government would like to have.
2. **Labor Supply and General Equilibrium**

In this section we analyze the behavior of workers, in order to determine the general equilibrium. The reduction of working time will change supply by changing the expected utility of workers. This section will mainly study the split of the population between skilled and unskilled workers. The next section will study the influence on growth.

2.1. **Worker's behavior**

2.1.1. *Utility function*

We suppose that preferences follow the axioms of Von Neuman-Morgenstern: the utility of a lottery is an expectancy of utility. Let $U(.)$ be the utility function of agents. It is assumed to be concave: agents are risk adverse. It mostly reflects an arbitrage relation between a risky situation (unskilled work) where agents can face unemployment, and a certain situation (skilled work) where they are hired with certainty.

Each worker compares the level of his indirect utility of either going on the skilled labor market or on the unskilled labor market. A threshold effect then arises: above a certain level of human capital, workers will choose to work as skilled workers and those under this threshold do not have a human capital stock high enough to valuate it on the skilled labor market and will prefer to go on the unskilled market.

Let $p$ be the probability to be hired when the agent goes on the unskilled labor market. The discounted utility $U_L$ for an adult is:

$$U_L = pU(d^\cdot\bar{w})$$

Implicitly we assume that utility when unemployment is equaled to zero.

For a skilled worker endowed with a human capital $h$, utility $U^H$ is given by:

$$U^H = U(\omega h)$$

The arbitrage relation between the two levels of the indirect utility gives the threshold of human capital $\bar{h}$ from which everyone decides to enter the skilled labor market:

$$U(\omega \bar{h}) = pU(d^\cdot\bar{w})$$

For a given behavior of the enterprise, i.e. for $\omega$ and $L$ given the reduction of $d$ induces a reduction in the threshold value. The income of unskilled workers is decreasing, so they have an incentive to invest further in education and to enter the skilled labor market. In this case reducing working time has a positive effect on unemployment because it creates a change in labor supply composition. Nevertheless the behavior of the enterprise is going to change, and that could lead to another change in supply, as it could change the probability of finding a job.
2.1.2. Human capital distribution

We suppose that the level of human capital of an individual $i$ is defined as the following:

$$ \tilde{h} = \Theta^i H $$  \hspace{1cm} (18)

where $H$ represents the general human capital and $\Theta^i$ the personal ability. In this section $H$ is given (in the following section discussing growth issues it will be endogenous).

Hence it is equivalent to determining $\bar{h}$ or $\bar{\Theta}$, as the threshold is also defined by:

$$ \bar{h} = \bar{\Theta} H $$  \hspace{1cm} (19)

Let $G(.)$ be the cumulative distribution function of $\Theta$. The total amount of human capital offered on the skilled labor market is:

$$ \int_{\bar{\Theta}}^{\infty} H \theta dG(\theta) = H \int_{\bar{\Theta}}^{\infty} \theta dG(\theta) $$  \hspace{1cm} (20)

the number of workers entering the unskilled labor market is:

$$ \int_{\bar{\Theta}}^{\infty} dG(\theta) = 1 - G(\bar{\Theta}) $$  \hspace{1cm} (21)

and the number of workers who go on the other is:

$$ \int_{0}^{\bar{\Theta}} dG(\theta) = G(\bar{\Theta}) $$  \hspace{1cm} (22)

2.2. General equilibrium

To determine the equilibrium we can proceed as follows. From equation (12), we have a decreasing relation between the relative demand of labor and the unskilled wage $\bar{w}$. What we would like to do is to build another relation summarizing the behavior of workers, and providing an increasing relation between the relative supply of labor and the unskilled wage. We therefore are going to show the specificity of this supply, as it depends on demand, and not only on wage. Existing unemployment affects the worker's decision through the probability of being hired.

