On the Structure of International Trade
Interdependence (*)

by

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This paper attempts a synthesis of previous empirical research on international trade by the author and by others. The theme is international trade interdependence. It will be shown that empirical research suggests a rather different view of the way in which world trade hangs together than that to which economists are accustomed.

In this paper we will try to show that empirical data and common sense suggest that it is meaningful to speak of an elasticity of exports with respect to GNP. Exports are thus determined not only by factors on the side of demand, but also by the rate of increase of supply. This means among other things that the often noted positive correlation between the rate of growth of exports and of GNP should not be interpreted in terms of exports causing growth (e.g. W. Beckerman, B. Balassa (1962, 1963, 1964)) but as growth inducing exports. The catch phrase of export propelled growth should be replaced perhaps by the notion of growth propelled exports (1).

The survey of empirical evidence also provides some empirical justification for a type of approach which has been used by different authors, often independently of each other, to analyse the structures of international trade matrices.

The « abstract model » with which the paper concludes is meant to have descriptive value mainly, as a convenient way of summing up the pattern of forces which empirical research and common sense suggest

(*) The present paper, written while the author was visiting professor at the Department of Economics of M.I.T., discusses the implications for foreign trade theory of quantitative research on foreign trade in which he has been engaged for several years in the Département d’Économie Appliquée de l’Université Libre de Bruxelles. The latter stages of this research have been supported by a grant for research on model building from the Fonds pour la Recherche Scientifique Fondamentale Collective. He is glad to acknowledge the ideas and stimulation he has received during this time in discussion with J. Benard, H. Gleiser, E. Kirschen, B. Marin Curtaud, E. Rosselle, J. Sandee, W. Tins, and J. Tinbergen. Valuable comments were received on a preliminary version of the paper from H. Gleiser, C.P. Kindleberger, E. Kirschen and M. Scott. These acknowledgements must not be interpreted as an artful dodge to shift to unsuspecting victims collective responsibility for weaknesses of this paper.

(1) As pointed out to me by C.P. Kindleberger, there is no incompatibility between the two notions: a mutual stimulation with exports propelling growth e.g. via economies of scale, and growth in turn propelling exports would be quite compatible with the facts as observed.
as determining international trade flows. The model can however be justified more rigorously in terms of profit maximazing behaviour of the firm. This problem will be treated in a forthcoming more mathematical article.

A) Setting of the problem

The setting of the problem is the matrix of international trade. This matrix has the form

\[
\begin{array}{ccc}
X_{11} & \cdots & X_{1n} \\
\vdots & \ddots & \vdots \\
X_{n1} & \cdots & X_{nn} \\
\hline
X_{01} & \cdots & X_{on} \\
\hline
X_{10} & & X_{10} \\
X_{no} & & X_{no} \\
\end{array}
\]

(1a)

when \( \sum_{j} X_{ij} = X_{io} \), \( \sum_{i} X_{ij} = X_{oj} \)

In this presentation, international trade is disaggregated by markets but not by commodities. It is not difficult to define a trade matrix with different commodities. It will take the form of disconnected blocks,

\[
\begin{array}{ccc}
X^{11} & 0 & \cdots & 0 \\
0 & X^{22} & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & X^{mm} \\
\hline
X^{01} & X_{02} & \cdots & X_{0n} \\
\hline
X^{10} & & & X^{10} \\
X^{no} & & & X^{no} \\
\end{array}
\]

(1b)

where each block \( X^{il} \) is of the same general form as the block of the elements \( X_{ij} \), and describes the matrix of trade of commodity \( (l) \).

The trade matrix can be transformed into a matrix model by expressing each of its elements as functions

\[
X_{ij} = X_{ij} (z_1 \ldots z_n) = X_{ij} (z)
\]

(2)

of other economic variables. In defining these functions Occam’s razor will be wielded with determination: only the simplest formulation consistent with observed facts is to be retained.

It may be worthwhile to draw attention in passing to the difference between the matrix approach adopted here and that which is usual in international trade theory. The latter, which centers attention on total exports and imports, is not so well suited as the matrix approach to trace out the repercussions of an exogenous change acting on any part of the system, because it does not describe interrelations between every pair of countries. The classical approach
is of course perfectly suited to study of the partial equilibrium and
two country models to which so many important and fruitful studies
have been devoted.

