A COBWEB MODEL OF HIGHER EDUCATION AND LABOUR MARKET DYNAMICS

CLAUDE DIEBOLT*
(CNRS, UNIVERSITÉ LOUIS PASTEUR DE STRASBOURG & HUMBOLDT-UNIVERSITÄT ZU BERLIN)

BACHIR EL MURR**
(UNIVERSITÉ LIBANAISE)

ABSTRACT:
As a continuation of preceding publications in cliometrics (Diebolt, 1994, 1997, 2001; Diebolt & El Murr, 2003, 2004; Jaoul, 2004), this article develops 'cobweb' modelling of education-labour market relations applied to higher education. The research is an extension of the seminal work of Freeman (1971) and the book by Ehrenberg & Smith (1994). Our approach differs from this pioneer work in that cobweb effects are studied not only for a single labour market but for the general case of supply allocated to several markets.

JEL CLASSIFICATION: I21, J24, J44.

KEYWORDS: cobweb models, education, career choice.

* Address: BETA/CNRS, Université Louis Pasteur de Strasbourg, Faculté des Sciences Economiques et de Gestion, 61 Avenue de la Forêt Noire, 67085 Strasbourg Cedex, France - cdiebolt@cournot.u-strasbg.fr
** B.P. 70-1055 Antelias, Lebanon - bach.murr@cyberia.lb
INTRODUCTION

This article aims at further developing cobweb modelling of the education-labour market relation applied to higher education in line with preceding publications in cliometrics (Diebolt, 1994, 1997, 2001; Diebolt & El Murr, 2003, 2004; Jaoul, 2004). It extends and generalises the works of J. Walsh (1935), G. Becker (1964), Y. Ben-Porath (1967), K. Arrow (1973), M. Spence (1973, 2002), J. Mincer (1974), and others. Like R. Freeman's recursive cobweb models (1971), the paper is aimed at establishing possible causal relations between educational development and the movements of the labour market, or rather the labour markets in the plural.

A two-fold hypothesis is put forward. Firstly, the behaviour of students in their choice of curriculum depends on the expected rewards. Indeed, the allocation of students to the various faculties depends on the comparative yields of the latter in terms of expected earnings and jobs in the corresponding professional sectors. Thus, the rewards expected by a student are represented by the earnings on the labour market at a given moment and that he or she considers to be sustainable in time. Secondly, an attraction phenomenon may appear for certain curricula when a shortage occurs in different professional sectors. Once the shortage has been made up, the demand effect continues as a result of delay in the perception of the situation by young people. This may gradually lead to comparative over-production of qualified university leavers. This unbalanced situation diverts new cohorts of students to other sectors of education and may cause a new shortage, finally resulting in a cyclical movement modulated according to job availability.

Our research forms an extension of the seminal work of Freeman (1971) and a publication by Ehrenberg & Smith (1994) who, in Chapter 9 of their book entitled Modern Labor Economics, present an original treatment of cobweb models in labour economics. Our approach differs from these pioneering publications in that cobweb effects are studied not only for a single labour market but for the general case of supply allocated to several markets.

In Part II we develop a formal model. That development allows us the present, in Part III, a graphic simulation. Part IV concludes.

1. THE MODEL

The model describes a stylised situation. It combines the behaviour of supply and demand for human capital and the structure of the related earnings on the various labour markets. We first address the mechanism causing the formation of stocks of human capital, first for one academic curriculum and then for many curricula. This makes it possible to identify the main factors in the supply movements leading to the accumulation of human capital. As a continuation, the comparison of supply and demand equations on the labour markets enables study of the characteristics of possible equilibria (or disequilibria) and the analysis of the growth pathways used (convergent, divergent or continuous cobweb).

Notations and hypotheses

Let A be an academic subject and X the corresponding occupation on the labour market. The duration of studies in subject A is a years of university. If 0 is the period of starting curriculum A, then the degree is gained in period a. At the start of the university cycle, every student will have i (= a + l - a) periods to complete before reaching professional retirement at date T,
when he will have worked for \((T - a)\) years; the new graduate will work for \((i - a)\) years.

We therefore describe the life cycle of a person from the date he starts at university as follows:

**Diagram 1.**

\[
\begin{array}{cccccccc}
0 & 1 & 2 & a & a+1 & i & T & t \\
\end{array}
\]

\(A_t\) represents the populations of students aspiring to university training in subject A during the period of time \(t\), and \(A_t\) represents the numbers of students who have completed their degree in subject A. Our approach ignores the effect of possible failure or abandon to change curricula during the university career; these are assumed to be rare. The number of students moving from one year to the next remains the same and the number of persons awarded a degree is the same as the number of students starting the subject a years previously. If, for example, \(A_0\) represents the flows of students wishing to start subject A during period 0, the number of holders of degrees in the subject after \(a\) years of university, \(A_a\), is \(A_0 \ (A_a = A_0 + a = A_0)\).