The arbitrage relation can be rewritten as follows:

$$ U(w, \bar{\Theta}H) = p U(d, \bar{w}) $$  \hspace{1cm} (23)

Let us now determine the value of $p$. We are going to express it as a function of $\bar{\Theta}$ and the relative demand of labor. The reason why we are looking for a relation involving $\bar{\Theta}$ is that given its value, the labor supply will be easy to determine, as at each date there are $G(\bar{\Theta})$ workers who go on the unskilled labor market, and $1 - G(\bar{\Theta})$ on the skilled one.
The value of $p$ can be defined as the following:

$$ p = \frac{L^D}{L^S} = \frac{(L/Q)^D}{(L/Q)^S} \quad (24) $$

The reason why is that as the skilled wage is competitive the skilled labor market clears, and we always have the following:

$$ Q^S = H \int_0^{\infty} \theta dG(\theta) = Q^d \quad (25) $$

The relative supply of labor is a simple function of $\Theta$:

$$ \left( \frac{L}{Q} \right)^S = \frac{G(\Theta)}{H \int_0^{\infty} \theta dG(\Theta)} \quad (26) $$

Combining equations (23) (24) and (26) gives the following arbitrage relation:

$$ \frac{G(\Theta)}{H \int_0^{\infty} \theta dG(\Theta)} U(\omega \Theta H) = \left( \frac{L}{Q} \right)^D U(d^{x-w}) \quad (27) $$

Introducing the value of the wage per unit of human capital (see equation (13)) gives:

$$ \frac{G(\Theta)}{H \int_0^{\infty} \theta dG(\Theta)} U(\Omega(d^{-1}w)\Theta H) = \left( \frac{L}{Q} \right)^D U(d^{x-w}) \quad (28) $$

as the integral is decreasing with $\Theta$, $G(.)$ and $U(.)$ increasing. The relation above gives an equilibrium value of $\Theta$ that is increasing with demand for labor as well as working time. A higher demand for unskilled labor will naturally increase supply by increasing the probability to be hired. A reduction of working time will reduce the wage for unskilled workers, thereby reducing the incentive to go on the unskilled labor market. This threshold determines the relative demand and supply of labor, as well as the level of production and the unemployment rate.

We can use the equation (28) to define the relative supply of labor:

$$ \left( \frac{L}{Q} \right)^S = \zeta \left( \frac{L}{Q} \right)^D \quad (29) $$

With this general formulation, it is not possible to determine a monotonic relation between the threshold and exogenous variables. This is natural, as the results will depend on risk aversion of agents. Hence, in the next two sub-sections we expose the general properties we want to emphasize, considering the supply and demand of unskilled labor.
2.2.1. Weak compensation

The case where the demand for unskilled workers is decreasing with working time is the natural case to be considered. This is the standard positive effect expected from such a measure: enterprises will increase their demand when working time is reduced. But this will induce a change in labor supply according to two mechanisms:

- On one hand a reduction in unskilled wage induces a downward pressure on supply of unskilled labor.
- On other hand, the increase in demand tends to increase supply.

At the equilibrium the reaction of supply is influenced by the action of enterprises: they will change the wage for skilled workers, as they stay on the efficiency frontier. The overall effect depends on the workers' degree of risk aversion. The more adverse they are, the more they valuate finding a job rather than earning better salary. Thus, improving hiring conditions in the unskilled sector will lead to an increase in supply. This increase will lead to a total compensation of demand, and will therefore have a negligible impact on unemployment.

Those two effects are summarized in the following diagram. The shifts of curves are due to a reduction in \( d \). From equation (12) it is possible to exhibit a decreasing relation between \( L \) and the unskilled wage. It is represented by the curve \((L/Q)^D\) on the diagram. Equation (28) gives an increasing relation between \( L \) and the unskilled wage, represented by the \((L/Q)^S\) curve. As \( \bar{w} \) is fix, the rule to determine the equilibrium is standard for disequilibria models:

\[
\frac{L}{Q} = \text{Min}\left(\frac{L}{Q}^S, \frac{L}{Q}^D\right)
\]

\(30\)

**Figure 1**
2.2.2. Strong compensation

It is also possible that compensation leads to an increase in the cost of unskilled labor. Then one finds an a priori undesirable case. The demand from enterprises decreases, which should lead to an upward pressure on unemployment. Nevertheless by reducing the hiring probability in the unskilled segment of the market, unemployment can be reduced if workers are sufficiently risk adverse and reduce their supply even more than enterprises reduce their demand. This is what the following diagram shows.

**Figure 2**

Workers have incentives to invest more in education, and this can have strong positive effects on long-run growth. Of course, it is again important to emphasize that this could of course be rather harmful for growth in the short run, as it could normally lead to a first increase in unemployment. Nevertheless it is important to emphasize the positive long-run effects, as those are too often omitted in public debates.