B) Hypothesis 1: Leontief proportionality

The grandfather of matrix models is the Leontief input output
table, so it is natural to start by examining whether a Leontief type
scheme can explain changes in the network of international trade.

The resulting model is easy enough to define and has been much
used. Define market shares of country i in the imports of country j
during year t

$$A_{tij} = X_{tij}/X_{tj}$$

These market shares are analogous to the Leontief technical coeffi-
cients, and like the latter may be assumed to be constant. Then
elements of the trade matrix will be the following functions of the
matrix's column totals

$$X_{tij} = A_{ij} X_{tij}$$

This equation will be designated as the L Model in honor of W.
Leontief.

The empirical validity of the equation (3) has been tested by
W. Tims and F.M. Meyer zu Schlochtern, D. Phan, and J.
Waelbroeck in studies using a comparable methodology (2). The
test of the formula is to check whether total exports as predicted
by equation (3) coincide with actual exports. Regression of actual
on predicted exports can then be used to verify whether the L model
is satisfactory.

Let

$$\hat{X}_{tio} = \sum_j A_{ij} X_{tij}$$

where in the studies referred to the A_{ij} s are market shares for year
(t - 1). Then computing the time series regression (3).

$$X_{tio}/X_{tio}^{t-1} = a \hat{X}_{tio}/X_{tio}^{t-1} + b + u_t$$

the L model will be verified if the values of a, b, and \(R^2\) are quite
close to 1, 0, and 1.

(2) See W. Tims and F.M. Meyer zu Schlochtern, (1962), D.L. Phan, (1965), and

(3) Or alternatively the regression

$$X_{tio}/X_{tio}^{a} = A \hat{X}_{tio}/X_{tio}^{a} + B_t + C + U$$

the test of validity of the L model is then A = 1, B = C = 0, \(R^2 = 1\). Use of the two
forms has in practice led to similar results. See J. Waelbroeck, (1962).
As shown by the table below the estimated values of the coefficient (a) are as expected, but the (b) coefficient is in some instances highly significantly different from zero. Market shares seem to be subject to a trend-like drift effect which requires explanation.

### TABLE 1

**Results of tests of the L model**

<table>
<thead>
<tr>
<th></th>
<th>Belgium - Luxemburg</th>
<th>Netherlands</th>
<th>France (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1926-38</td>
<td>1953-60</td>
<td>1926-38/1953-60</td>
</tr>
<tr>
<td>a</td>
<td>0.88</td>
<td>0.90</td>
<td>0.96</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.13</td>
<td>0.24</td>
<td>0.06</td>
</tr>
<tr>
<td>b</td>
<td>1.9</td>
<td>-2.2</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.36</td>
<td>1.91</td>
<td>0.83</td>
</tr>
</tbody>
</table>

(a) In the French study coefficients were estimated for the two periods 1926-38 and 1953-60, using dummy variables to distinguish constant terms for the pre-war and post-war period. There is thus only one regression coefficient for the two periods but two constants.

Analysis of the residuals indicates also that they are not random. In the Belgian study this is shown by an analysis of variance showing that year to year changes in market shares tend to be all positive or all negative for a given year on all geographic markets and for all commodity groups exported. In the Dutch study a strong correlation is found between fluctuations in market shares and domestic inflationary pressure, as measured by the unemployment percentage. In France the devaluations and the Pinay and Rueff stabilization schemes led to sharp spurts in exports and to sudden increases in France's market shares. In his very interesting study Phan shows that the fit is substantially improved by inclusion of relative prices in the regression equation (4).

The L model is not wrong but it is incomplete. It does not account for two important phenomena:

- the gradual drift of market shares which sometimes continues for many years;
- the sawtooth fluctuations of those shares produced by devaluations and by domestic inflationary maladjustment.