The flows of degree-holders during period \(t\) will be added to the numbers of employed persons, \(X_t\), on the corresponding labour market, forming the total supply of labour, referred to as \(X_t\). It is assumed that \(X_{t=0} = 0\), ignoring the initial stock effect that is not of particular interest for our analysis. Indeed, what is important for us is the evolution of the flows of degree-holders, hence of future employed persons, from period \(a\). Thus, the number of employed persons at time \(t = a\) will be the new graduates from period \(a\), \(A_{a}\), who started subject A \(a\) years previously \((X_a = A_a = A_0)\). It is in a way the stock of human capital on date \(a\).

The evolution of populations of graduates and employed persons during the subsequent period can be described by the following diagram:

**Diagram 2**

1.1. **Earnings and Labour Supply: The Case of a Single Sector**

The assumption is made that individual choice is doubly correlated with collective choice. We consider that the set of individual decisions results in an assortment of collective choices shared between the different curricula and affecting the situation on the various labour markets. Furthermore, we hold that the making of individual decisions is based essentially on the opinion that can be made of the state of the market in various occupations and often identified by the comparison of the relative earning levels. We consider here that a student chooses between two curricula according to the salary that he hopes to earn by practising—
throughout his life—the occupation corresponding to the best thought-of curricula at the time he makes his academic choice. We use a progressive approach to be able to formulate all the interactions between the individual choice of academic subjects on the one hand and the situation on the labour markets on the other. This should enable a clear representation of the nature and functioning of decision processes and the pathways of the evolution of flows of human capital on the labour markets.

Attention is focused first on the formation of the numbers studying a single subject, thus considering that it is typical. We therefore only take into account the impact of the variation of earnings in a single occupation on the flows of diplomas in the corresponding sector. This formalisation is then extended to two or more curricula and occupations.

The term $w_t^X$ indicates the earning level during period $t$ in occupation $X$, the outlet for curriculum $A$. As the variation of earnings in $X$ stimulates student entry in curriculum $A$, each time that the earnings vary the number of students starting curriculum $A$ would change proportionally by $\alpha_a w_t^X$ (where $0<\alpha_a<1$ coefficient of variation). The number of students starting an academic year is therefore multiplied by $(1+\alpha_a w_t^X)$, as is the number of graduates $a$ years later.

Following this variation in earnings, the evolution of populations of students completing their degrees can be presented as follows:

<table>
<thead>
<tr>
<th>Period</th>
<th>New graduates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$A_a = A_0$</td>
</tr>
<tr>
<td>$a+1$</td>
<td>$A_{a+1} = A_a (1 + \alpha_a w_{(a+1)-a}^X) = A_0 (1 + \alpha_a w_t^X)$</td>
</tr>
<tr>
<td>$a+2$</td>
<td>$A_{a+2} = A_{a+1} (1 + \alpha_a w_{(a+2)-a}^X) = A_0 (1 + \alpha_a w_{(a+1)-a}^X)(1 + \alpha_a w_{(a+2)-a}^X)$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$t$</td>
<td>$A_t = A_{t-1} (1 + \alpha_a w_{t-a}^X) = A_0 \prod_{i=a}^{t} (1 + \alpha_a w_{t-a}^X)$</td>
</tr>
</tbody>
</table>

It is possible to use the relative variation in earnings in occupation $X$ (on date $a$) to have an idea of the number of persons present on the labour market after several periods, as is shown in the table below.
<table>
<thead>
<tr>
<th>Period</th>
<th>Working population</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$X_a = A_a = A_0$</td>
</tr>
<tr>
<td>$a+1$</td>
<td>$X_{a+1} = A_{a+1} + X_a = A_0 \left(1 + \alpha_a w_{a+1-a}^x\right) + A_0$</td>
</tr>
<tr>
<td>$a+2$</td>
<td>$X_{a+2} = A_{a+2} + X_{a+1}$</td>
</tr>
<tr>
<td></td>
<td>$= A_0 \left(1 + \alpha_a w_{a+2-a}^x\right) \left(1 + \alpha_a w_{a+1-a}^x\right) + A_0 \left(1 + \alpha_a w_{a+1-a}^x\right) + A_0$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$t$</td>
<td>$X_t = A_t + X_{t-1} = A_{t-a-1} \left(1 + \alpha_a w_{t-a}^x\right) + X_{t-1} = A_0 \sum_{j=a}^{t-a} \prod_{i=a}^{j} \left(1 + \alpha_a w_{i-a}^x\right)$</td>
</tr>
</tbody>
</table>

As the student flow in a particular curriculum is equal to the flow of graduates in this subject $a$ years later, $A_t = A_{t-a}$, and hence $A_{t-1} = A_{t-a-1}$.