In the case of South Africa, it is rather unlikely that workers will be risk adverse. Once they become unemployed they can work in the informal sector, which will provide some minimum income. But the fact that they participate in the formal sector is crucial as resources can be used to further invest in education (see following section). As this will tend to lower the relative supply of unskilled labor, the outcome on unemployment will be lowered, but this will also give more incentive for people to invest in education.

3. Growth

The impact on reducing working time on growth depends on the human capital accumulation function. In this section we integrate this key element, and then study the long-run effects of such a measure.
3.1. The Accumulation of Human Capital

It is necessary to add an explicit intertemporal model. To do so, we will use an overlapping generations model over two periods. In the first period of life individuals make their decision on education, regarding their expected future income, and do not make any other economic decisions. In the second period of life they work and allocate their income to consumption expenditures and education financing of their unique child. This supposes that the population is constant: every individual gives birth to a unique offspring. The population consists in a continuum on the interval $[0;1]$.

In the first period every individual invests in education. The law of motion of human capital of an individual $i$ is given by the relation:

$$h_{i+1}^i = \theta^i_i H_i$$  \hspace{1cm} (31)

where $\theta^i$ is the personal aptitude. $H_i$ is the quality of the school. We suppose that the system is public: every child benefits from exactly the same quality of education. This allows us to distinguish the effect of reducing working time:

• As all individuals of the same generation have benefited from the same education quality, the split of population between skilled and unskilled depends only on the aptitude, not on the education system itself. Then the analysis presented in the previous section applies for the first generation.

• Nevertheless this repartition will have an impact on the following generations, which is further investigated in the following section.

$H_i$ is supposed to be a function of a parameter of productivity, $A$, representing education quality, and the total expenditure for schooling, $D_t$. $A$ is supposed to be exogenous and constant, while $D_t$ is endogenous, and depends on wages received:

$$H_i = AD_t$$  \hspace{1cm} (32)

3.2. Partial Equilibrium of Workers

Education expenditures are financed by a proportional tax on workers. The choice to work as skilled or unskilled is then a function of the taxation rate. Let $\tau$ be this rate. The arbitrage relation becomes:

$$U(\theta^i_i \omega h^i_i) = pU((1-\tau)l^i \bar{w})$$ \hspace{1cm} (33)

To provide an explicit analysis we suppose that the utility function is $U(x) = x^\beta$ (with $\beta > 0$), hence we have:

$$\left((1-\tau)\omega h^i_i\right)^\beta = p \left((1-\tau)l^i \bar{w}\right)^\beta$$ \hspace{1cm} (34)
3.3. PARTIAL EQUILIBRIUM OF ENTERPRISE

We also assume a Cobb-Douglas production function:

\[ Y = (dL)^a Q^{1-\alpha} \]  (35)

the labor demand is defined by the first order condition giving the optimal condition on the unskilled labor market:

\[ d^c \bar{w} = \alpha d^a \left( \frac{L}{Q} \right)^{\alpha-1} \Leftrightarrow \left( \frac{L}{Q} \right)^{1-\alpha} = \left( \frac{\bar{w}}{\alpha} \right)^{\alpha-1} \]  (36)

and the skilled wage is given by:

\[ \omega = (\ell - \alpha) \left( \frac{dL}{Q} \right)^{\alpha} \]  (37)

integrating relation (36) into (37) provides the following:

\[ \omega = (\ell - \alpha) \left( \frac{\bar{w}}{\alpha} \right)^{\alpha-1} d^{\alpha \ell \alpha-1} \]  (38)

3.4. GENERAL EQUILIBRIUM

The impact of reducing working time on the structure of labor supply has been laid out. It is worth then to analyze the consequences on growth. This will depend on the evolution of expenses. They are defined as:

\[ X = \tau \omega L + \tau \omega Q \]  (39)

Replacing \( \omega L \) and \( \omega \) by their values, and integrating the relative demand for labor, again using the property \( Q^S = Q^D \) leads to the following:

\[ X = \tau \left( d^c \bar{w} \left( \frac{\bar{w}}{\alpha} \right)^{\alpha-1} \right) + \left( 1 - \alpha \right) \left( \frac{\bar{w}}{\alpha} \right)^{\alpha-1} d^{\alpha \ell \alpha-1} Q^S \]  (40)

And replacing \( \left( \frac{\bar{w}}{\alpha} \right)^{\alpha-1} \) by its value given by equation (36) gives:

\[ X = \tau \bar{w}^{\alpha \ell \alpha-1} d^{\alpha \ell \alpha-1} Q^S \]  (41)

Hence the variation of \( X \) depends only on the variation of the quantity \( d^{\alpha \ell \alpha-1} Q^S \).