(4) A similar result has been obtained for Belgium by Tibiri in an unpublished study.
C) Hypothesis 2: The gravity model: distance, export push and import pull

Further light is thrown on trade interdependence by the recent studies undertaken independently by J. Tinbergen and by the Finnish authors K. Pöyhönen and P. Pulliainen (6). Those men have set out to estimate the following unconventional model. The version used by Tinbergen is:

\[ X_{ij} = C Y_i^a Y_j^b/d_{ij}^d \]  

(6)

while the Finnish model is written

\[ X_{ij} = C C_i C_j Y_i^a Y_j^b/d_{ij}^d \]  

(7)

where \( Y_i \), \( Y_j \) are the gross national products of the exporting and of the importing country and \( d_{ij} \) the geographic distance between their main economic centers. The exponents \( a, b, \) and \( -d \) are the elasticities of trade with respect to GNPs of the exporting country, of the importing country, and to distance. \( C \) is a scale factor while the coefficients \( C_i \) and \( C_j \) which set apart the Finnish from the Dutch formulae describe the openness to trade of countries (influenced for example by tariff levels, presence or absence of natural resources).

Estimation of model (6) and (7) is done from crosssection data relating each item of the trade matrix (1a) to GNPs and distances. In both studies the values of \( a, b, \) and turned out to be quite close to 1. The \( C_i \) values of the Finnish study were not published.

There is, as has been pointed out, an odd similarity between formulae (6) and (7) and the law of gravity, with \( Y_i \) and \( Y_j \) playing the role of masses, and this justifies the christening the model as the gravity, or G model. The reader who remembers his highschool physics will observe that \( d \) is not equal to 2 as in a Newtonian field. Trade flows seem to be described by a field of logarithmic potential (8).

A more important remark from the present point of view is that the G model implies that there exists a positive elasticity of exports with respect to GNP. If this is true, as the author thinks, then this is a fact which has not received enough attention in international

(6) Both Tinbergen and Pöyhönen and Pulliainen present more refined formulae but no allusion will be made to them since the refinements are not relevant to construction of the present model. The improvement of statistical fit caused by these refinements is in any case rather small.

(8) There should be no difficulty from a mathematical point of view in extending the formulae to cover continuous mass distributions using the classical methods of potential theory. This could lead to interesting developments in location theory, but is outside the scope of this study.
trade theory. It implies that a big country can export more just because it is bigger, and so has more diversity of production, a more extensive selling network abroad, more salesmen visiting foreign clients and trying to match orders from competitors. We will return to this point in the next section, and merely note here that the Tinbergen and Pöyhonen - Pulliainen results imply that the catch phrase "export propelled growth" which has attracted attention off and on should perhaps be replaced or possibly complemented by the notion of "growth propelled exports".

The G model does provide at least a partial explanation of the drift in market shares noted in section B. Thus if a country (1) grows at a rate \( \rho_1 \), while its competitors grow at the rate \( \rho_2 \neq \rho_1 \), and if \( A_{01j} \), \( A_{02j} \) are the shares of the country and of its competitors in the imports of country j in year 0, it turns out that according to the gravity model the market shares of country 1

\[
A^t_{ij} = \frac{1}{1 + \left( \frac{Y^o_2}{Y^o_1} \right)^a \left( \frac{d_{1j}}{d_{2j}} \right)^d e^{(\rho_2 - \rho_1)t}}
\]

will drift slowly upward or downward according to whether country 1 grows faster or more slowly than its competitors. Thus the relatively slow Belgian economic growth in the postwar period is very probably the main cause of the gradual reduction of her world market shares in recent years (?).

D) **Hypothesis 3: a price sensitive model**

(i) *The specification used for estimation purposes*

Prices are absent from both the L and G models. This is certainly not an acceptable simplification of the facts, since in practice countries sell very similar goods, so that it does not make sense to exclude prices from a general model of foreign trade. The exclusion of prices from the G and L models implies also that the state of balances of payments are determined solely by income effects, countries being unable to control surpluses and deficits otherwise than by modifying their rates of growth.

(? For prewar years however the favorable trend of Belgium exports reflects very low domestic costs rather than rapid economic growth. The distinction between growth and competitive effects is not quite so cut and dry as will be implied by the abstract model to be constructed later in the paper.
A natural way to include prices on the G model is to assume

$$X_{ij} = C \frac{Y_i^a Y_j^b}{d_{ij}^e} \left( \frac{P_i}{P_j} \right)^{e_i} \tag{9 a}$$

We have only indirect evidence that the model (9 a) is empirically founded. This evidence is

a) based on analysis of total exports of countries and not of trade flows between pairs of countries; hence appropriate aggregation conditions must be fulfilled if results obtained are to be interpreted as describing the forces governing bilateral trade flows between countries.

b) in the study quoted no satisfactory way was found of measuring the export push factor $Y_i$. It is reflected in the regression constants obtained.