Simple replacement is used to find the equations addressing the interaction between populations of students and the working population on the labour market:

$$X_t = A_t + X_{t-1} = A_{t-a-1} \left(1 + \alpha_a w_{t-a}^x\right) + X_{t-1} = A_0 \sum_{j=a}^{t-a} \prod_{i=a}^{j} \left(1 + \alpha_a w_{i-a}^x\right) \quad (1)$$

At this stage we also introduce a parameter allowing for the impact of population growth on the total annual numbers desiring a university education. Indeed, the combination of the demographic aspect and the relative earnings hypothesis gives a better description of the distribution of student flows among the various curricula. It is true that given the input of student populations, the limits of places and the number of curricula and occupations, the availability of places in all the curricula as a whole and jobs on all the labour markets acquires a dominant role, but the most attractive curricula and occupations (ceteris paribus) will always boast the largest numbers.

If $\delta_t$ is the growth rate of the working population leaving higher education, the numbers available (employed or not) on a given date can be shown as follows:

$$X_1 = \begin{cases} 
A_1 + X_{t-1} \\
A_{t-a-1} \left(1 + \alpha_a w_{t-a}^x\right) \left(1 + \delta_{t-a}\right) + X_{t-1} \\
A_0 \sum_{j=a}^{t-a} \prod_{i=a}^{j} \left(1 + \alpha_a w_{i-a}^x\right) \left(1 + \delta_{t-a}\right) 
\end{cases} \quad (2)$$

413
These equations make it possible to observe the evolution of the working population dictated by both the variation in earnings and the growth of the student population several years previously. They have the merit of providing information about the effects of the stock of human capital during a specific period, given the evolution observed in the preceding years in both earning levels on the labour market and the level of increase of the student population. They show the cumulated effect of the working population resulting from the behaviour of populations of students drawn a years before by a particular curriculum because of the attraction of the salary levels of the corresponding occupation.

These formulas also have the advantage of showing a direct correlation between the working population on the labour market at a given period and the variation in earnings a periods previously, thus engendering clearly identified cobweb behaviour:

\[ w_t \rightarrow X_{t+a} \rightarrow w_{t+a} \rightarrow X_{t+2a} \rightarrow \ldots \]

They also show the natural link between the stock of human capital during a given period and the flows of students aiming for the curriculum corresponding to the occupation in question a periods previously.

1.2. THE EVOLUTION OF EARNINGS: THE CASE OF TWO SECTORS

The model is extended to the case of two 'competing' curricula, A and B, and the corresponding labour market occupations X and Y to approach the real case of the existence of several curricula and occupations that share the human capital available. Given the segmentation of the labour market, the supply of human capital in an occupational sector depends not only on the earnings in the sector but also on the movement of earnings in the other occupations several years previously. When subjects are chosen, ultimately determining the size of the flow of graduates and hence the supply of human capital by occupational sector, all decisions are made with considerable attention paid to the earning structures in existing occupations and not only in one or other of the alternate occupations at the moment the decision is made. A curriculum thus appears to be more competitive according to the increase in earnings on the corresponding labour market in comparison with the lesser earning growth in the other occupational sectors.

In formal terms, we express the competitiveness of the curricula as the ratio between the earnings in the two corresponding occupations X and Y, denoted \( \sigma_t = \frac{w_t^X}{w_t^Y} \).

This ratio reflects in a way the earning-competitiveness of the two competing occupations and of the university curriculum associated with them during a given period. When they take a decision, student populations will focus on the evolution of this ratio rather than on the earning level in a given occupation. We thus put forward the hypothesis that what is important is not the earning level in an occupation, but the relative earning growth pathways in the various occupations available, generally dictated by the variation
in the competitiveness ratio. If the earning ratio, \( \omega \), also referred to as competitiveness ratio of the curricula, varies, indicating a noteworthy change in the relative earning structure of the two occupations available, there will be an increase in the flow of students \( \beta_a \omega \) and \( \beta_b \omega \) in curricula A and B respectively (0<\( \beta_a \), \( \beta_b \) <1). This will have a direct impact on the flow of graduates entering the labour market in occupations X and Y several years later. This hypothesis is formalised as follows:

\[
X_t = \begin{cases} 
A_{1-t} + X_{t-1} \\
A_{1-a} \left(1 + \beta_a \omega_{1-a}\right)\left(1 + \delta_{1-a}\right) + X_{t-1} \\
A_0 \sum_{j=0}^{1} \prod_{i=0}^{j} \left(1 + \beta_a \omega_{1-i}\right)\left(1 + \delta_{1-i}\right) 
\end{cases}
\]  

(3)

\[
Y_t = \begin{cases} 
B_{1-t} + Y_{t-1} \\
B_{1-a} \left(1 - \beta_b \omega_{1-a}\right)\left(1 + \delta_{1-a}\right) + Y_{t-1} \\
B_0 \sum_{j=0}^{1} \prod_{i=0}^{j} \left(1 - \beta_b \omega_{1-i}\right)\left(1 + \delta_{1-i}\right) 
\end{cases}
\]  

(4)

These developments enabled us to write a formula relating the increase in the stock of human capital to the evolution of earnings in the occupations and the growth of the student population a periods previously. The supply of jobs in a given occupation is directly related not only to the evolution of earnings in the two occupations a periods previously, but also to the rate of population increase in the student generations aiming at higher education a periods previously. The sharing of students between the different curricula matches the attraction that these curricula may have at the beginning of each academic year. Thus each subject will have its share of students engendered by demographic growth according to the degree of attractiveness of the corresponding occupation on the labour market.