The term \( d^{\alpha \ell \alpha-1} \) measures the effect of wages, and summarizes the behavior of the enterprise. The term \( Q^S \) summarizes the behavior of workers.
When working time is reduced, the influence on wages is always negative, which is harmful for growth. This is due to the fact that whether reducing working time induces higher costs for enterprises or not (depending on the compensation degree), these ones will always adjust their demand to maintain their profitability, which will not induce an increase in the global payroll.

The impact on $Q^S$ depends on wage compensation. If compensation is strong ($c<\alpha$), then a decrease in working time will induce a decrease in the threshold $\bar{\Theta}$, and an augmentation of $Q^S$. In this case the impact on growth is ambiguous. If compensation is weak ($c>\alpha$) then it is the opposite, and reducing working time has a negative effect on growth (as there is negative effect on wages combined with a negative effect on workers' repartition).

**CONCLUSION**

The impact of reducing working time on unemployment and growth depends a lot on the degree of workers' aversion to risk:

- it is important to resist artificial wage compensation (i.e. low $c$) in order to allow unemployment to fall if workers have a weak aversion to risk, and then the effect on growth is negative;

- it is necessary to compensate if they have a strong aversion, and then the impact on growth might be positive.

As we mentioned earlier in the case of South Africa, workers are likely to have a rather low degree of risk aversion. Therefore the major problem facing South Africa in implementing such a measure would be to jeopardize long-run growth. Therefore it will be particularly important to implement this measure together with other measures aimed at promoting education and employment. As an example, in the recent past the Government introduced a program called 'Spatial Development Initiative', or SDI. This program is aimed at creating jobs locally, but has two drawbacks: (i) it takes time to implement these programs in a sustainable way (as the Government is also trying to keep those programs attractive to private investors even without fiscal advantages) and (ii) it may also contribute to reduce investment in education by developing low-skill jobs. Reducing working time would probably bring additional help to those initiatives that are aimed at reducing unemployment at a faster pace.
REFERENCES


ANNEXES

1. THE BEHAVIOR OF ENTERPRISES

The optimal behavior of the enterprise regarding unskilled labor gives the following relation:

\[ d^{c-1}\overline{w} = F'_1(dL, Q) \]  \hspace{1cm} (42)

As the production function is homogenous of degree 1, the derivative is homogenous of degree 0. We can then write:

\[ d^{c-1}\overline{w} = F'_1(dL/Q, l) \]  \hspace{1cm} (43)

Define the function \( H(., .) \) as the following:

\[ T(x) = F'_1(x, l) \]  \hspace{1cm} (44)

And introducing it in the previous relation gives:

\[ \frac{dL}{Q} = T(d^{c-1}\overline{w}) \]  \hspace{1cm} (45)

or:

\[ \left( \frac{L}{Q} \right)^D = \frac{1}{d} T(d^{c-1}\overline{w}) \]  \hspace{1cm} (46)

2. ELASTICITY OF THE RELATIVE LABOR DEMAND

Consider the differential of equation (12) gives the following:

\[ (c - 1) \frac{d}{d} = \frac{F'^*_L}{F'_L} \left( \frac{\partial d}{\partial L} \right) (dL/Q) \]  \hspace{1cm} (47)

hence:

\[ (c - 1) \frac{d}{d} = \frac{F'^*_L}{F'_L} \left( \frac{\partial d}{\partial L} \right) (dL/Q + d\partial(L/Q)) \]  \hspace{1cm} (48)

Let \( \varepsilon = \frac{dL}{Q} \frac{F'^*_L}{F'_L} \left( \frac{\partial d}{\partial L} \right) \) it follows:

\[ \frac{\partial (L/Q)}{L/Q} = \frac{c - 1 - \varepsilon}{\varepsilon} \frac{d}{d} \]  \hspace{1cm} (49)