The connection between the empirical results quoted and the matrix model considered in this paper can be made precise as follows.

Assume that for all commodities (s) in a group (l) export demand is determined by the equation

$$X_{isl} = C_l \frac{Y_i^{ql} Y_j^{bl}}{d_{il}^e} \left( \frac{P_{is}}{P_{js}} \right)^{-e_l} \tag{9 b}$$

where $Y_{il}$ is an indicator of the production capacity of industry $l$ in the exporting country, $Y_{jl}$ an indicator of demand for the products of this industry in the importing country, $P_{is}$ and $P_{js}$ the prices of good s in countries i and j.

To make aggregation valid it is necessary to restrict attention to a group K of countries which are reasonably close together. Then in considering exports of each country of the group to third countries it is not unreasonable to suppose that only minor distortions will result if the exact geographic location of each is disregarded — i.e. the $d_{ij}$s are replaced by an appropriate set of $d_{js}$.

If distance effects are uniform $X_{ij}$ splits up into the product of a function of i and a function of j — say

$$X_{ij} = g(i) h(j)$$

so that as may readily be seen, defining (we drop the group and product indices l and s to lighten notation)
\[ A_{ij} = \frac{X_{ij}}{X_{0j}} = \frac{g(i)}{\Sigma g(i)} \quad (i \in k) \]
\[ A_{io} = \Sigma_{j} \frac{X_{ij}}{\Sigma_{i,j} X_{ij}} = \frac{g(i)}{\Sigma g(i)} \quad (i \in k) \]
\[ A_{ij} = A_{io} \]

hence horizontal aggregation of trade flows is valid and the behaviour of the \( A_{ij} \) can be inferred from knowledge of that of \( A_{io} \).

Defining \[ Y_{i}^{a}/\Sigma Y_{j}^{a} = g_{i} \quad (i \in k) \] (10)
and defining \( p \) as the price index
\[ P_{i} = \Sigma (g_{i} P_{i}^{a-e})^{-\frac{1}{2}} \quad (i \in k) \] (11)
leads finally to the simple result
\[ A_{io} = A_{ij} = g_{i} (P_{i}/P)^{-e} \] (12)
in which \( g_{i} \) describes the economic weight of country \((i)\) as defined by (10) and \( P_{i}/P \) the competitiveness of its exports with those of other countries of the group.

Strictly speaking the formula (12) can be estimated only iteratively, as the form of the price index makes it non-linear. In practice it probably does not distort the results very much to replace the index (11) by an index of average unit prices of total exports of countries of the group \( k \), and this was the solution adopted.

(ii) Estimation results

It only remains to describe estimation procedures of (12). The results quoted are from a study by E. Roselle and J. Waelbroeck, (1964 e.) who used a refinement of the cross-section approach pioneered by G.D.A. McDougall in his well-known 1951 and 1952 articles.

In this approach individual observations relate to relative quantities and prices of exports of different commodities for a given year, so that the regression is a cross-section over commodities. Whereas McDougall pooled his observations into a single sample, Roselle and Waelbroeck divided theirs into subsamples using various classification criteria and used analysis of covariance to test for differences in the values of \( g \) and \( e \) between countries and broad commodity groups. The sample investigated was 1958 exports of some 400 commodities by France, Germany, Belgium-Luxemburg, Italy, and Holland. The choice of countries ensures that the aggregation condition of geographic proximity is satisfied as closely as can be hoped for in practice.
Table (2) describes the part of the results which is most relevant to the present study.