1.3. DEMAND FOR LABOUR

Full identification of cobweb behaviour and possible glutting also requires analysis of the demand for labour. This demand for human capital is influenced by several micro and macroeconomic factors. We consider here that demand for labour is conditioned essentially by the earnings on all the labour markets and by the economic situation. The hypothesis is also put forward that the important feature is not the nominal earning level in a sector but rather the relative earning level in comparison with similar sectors. Our analysis here was limited to two sectors of activity and the competitiveness ratio was chosen as the determinant factor in the decision to hire. If earnings rise in occupation
$X_t$, ratio $\omega_t (= \frac{w_x}{w_y})$ also increases, causing employers to be more reticent to take on personnel in sector $X$ than in sector $Y$ and vice versa. In a generalisation of the analysis, it is possible to conceive several competitiveness ratios allowing correct positioning with regard to the various occupations.

On a formal basis, demand for labour in occupations $X$ and $Y$ can therefore be expressed explicitly in terms of recurrence as follows:

$$X_t = \begin{cases} 
X_{t-1} \left( 1 - \beta_x \omega_t \right) \left( 1 + \mu_x \tau_t \right) + X_{t-1} \\
X_0 \sum_{j=a}^{t} \prod_{i=a}^{j} \left( 1 - \beta_x \omega_i \right) \left( 1 + \mu_x \tau_i \right)
\end{cases} \quad (5)$$

$$Y_t = \begin{cases} 
Y_{t-1} \left( 1 + \beta_y \omega_t \right) \left( 1 + \mu_y \tau_t \right) + Y_{t-1} \\
Y_0 \sum_{j=a}^{t} \prod_{i=a}^{j} \left( 1 + \beta_y \omega_i \right) \left( 1 + \mu_y \tau_i \right)
\end{cases} \quad (6)$$

where: $X_t$ and $Y_t$ are the total demand for work until period $t$ in occupations $X$ and $Y$ respectively. This is all the human capital used or to be employed in the two occupational sectors.

$X_t$ and $Y_t$ are the demand for labour displayed during period $t$ in occupational sectors $X$ and $Y$ respectively.

$G_t$ is the gross domestic production (possibly sectoral) in period $t$.

$\tau_t$ is the expected economic growth rate.

$\mu_x$ and $\mu_y$ are positive parameters expressing the sensitivity of demand for human capital in markets $X$ and $Y$ to economic growth.

$\beta_x$ and $\beta_y$ are positive parameters expressing the sensitivity of demand for human capital to variations in the earning structure in markets $X$ and $Y$.

Demand for labour is therefore a function of the existing state of the earning structure on the different markets, expressed by the current competitiveness ratio. The supply of human capital depends on the earning structure $a$ years previously:
Demand for labour:

\[
(-) \quad (+) \quad \quad (+) \quad (+)
\]
\[
X_t = f \left( \omega_t = \frac{w_t^x}{w_t^y}, G_t \right) \quad \quad Y_t = f' \left( \omega_t = \frac{w_t^x}{w_t^y}, G_t \right)
\]  \hspace{1cm} (7)

Labour supply:

\[
(+ \quad (+) \quad (-) \quad (+)
\]
\[
X_t = h \left( \omega_{t-a} = \frac{w_{t-a}^x}{w_{t-a}^y}, \vartheta_{t-a} \right) \quad \quad Y_t = h' \left( \omega_{t-a} = \frac{w_{t-a}^x}{w_{t-a}^y}, \vartheta_{t-a} \right)
\]  \hspace{1cm} (8)

The phases of the cobweb process are thus easily detectable (see Diagram 3).

**Diagram 3.**

1.4. Equilibrium and glutting of labour markets

The identification of the parameters and pathways of possible equilibria can be envisaged once supply and demand have been determined. A situation of equilibrium is used as reference for appraisal of the scale of any disequilibrium that may occur following a change in the structure of earnings. A disequilibrium is understood as any saturation or shortage on the labour market.

We sought both equilibrium on each labour market and overall equilibrium of all the labour markets. Equilibrium on each labour market is found by balancing out the supply of human capital resulting from existing stocks of working population and the flow of graduates from the corresponding curriculum with the demand for labour on the concerned market. By virtue of the interdependence of markets, our approach means that earnings are determined by the equilibrium between supply and demand for jobs on each of the markets.

Equilibrium on the two labour markets considered separately is characterised by:

\[
X_t = X_t
\]
\[
Y_t = Y_t
\]  \hspace{1cm} (9)
Equilibrium of all the labour markets can be obtained as follows:

$$X_t + Y_t = X_t + Y_t$$  (10)

The issue is thus no longer in analysis of supply and demand on the labour market for a single occupation but in the breaking down of supply between academic curricula according to the attractiveness of the corresponding professions. Whence the advantage of our approach, that can address such more realistic considerations by using effects of substitution and/or complementarity between the different academic curricula on the one hand and between the different occupational sectors on the other. Excess supply on one market can be taken up by excess demand on another more attractive market via the orientation of populations of students to the most attractive curricula. The glutting observed in a specific field of activity can easily be handled if the input becomes smaller and smaller and output increasingly frequent. The substitution phenomenon between academic curriculum is indeed such as to lead to equilibrium on the labour markets as a whole. The occupation with surplus labour suffers a decrease in earnings and thus becomes less attractive to the populations of students aiming at higher education. The corresponding academic curriculum will soon be replaced by another whose target occupation displays surplus demand for labour and thus pays higher salaries. Access to this new academic curriculum remains attractive as long as the relative earnings in the occupational sector targeted are higher than those in the initially abandoned sector at the time new students make their choice at the beginning of the academic year.

The phenomenon can be shown in the form of a glutting cycle:

**Diagram 4**
Examination of this diagram reveals alternate periods of shortage and saturation on each labour market. Surplus supply in relation to demand on a sector labour market causes saturation and the competing labour market becomes more attractive (with a shortage) as relative earnings are higher because demand for labour exceeds supply. However, this situation is not stable insofar as the most attractive sector attracts most school leavers to the corresponding academic curriculum, causing the trend to be reversed several years later. Indeed, the race to the most attractive sector—driven by the increase in relative earnings—increases the flow of graduates in the corresponding specialisation, causing glutting. This phenomenon is reiterated continuously, revealing the pattern of a cobweb model engendered by the alternation of the two opposing phenomena saturation and shortage on the various markets. The implicit competition between the various markets means that no market conserves a dominant position indefinitely, attracting crowds of secondary school leavers. The variation in relative earnings following a reversal of the situation in the employment market means that the attractive occupation and academic curriculum have to give up their dominant position at a certain moment. The repeated alternate gaining of the dominant position leads to cyclic behaviour in the long term, with each occupation able to lead.

In the case with two curricula and two occupations examined here, the cycles move in opposite directions: when occupation X approaches saturation in terms of qualified persons, a shortage develops in occupation Y and subsequently changes the relative earning position and re-launches the process of increased enrolment for the academic subject corresponding to Y and hence an increase in the supply of workers available in Y. The phenomenon is repeated until a situation similar to the initial one is observed, that is to say saturation in occupation X and a shortage in occupation Y and so on and so forth. The phenomenon can be represented as a cyclic reiteration, assumed here to be stable (Diagram 5).

**Diagram 5**
1.5. THE FORM OF THE COBWEB MODEL

The form of the cobweb model identified above depends on its equilibrium parameters. It is true that the equilibrium situation is a reference situation around which the cyclic movements form. But here, in contrast with equilibria attained instantaneously calculated from equations whose components are indexed to the same period, adjustment towards equilibrium takes time because of the heterogeneity of the reaction times of supply and demand with regard to earning fluctuations. As is shown by the functional equations (7) and (8), supply is slow to react to the variation in relative earnings whereas the reaction of demand is instantaneous. The earning structure is generally determined during the subsequent period from the demand equation. This variation in the earning structure causes a change in supply several periods later. The surplus (or shortage) of supply thus observed on the market leads once again to a decrease (or increase) in earnings. The process continues until equilibrium or saturation is attained. Recognition of the type of cobweb model (with a convergent, divergent or continuous cycle), depends on the slopes of the curves of supply and demand for human capital.

These enable us to identify the appropriate cobweb model type and then to know whether the reiteration in time of the ascending periods (causing glutting) and descending periods (causing a shortage of the working population) finally leads to a market growth pathway in equilibrium (or disequilibrium). Indeed, if the demand curve is narrower than that of supply (elasticity of demand in relation to the earning ratio higher than that of supply), the cyclic movement converges towards the point of equilibrium. If the opposite occurs, the cyclic movement explodes and diverges infinitely from the point of equilibrium. If the two slopes are the same, the cyclic movement is stable and revolves at the same distance around the point of equilibrium, as is shown in Diagram 6.

**Diagram 6**

Starting from the equilibrium equations (9), simple replacement and mathematical resolution (see Annex) can be used to easily obtain the recursive form of the cobweb model (the highlighted variables are the mean values) through the clearly visible relation between the competitiveness ratios at instant t and those of period (t - a):

420
\[
\overline{\omega}_t = \left( -\frac{\beta_a}{\beta_x} \right) \overline{\omega}_{t-a} - \left( \frac{1}{\beta_x} \right) \overline{\theta}_{t-a} + \left( \frac{\mu_x}{\beta_x} \right) \overline{\tau}_t - \frac{\log(z_0/x_0)}{(t-a)\beta_x} \\
\overline{\omega}_t = \left( -\frac{\beta_b}{\beta_y} \right) \overline{\omega}_{t-a} + \left( \frac{1}{\beta_y} \right) \overline{\theta}_{t-a} - \left( \frac{\mu_y}{\beta_y} \right) \overline{\tau}_t + \frac{\log(y_0/y_0)}{(t-a)\beta_y}
\]

(11)

Beyond the impact that the economic situation and the growth of the working population may have on the earning structures of competing occupations shown by these equations, it is easy to see the recursive form of the key variable, the competitiveness ratio, making it possible to obtain an idea of the possible forms of the cobweb model engendered by the time lag between supply and demand for human capital on the labour markets of the two occupations concerned.