**TABLE 2**

Result of cross-section estimate of export demand curves for Common Market countries (a)

<table>
<thead>
<tr>
<th></th>
<th>Equipment Goods</th>
<th>Consumption Goods</th>
<th>Metals</th>
<th>Other Manufactures</th>
<th>Textiles</th>
<th>Chemicals</th>
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</thead>
<tbody>
<tr>
<td>Germany</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\hat{e}$</td>
<td>$+0.46$</td>
<td>$-2.09$</td>
<td>$-2.09$</td>
<td>$-0.34$</td>
<td>$-3.15$</td>
<td>$-1.19$</td>
</tr>
<tr>
<td>$-\hat{e}/\hat{\sigma}e$</td>
<td>$1.25$</td>
<td>$-4.01$</td>
<td>$-3.44$</td>
<td>$-0.91$</td>
<td>$-4.48$</td>
<td>$-2.56$</td>
</tr>
<tr>
<td>$\hat{g}$</td>
<td>$0.42$</td>
<td>$0.13$</td>
<td>$0.31$</td>
<td>$0.18$</td>
<td>$0.20$</td>
<td>$0.42$</td>
</tr>
<tr>
<td>$\log \hat{g}/\hat{\sigma} \log g$</td>
<td>$8.7$</td>
<td>$9.6$</td>
<td>$8.8$</td>
<td>$11.5$</td>
<td>$5.2$</td>
<td>$7.2$</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\hat{e}$</td>
<td>$-0.69$</td>
<td>$-1.40$</td>
<td>$-2.59$</td>
<td>$-1.83$</td>
<td>$-1.94$</td>
<td>$-1.39$</td>
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<tr>
<td>$-\hat{e}/\hat{\sigma}e$</td>
<td>$-3.08$</td>
<td>$-4.14$</td>
<td>$-3.05$</td>
<td>$-5.79$</td>
<td>$-5.23$</td>
<td>$-5.76$</td>
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<tr>
<td>$\hat{g}$</td>
<td>$0.14$</td>
<td>$0.19$</td>
<td>$0.16$</td>
<td>$0.18$</td>
<td>$0.25$</td>
<td>$0.17$</td>
</tr>
<tr>
<td>$\log \hat{g}/\hat{\sigma} \log g$</td>
<td>$16.7$</td>
<td>$20.6$</td>
<td>$20.2$</td>
<td>$7.32$</td>
<td>$11.5$</td>
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</tr>
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<td></td>
</tr>
<tr>
<td>$-\hat{e}$</td>
<td>$-1.73$</td>
<td>$-1.92$</td>
<td>$-4.08$</td>
<td>$-0.98$</td>
<td>$-2.08$</td>
<td>$-1.83$</td>
</tr>
<tr>
<td>$-\hat{e}/\hat{\sigma}e$</td>
<td>$-3.62$</td>
<td>$-3.84$</td>
<td>$-5.28$</td>
<td>$-2.73$</td>
<td>$-3.31$</td>
<td>$-3.21$</td>
</tr>
<tr>
<td>$\hat{g}$</td>
<td>$0.05$</td>
<td>$0.05$</td>
<td>$0.04$</td>
<td>$0.03$</td>
<td>$0.09$</td>
<td>$0.04$</td>
</tr>
<tr>
<td>$\log \hat{g}/\hat{\sigma} \log g$</td>
<td>$15.1$</td>
<td>$12.2$</td>
<td>$11.0$</td>
<td>$14.0$</td>
<td>$8.15$</td>
<td>$11.9$</td>
</tr>
<tr>
<td>Holland</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\hat{e}$</td>
<td>$-1.27$</td>
<td>$-0.24$</td>
<td>$-2.01$</td>
<td>$-0.98$</td>
<td>$-1.38$</td>
<td>$-2.00$</td>
</tr>
<tr>
<td>$-\hat{e}/\hat{\sigma}e$</td>
<td>$-3.25$</td>
<td>$-0.58$</td>
<td>$-1.94$</td>
<td>$-2.18$</td>
<td>$-3.01$</td>
<td>$-3.85$</td>
</tr>
<tr>
<td>$\hat{g}$</td>
<td>$0.04$</td>
<td>$0.15$</td>
<td>$0.03$</td>
<td>$0.08$</td>
<td>$0.09$</td>
<td>$0.07$</td>
</tr>
<tr>
<td>$\log \hat{g}/\hat{\sigma} \log g$</td>
<td>$19.4$</td>
<td>$11.5$</td>
<td>$12.3$</td>
<td>$13.8$</td>
<td>$10.6$</td>
<td>$9.5$</td>
</tr>
<tr>
<td>Belgium-Luxemburg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-\hat{e}$</td>
<td>$-0.26$</td>
<td>$-1.30$</td>
<td>$-0.24$</td>
<td>$-0.19$</td>
<td>$-0.96$</td>
<td>$-0.78$</td>
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<td>$-\hat{e}/\hat{\sigma}e$</td>
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<td>$-0.60$</td>
<td>$-0.98$</td>
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</tr>
<tr>
<td>$\hat{g}$</td>
<td>$0.04$</td>
<td>$0.05$</td>
<td>$0.15$</td>
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</tr>
<tr>
<td>$\log \hat{g}/\hat{\sigma} \log g$</td>
<td>$20.3$</td>
<td>$21.3$</td>
<td>$7.8$</td>
<td>$16.4$</td>
<td>$8.3$</td>
<td>$11.4$</td>
</tr>
</tbody>
</table>