<table>
<thead>
<tr>
<th>Nature of cobweb cycle</th>
<th>Occupation X</th>
<th>Occupation Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convergent</td>
<td>(</td>
<td>-\beta_a</td>
</tr>
<tr>
<td>Divergent</td>
<td>(</td>
<td>-\beta_a</td>
</tr>
<tr>
<td>Continuous</td>
<td>(</td>
<td>-\beta_a</td>
</tr>
</tbody>
</table>

Indeed, the cycle is convergent (divergent) if the sensitivity of supply of human capital in absolute terms is lower (greater) than the sensitivity of demand in absolute terms. In contrast, if supply and demand are equally elastic in relation to the competitiveness ratio, the representative curve of the cobweb model revolves continuously and at the same distance around the point of equilibrium.

The characteristics of the various markets, reflected by the values of the sensitivity parameters, will thus show the pattern of the saturation and shortage phases, leading growth pathways to situations of equilibrium or disequilibrium. The glutting effect is caused, in fact, by the intertemporal interaction between earning structure and the supply of human capital in the competing professions throughout the growth pathways. It causes a cyclical oscillation phenomenon with regular alternate in both earning structure and the numbers employed in the competing professions. It results from the simultaneous combination of two effects, one upstream and the other downstream. The substitution effect is an upstream diversion of the supply of human capital from a saturated occupation that has therefore become less attractive than an alternative occupation with a shortage. The saturation effect occurs downstream, reflecting the intention or capacity of companies to take on personnel. Companies hire less in the sectors with excess human capital and where earnings are relatively high.
2. Graphic Simulation of the Cobweb Model: The Case of Four Sectors

We show the application of the model as an extension to previous publications on the cyclic pattern of numbers enrolled in law, medicine, protestant theology and literature faculties in Prussia before World War 2 (for a summary see Diebolt, 2004). The first reason to collect such data was to discuss the erroneous representation of the glutting crisis in German and Prussian universities from 1885-1890. The hypothesis was put forward, that this glutting was manipulated and augmented by the conservative Prussian government to prevent members of the lower social classes from studying and to restore the social exclusivity of the Prussian Universities. This crisis had a large impact on the public discussion surrounding academic education and culminated in references to a growing "academic proletariat". During a great debate in parliament the Prussian minister of culture (Gossler) asked, with the acclamation of the conservatives, How many students are necessary to maintain the size of the governing classes? A secret prognosis of student enrolments, which had been made by Wilhelm Lexis was used to successfully push through political measures designed to diminish student populations. But all these efforts were unsuccessful and after a short cyclical recession, student populations grew at a much faster rate than they had beforehand. Therefore it became clear that the dynamic of the process that led to glutting was independent of political influences. As well, the hypothesis that the glutting crisis was manipulated by particular interest groups, could not be sustained. Therefore further investigations into the nature of the generated dynamics present in the time series were necessary. As a result of these investigations, it was possible to publish a complete stock of data on the changing number of students for all subjects at all universities. This data, together with other information such as age, sex, and social origin of the students, has been publish in three books (Titze et al., 1987, 1995, Diebolt, 1997).

In our illustration, a distinction is made on the one hand between the numbers enrolled at the faculties of protestant theology (T), law (D), medicine (S) and literature (L). On the other, the corresponding professions, that is to say Pastors (P), the legal professions (A, lawyers and judges), doctors (M) and secondary school teachers (E).

Presentation is in the form of a two-dimensional graph with four independent quadrants grouped in such a way that the axes of the coordinates serve two purposes. The numbers studying two academic curricula and the corresponding occupations are displayed on either side of a single x-axis. Likewise, two distinct ratios are set out on either side of the y-axes, each consisting of the earning ratios in two different professions. The curves for supply and demand for human capital are thus observed in each quadrant. As supply is from the academic subjects studied, the notations for the various curricula are combined with the supply in the occupations concerned. The demand curves are marked with occupation notations insofar as businesses instigate employment.
We use the assumption of the accidental increase in demand in a given occupation (A, for example) that causes an increase in earnings in the sector. This can change the earning structure in various occupations. We therefore analyse the dynamic behaviour of supply and earnings from this moment onwards on the various labour markets.