(a) In the table $-\hat{e}$, $\hat{g}$ refer to the estimates of the negative of the elasticity of substitution and of the antilog of the regression constant; $-\hat{e}/\hat{\sigma}e$ and $\log \hat{g}/\hat{\sigma} \log g$ to the ratios of $\hat{e}$ and of $\log \hat{g}$ to their standard errors of estimate.
The reader will note the rather low values of the elasticities of substitution. Most of them are significant. This does not mean that they should be accepted at face value, since as McDougall's careful analysis of statistical problems shows the estimates are strongly biased and all the biases tend to bias values of elasticities downwards. Roselle and Waelbroeck expressed the opinion that these results are consistent with an elasticity of the order of—4, which is also McDougall's best guess.

The most interesting result of the study however is its confirmation of the gravity hypothesis. As values of g show economically stronger Germany and France are able to sell much more at given prices than Italy, Belgium-Luxemburg, and Holland (8). The differences are not explained by more favorable geographic locations. They are much too large to be explained by differences in quality. These may of course be important for individual commodities but can be of marginal significance only for overall exports of countries which like the Common Market countries have rather equal technical capabilities.

Note also that for each country the g coefficient is highest for products in the export of which that country has come to be specialized. This suggests that in the long run countries enjoying a comparative advantage for some type of goods are able to consolidate their position so that they become able to sell more without any need to cut prices below those of competitors.

E) The Growth propelled exports hypothesis and its common sense justification

The idea that economic growth of a country induces a shift in demand for its exports is not a usual one in international trade theory. Economists, accustomed as they are to build-up market demand curves by adding up demand of individual consumers, usually think of the demand curve as something which sellers must reckon in their calculations as determined independently of their own decisions. There are, it is true, discussions in the literature of the influence of publicity on market demand; but their discussions are largely confined to the literature on monopolistic competition. It is in any case not to publicity that we shall look mainly to account for the effect of economic growth on exports.

(8) Careful readers may note that the g coefficients do not add up to one. This is because logarithmic regressions do not have the additivity property. It is the relation of the g9 to each other which should be looked at rather than their actual magnitude.
The first and perhaps the most important explanation of the phenomenon is that except for primary commodities international trade takes place almost in an information vacuum. Buyers may know more or less what sellers in their own country have to offer; but they are quite unable to know well more than a tiny fraction of potential foreign suppliers. They do not buy blindly, of course, but they do buy on the basis of very incomplete knowledge of what is for sale.

In such a situation, it is very difficult to think of demand as something exogenous from the point of view of sellers. The location of the demand curve depends on how much effort a seller puts into spreading knowledge about what he has to sell. This means, of course, publicity in trade and other journals; but it also means sending salesmen to prospective customers, distributing samples, making demonstrations, creating selling subsidiaries abroad, etc.

From the point of view of an individual firm what is to be decided at every moment is whether to adjust to changing market conditions by changing prices-moving along the demand curve—or by changing the selling effort and thus shifting the demand curve to the right, at the cost of increased overhead.

The international trade specialist cannot observe these marginal decisions of each firm. But he can make a pretty good guess that if there are many firms in a country, with a large production capacity, there will also be large resources invested in selling what is produced. And he can guess also that if the country's capacity to produce increases, its enterprises will adjust their selling network, manage to contact more foreign buyers and very likely manage to sell more at the old price. This is what Germany and other fast-growing countries have done after the war.

This conclusion is reinforced by the diversity and constant change of the assortment of goods traded on world markets. The fast-growing country, with a high rate of investment and enterprising businessmen, will be taking advantage of the profit opportunities created by economic change by stepping into many of the new markets which open up. Thus through an aggregation effect the demand curve for its exports will shift to the right even if demand for goods already in production is stable.

F) Summing up the results by an abstract model of international trade flows

It is time to bring together the threads of the argument and see what type of international trade model emerges. It will be clear
that as often happens in econometric work the studies cited describe a certain interplay of forces rather than functional relations between specific variables. Thus push and pull effects may be associated either with GNPs, or export and import totals, or time trends of market shares, or regression constants. Likewise the competitive edge enjoyed by some countries over others has been associated with differences in inflationary pressure (model L) and with prices (model C).

It also would be unwise to attach too great importance to the actual values of the parameters estimated. In most cases substitution for example of GNP for imports as an explanatory variable, or change in definition of a price index, or switch from non-deflated to deflated series might well change parameter values. In at least one case — the price elasticities — the estimated parameter values are known to be strongly biased.

We shall finally abstract from transient factors, to retain only the long term forces on which emphasis has been laid in this study. Transient factors are however important and we hope to investigate these more closely in future. Thus a new commodity will not become known immediately, there will be an adaptation delay. Another type of short run dynamic interaction that will be neglected for simplicity's sake is the effect of domestic inflationary pressures on the level of exports, whose importance was revealed by the tests of the (L) model.

The best way of summing up the results quoted is thus to express them as what may be called an abstract model of international trade (the "A" model). The adjective "abstract" alludes to the two characteristics mentioned. First the variables do not refer to GNP, or export and import totals; or to relative prices and to relative inflationary pressures; or to geographical distance; but to the complexes of variables, some of them difficult to measure, associated with size, competitive edge, and international transfer costs of commodities. The fact that to simplify matters for the reader and to simplify comments on properties the same letters and names are used to designate variables of the A model as those which usually designate specific economic magnitudes (e.g. $Y_i$, income, $d_{ij}$, distance) should thus not be misinterpreted.

The second characteristic is that all parameters of the abstract model are taken to be equal to 1 or $-1$. 
TABLE 3

Pattern of interrelations in the studies cited and in the A model

<table>
<thead>
<tr>
<th>Types of interrelations</th>
<th>Distance</th>
<th>Seller’s push</th>
<th>Buyer’s pull</th>
<th>Competitive edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model L</td>
<td>- - -</td>
<td>b</td>
<td>X_{oj}</td>
<td>u</td>
</tr>
<tr>
<td>Model G</td>
<td>d_{ij}^{-d}</td>
<td>C_i Y_i^{a}</td>
<td>C_j Y_j^{b}</td>
<td>- - -</td>
</tr>
<tr>
<td>Model C</td>
<td>- - -</td>
<td>G_i</td>
<td>X_{oj}</td>
<td>\left( \frac{P_i}{P_j} \right)^{-e}</td>
</tr>
<tr>
<td>Model A</td>
<td>d_{ij}^{-1}</td>
<td>Y_i</td>
<td>Y_j</td>
<td>\frac{P_j}{P_i}</td>
</tr>
</tbody>
</table>

In the notation, the abstract model is thus written

\[ X_{ij} = \frac{Y_i Y_j P_i}{d_{ij} P_j} \tag{13} \]

where \( Y_i, Y_j \) can be interpreted in terms of different measures or of a combination of measures of economic weight (in constant prices); \( d_{ij} \) refers not to geographic distance but to all transfer costs incurred in shipping commodities between countries (including tariffs); this parameter can also be understood as reflecting the degree of complementary of economic structure of pairs of countries; \( P_i, P_j \) are related not only to prices but also to differences e.g. in delivery delays.

G) The abstract model and analysis of the structure of international trade matrices

The model (13) provides some justification for the use by the author and by others of the RAS model used by A. Brown and R. Stone in input-output research (1962) for the analysis of international trade flows \(^9\).