The situation of equilibrium assumed to be initially established at date 0 is marked by relative earning ratios $\omega_0$, $\omega'_0$ and $\omega''_0$, and human capital supply levels $D_0$, $S_0$, $L_0$ and $T_0$. An increase in earnings in occupation A following increased demand results in an increase in these ratios (earning in occupation A is among the numerators) to $\omega_1$, $\omega'_1$ and $\omega''_1$. These levels are much higher than the new long term equilibrium logically assumed to become established after the advent of the structural change observed in occupation A. This leads a years later to a variation in the supply of graduates to $D_1>D_0$, $S_1<S_0$, $L_0$ and $T_1<T_0$.

The decrease thus observed in the supply of graduates in health, arts and theology to below their long term equilibrium levels $S_e$, $L_e$ and $T_e$ results in a decrease in the number of doctors, teachers and pastors on the labour market and induces respective increases in the salaries for these occupations. This results in a change in earning structure, supported by the decrease of those in occupation A following the increase in supply in the sector. The earning ratios thus decrease to levels $\omega_2$, $\omega'_2$ and $\omega''_2$; these are lower than their long terms equilibrium levels ($\omega_2<\omega_e$, $\omega'_2<\omega'_e$ and $\omega''_2<\omega''_e$). The attraction of health, the arts and theology would thus win over law, implying a flow of students to these curricula, a larger supply graduate doctors, teachers and pastors and a decrease in the supply of lawyers a years later to levels $D_2>D_e$, $S_2>S_e$, $L_2>L_e$ and $T_2>T_e$. The reiteration of this kind of behaviour by students in the face of subsequent changes in earning structures in the various occupations leads to the simultaneous development of four cobwebs (Diagram 7); these are convergent (if the slope of the supply curve is greater than that of the demand curve), divergent (if it is smaller) or continuous (if the slopes are the same).
CONCLUSION

This article leads on from previous work. Indeed, we demonstrated in a preceding paper the cyclical behaviour of populations of students and working populations engendered by successive saturation and shortage of human capital on the different labour markets. Here, we have used tools from labour economics to develop a formal theoretical framework that can explain the recursive movements of flows of students and working populations.

Indeed, given the recursive character of the phenomenon emerging from the relation between the supply of human capital and earning structure characterising the different labour markets several years previously, the cobweb model appears to be appropriate for addressing the resulting pathways of equilibrium. Although an econometric study based on
the chronology of data observed on particular markets is imminent, three alternative forms of the cobweb model emerge, depending on parameters of sensitivity of labour supply and demand to earning structure on the different markets.

It were as if the combination of an upstream substitution effect and a downstream saturation effect finally causes alternating saturation and shortage phases whose patterns are conditioned by the identification of the parameters of the cobweb model, consisting of the sensitivity of supply and demand for human capital to movement of the earning structure in alternative occupations. The intersectoral mobility of human capital resulting from the substitution effect, combined with the absorption capacity of the various markets leads via alternate saturation and shortage cycles in competing markets either to a balanced growth pathway (a self-regulation effect) if the elasticity of demand is sufficient, or to an unbalanced growth pathway (a self-explosion effect) if elasticity of demand is weak in relation to elasticity of supply. If the substitution effect is not strong enough to enable the gradual diversion of populations of students from the subject corresponding to the occupation with excess numbers accumulated in the past (glutting) to the curriculum whose occupation suffers from a growing shortage of human capital (a gradually increasing shortage), stringent variations must be observed in the earning structure, such as to be able to mobilise a sufficient supply of human capital to meet the precise requirements of the different labour markets, thus leading to divergent cyclic behaviour and hence to a disequilibrium growth pathway. This phenomenon is all the more marked insofar as the occupations can be less and less substituted (are less competitive) and even complementary. In the latter specific case, the complementarity of two occupations means that they have the same nature and the same direction of cycle.

If, however, the substitution effect is strong, the surplus numbers of persons accumulated in time will be absorbed more rapidly thanks to variations increasingly cushioned by the earning ratio cycle, thus creating a convergent spiral leading the growth pathway to a situation of equilibrium. Furthermore, if the saturation effect displays a limited absorption capacity on certain labour markets, this reduces opportunity for employment and increases the risk of laying off, requiring increasingly large variations, towards decreasing earnings to face the rigidity of the corresponding labour markets and rapidly leading to divergent growth pathways.

The advantage of the model lies in the prediction capacity that it can provide for decision makers involved in the management of human resources and labour markets at the national level. Indeed, with knowledge of the pattern of cyclical behaviour drawn from the prior identification of the parameters of the cobweb models, a decision maker can fairly easily find the positions in the phases of the cycles of the different occupations, forecast future trends and provide the appropriate guidance for the populations of students entering universities in the form of measures of information, dissuasion or encouragement; this upstream action can cushion or even prevent risks of glutting and hence future unemployment.
REFERENCES


APPENDIX

This appendix extends the model to solve equilibrium equations.