The RAS model projects
\[ X_{tij} \text{ by } \hat{X}_{tij} = X_{oij} \cdot r_i \cdot s_j \]
where \( r_i \) and \( s_j \) are defined by the conditions
\[ \sum_i \hat{X}_{tij} = X_{toj} \]
\[ \sum_j \hat{X}_{tij} = X_{tio} \]
where \( X_{oij} \) are matrix flows in some base year, and \( X_{toj} \) and \( X_{tio} \) are total imports and exports of countries \( j \) and \( i \) in year \( t \).

The \( r_i \) and \( s_j \) parameters are estimated by an iterative method developed by Stone and Brown.

According to the model (13) trade flows will change according to
\[ X_{tij} = X_{oij} \cdot Y_{ti} \cdot Y_{tj} \cdot P_{ij} / P_{ti} \]  \hspace{1cm} (14)
where the \( Y^t \) and \( P^t \) are indices with year (o) as base year. The \( r_i \) and \( s_j \) multipliers of the RAS procedure can obviously be interpreted as representing the \( Y^t_{ij}/P^t_i \) and \( Y^t_{tj} P^t_j \) factors of (14).

Following this line of reasoning the RAS method can be used to study the intensity of trade connections between pairs of countries. For this purpose a rough procedure is to use the RAS procedure to estimate what trade flows would be if total exports and imports of all countries were all equal to \( n \). This means applying the Stone-Brown procedure to the actual trade matrix \( X_{ij} \) under the conditions
\[ \sum_j X_{ij} \cdot r_i \cdot s_j = n \]
\[ \sum_i X_{ij} \cdot r_i \cdot s_j = n \]
estimating the hypothetical trade flows \( \hat{X}_{ij} \) as
\[ \hat{X}_{ij} = X_{ij} \cdot r_i \cdot s_j \]  \hspace{1cm} (15)

If the intensity of trading connections between countries was uniform it is clear that if their exports and imports were equal to \( n \)
all bilateral trade flows would be equal to 1. Thus the $\hat{X}_{ij}$ values can be taken to be indicators of the intensity of bilateral trading relations; values in excess of or below 1 mean that the intensity is above or below average.

To determine whether the intensity of relations has increased or decreased between a base year 0 and a current year $t$ it is possible to apply the Stone-Brown procedure to the matrix $X_{0ij}$ under the conditions:

$$\sum_j X_{0ij} r_i s_j = X_{tio}$$

$$\sum_i X_{0ij} r_i s_j = X_{toj}$$

The change in intensity of relations, $\hat{X}_{ij}$ being defined as above, is then given by the ratio

$$X_{tij} \div \hat{X}_{tij}$$

The A model is meant to be no more than a convenient but rough summing up of the interplay of forces which shapes trade flows; hence the application of the RAS procedure to the study of the international trade matrix is a convenient rule of thumb rather than a theoretically satisfactory procedure.

H) The abstract model and the impact of growth on balances of payments

Perhaps the most important implication of the paper is a more optimistic view of the relation between economic growth and balance of payments equilibrium than that implied by the usual approach to the problem of international trade interdependence. We have already mentioned this and will in this section confine ourselves to comparing in mathematical terms the usual approach to ours.

The L model can be taken as a typical example of the demand determined models of international trade interdependence. According to this model the balance of payments $S_i$ is given by

$$\sum_j A_{ij} X_{oj} - X_{oi} \sum_j A_{ji} = S_i$$
and clearly if imports of country i are an increasing function of its income an increase in output of country i must bring about a worsening of its balance of payments by an amount equal to

\[ \frac{\partial S_i}{\partial Y_i} = - \frac{\partial X_{oi}}{\partial Y_i} (\Sigma A_{ji}) < 0. \]

According to the A model there is as much chance that the balance of payments will improve as that it will deteriorate. It is readily checked that for that model (see e.g. (13))

\[ \frac{\partial S_i}{\partial Y_i} = \Sigma (X_{ij} - X_{ji}) / Y_i \]

the sign of which is as likely of being positive as of being negative.

This implication, we think, is a sign that the model, rough as it is, is on the correct lines, at least to the extent that it purports to describe the long term forces to which balances of payments are subjected. There is no evidence whatsoever that countries which grow rapidly have persistent balance of payments difficulties. The contrast between the postwar experience of the United Kingdom and Germany should be enough to convince of this.

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