Equating as shown in equation (9) the equations for annual supply and demand for human capital on the two occupation markets, ((3) = (5) and (4) = (6)) gives:

\[
A_0 \sum_{j=a}^{t} \prod_{i=a}^{j} (1 + \beta_a \omega_{i-a})(1 + \delta_{i-a}) = X_0 \sum_{j=a}^{t} \prod_{i=a}^{j} (1 - \beta_x \omega_i)(1 + \mu_x \tau_i)
\]

\[
B_0 \sum_{j=a}^{t} \prod_{i=a}^{j} (1 - \beta_b \omega_{i-a})(1 + \delta_{i-a}) = Y_0 \sum_{j=a}^{t} \prod_{i=a}^{j} (1 + \beta_y \omega_i)(1 + \mu_y \tau_i)
\]

This is the equivalent of:

\[
A_0 \prod_{i=a}^{t} (1 + \beta_a \omega_{i-a})(1 + \delta_{i-a}) = X_0 \prod_{i=a}^{t} (1 - \beta_x \omega_i)(1 + \mu_x \tau_i)
\]

\[
B_0 \prod_{i=a}^{t} (1 - \beta_b \omega_{i-a})(1 + \delta_{i-a}) = Y_0 \prod_{i=a}^{t} (1 + \beta_y \omega_i)(1 + \mu_y \tau_i)
\]

A condition for the result of each of the sums to be null is that all their components are null. This means writing \((i = a, \ldots, j ; j = a, \ldots, t)\):

\[
A_0 \prod_{i=a}^{t} (1 + \beta_a \omega_{i-a})(1 + \delta_{i-a}) = X_0 \prod_{i=a}^{t} (1 - \beta_x \omega_i)(1 + \mu_x \tau_i)
\]

\[
B_0 \prod_{i=a}^{t} (1 - \beta_b \omega_{i-a})(1 + \delta_{i-a}) = Y_0 \prod_{i=a}^{t} (1 + \beta_y \omega_i)(1 + \mu_y \tau_i)
\]

These equations can easily be linearised using their Napierian logarithm:

\[
\log \frac{A_0}{X_0} + \sum_{i=a}^{t} \log (1 + \beta_a \omega_{i-a}) + \sum_{i=a}^{t} \log (1 + \delta_{i-a}) = \sum_{i=a}^{t} \log (1 - \beta_x \omega_i) + \sum_{i=a}^{t} \log (1 + \mu_x \tau_i)
\]

\[
\log \frac{B_0}{Y_0} + \sum_{i=a}^{t} \log (1 - \beta_b \omega_{i-a}) + \sum_{i=a}^{t} \log (1 + \delta_{i-a}) = \sum_{i=a}^{t} \log (1 + \beta_y \omega_i) + \sum_{i=a}^{t} \log (1 + \mu_y \tau_i)
\]

As the terms \(\beta, \omega, \mu, \tau\) and \(\delta\) are very small, it is possible after several simplifications to replace certain terms by their limited development to the first order:
\[
\log \frac{A_0}{X_0} + \beta_a \sum_{i=a}^{t} \bar{\omega}_{i-a} + \sum_{i=a}^{t} \bar{\vartheta}_{i-a} = -\beta_x \sum_{i=a}^{t} \bar{\omega}_i + \mu_x \sum_{i=a}^{t} \bar{\tau}_i \\
\log \frac{B_0}{Y_0} - \beta_b \sum_{i=a}^{t} \bar{\omega}_{i-a} + \sum_{i=a}^{t} \bar{\vartheta}_{i-a} = \beta_y \sum_{i=a}^{t} \bar{\omega}_i + \mu_y \sum_{i=a}^{t} \bar{\tau}_i 
\]

By dividing by \( t - a \), these equations can be rewritten in terms of mobile averages, noted with bars, so that:

\[
\beta_a \bar{\omega}_{t-a} + \bar{\vartheta}_{t-a} + \log \left( \frac{A_0}{X_0} \right) / (t-a) = -\beta_x \bar{\omega}_t + \mu_x \bar{\tau}_t \\
-\beta_b \bar{\omega}_{t-a} + \bar{\vartheta}_{t-a} + \log \left( \frac{B_0}{Y_0} \right) / (t-a) = \beta_y \bar{\omega}_t + \mu_y \bar{\tau}_t 
\]

Isolating in the \( t \) periods the mean value of the competitiveness ratio \( \bar{\omega}_t \) (also the equilibrium value in period \( t \)) gives:

\[
\bar{\omega}_t = \left( \frac{-\beta_a}{\beta_x} \right) \bar{\omega}_{t-a} - \left( \frac{1}{\beta_x} \right) \bar{\vartheta}_{t-a} + \left( \frac{\mu_x}{\beta_x} \right) \bar{\tau}_t - \frac{\log \left( \frac{A_0}{X_0} \right)}{(t-a)\beta_x} \\
\bar{\omega}_t = \left( \frac{-\beta_b}{\beta_y} \right) \bar{\omega}_{t-a} + \left( \frac{1}{\beta_y} \right) \bar{\vartheta}_{t-a} - \left( \frac{\mu_y}{\beta_y} \right) \bar{\tau}_t + \frac{\log \left( \frac{B_0}{Y_0} \right)}{(t-a)\beta_y} 
\]