The role of capital in long-term projection models

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You have asked me, Mr Chairman, to expand upon the role of capital in macroeconomic models. The best way, I think, to give your question the required perspective is to compare the influence of capital upon the growth of national income as it can be deduced from the solution of different types of models. For purposes of illustration, therefore, three different models are chosen, two of which are based upon the assumption of complementarity, whereas the third presupposes substitution.

The first of these models deals with a very simple and imaginary case that allows for only one factor of production.

If we take capital to be this factor and if we suppose the price elasticity of supply to be zero, the supply of new capital is determined by the propensity to save and by the level of national income:

$$\Delta k_t = \alpha y_t,$$

(1)

with $\Delta k_t$ standing for the expansion of the capital stock, $\alpha$ for the propensity to save and $y_t$ for the level of national income.

Capital demand, on the other hand, is the stock of capital required to produce national income at a given level:

$$k_t = \kappa y_t,$$

(2)

$\kappa$ being the capital-output ratio.
The capital stock available at a given point of time is, of course, determined not only by investments in the current year but also by the previous investments. The supply equation, therefore, is:

\[ k_t = \int_0^t \alpha y_s \, dt + k_0 \]  

(3)

\( k_0 \) being the capital stock existing in the base year.

According to the well-known theorem of Harrod and Domar, the equilibrium rate of growth, i.e. the rate of growth that implies neither a shortage nor a surplus of capital, can be deduced from these equations:

\[ \frac{\Delta y}{y} = \frac{\alpha}{\kappa} \]  

(4)

that is to say that the percentage increase — is to be equal to the propensity to save divided by the capital-output ratio. If the rate of saving is 10%, i.e. \( \alpha = 0.10 \), and the capital-output ratio equals 4, \( \kappa = 4 \), then the \( \text{required rate of growth} \) is \( \frac{0.10}{4} = 0.025 \) or 2.5%. It is important to note that this expression (4), representing a basic equilibrium condition of the economy, will also be found in the two other models.

As to our one-factor case, nothing here guarantees that the equilibrium condition (4) shall actually be fulfilled. This is a consequence of the fact that new investments are considered as a predetermined variable:

\[ \Delta k_t = u_t \]  

(5)

where \( u_t \) is an exogenous factor, determined e.g. by entrepreneurial expectations regarding future profits. Hence, according to Eq. (1), we have:

\[ \frac{1}{\alpha} \Delta k_t = \frac{1}{\alpha} u_t \]  

(6)

The actual rate of increase of \( y \) will, therefore, not necessarily satisfy the equilibrium condition as given by Eq. (4). Considering \( \kappa \)
as a technical constant, maintenance of equilibrium with regard to capital requires that either — will be adapted to a level that is compatible with the existing values of \( \kappa \) and \( \alpha \), or, conversely, that \( \alpha \) be adjusted to the prevailing level —. In this very simplified case we have, therefore, two possibilities: either we adjust the rate of growth of national income, or we adjust the rate of saving. This freedom of choice does not exist any longer if we assume the existence of two factors instead of one, while maintaining the hypothesis of complementarity. Let this second factor be labour. We then have to add one labour-demand equation:

\[
a = ye,
\]

(7)

and one equation representing labour supply. Assuming that the labour force increases at fixed percentage rate, labour supply is given by:

\[
a = e^{\pi t},
\]

(8)
e being the base of the natural logarithms and \( \pi \) the expected annual rate of growth per year of the labour force.

Since we have two factors, labour and capital, and we assumed, moreover, that there is no possibility of replacing the former by the latter, or vice versa, it is clear that we have to face now not only an equilibrium condition with regard to capital, but also another one with respect to labour. For regardless to what is happening to capital, the equilibrium condition of national income with respect to labour is uniquely determined by the demand and supply functions for labour. It is found by equating the right-hand sides of the formula:

\[
\frac{\Delta y}{y} = \frac{\pi}{\rho}.
\]

(9)

With regard to capital, the Harrod-Domar theorem still remains valid:

\[
\frac{\Delta y}{y} = \frac{\alpha}{\kappa}.
\]

(10)
If it is desired to maintain equilibrium with regard to labour as well as to capital, economic policy ought to be chosen in such a way that the two equilibrium conditions are satisfied simultaneously. The right-hand side of Eq. (10) ought, therefore, to equal the right-hand side of (9):

$$\alpha = \frac{\pi}{\rho}.$$  \hspace{1cm} (11)

Eq. (11) represents, therefore, the «overall» equilibrium condition in the case of two factors of production if there is no possibility of substitution between the two factors concerned.

This equilibrium condition corresponds more or less with our a priori expectations as to the actual world. For we would expect the rate of saving (α) to vary proportionally to the rate of growth of the working population (π). Also it is reasonable to expect that the rate of saving is dependent upon the capital requirements per unit of output (κ). Finally, it seems only natural that α would vary inversely with the relative labour requirements as given by ρ, the exponent of the labour-demand equation (7). Considering κ, the capital-output ratio and ρ, the elasticity of labour demand as technical constants, whereas π is given by demographic conditions, the full burden of adaptation falls upon α. In a world that is altogether, or largely, governed by complementarity, the possibility of choice between different types of policies, appears therefore rather restricted.

The third model to be discussed allows for the possibility of factor-substitutability. In this type of models mathematics play a fairly important role. For purposes of exposition I will, therefore, rely only upon the final formula's since the derivations are already given in my paper (1). For the same reason the occurrence of technical development will be discarded for the moment (2).

In dealing with substitutability, we may retain not only the investment-supply equation:

$$\triangle k = a y,$$ \hspace{1cm} (12)

but also the supply equation for labour:

$$a = e^{\pi t}.$$ \hspace{1cm} (13)

(1) Presented in this volume, p. 59.
(2) The role of technical development is, however, shown fully in the paper just mentioned.
But the two demand equations are necessarily to be suppressed because of the assumption of substitutability. They are replaced by one production function, indicating the level of production to be expected for different combinations of capital and labour. We will use as such the Cobb-Douglas function:

\[ y = \beta a^\lambda k^\mu. \]  

(14)

This formula, I think, recommends itself not only because of its old standing but also for its convenient and well-known properties.

One important difference with complementarity is the fact that we now are free in our choice of the rate of saving without impairing the feasibility of long-term equilibrium. On the other hand, and this is certainly quite a difference with complementary models, there is no reason any longer for supposing the capital-output ratio to be a technological constant. In the present case, \( \kappa \), consequently, will be considered as an endogenous variable. For these two reasons, maintenance of long-term equilibrium leaves a much wider scope for differences in economic policy here than in the case of complementarity.

Presupposing again, that full employment has to be maintained for both labour and capital, the equilibrium rate of growth can be derived:

\[ \frac{\Delta y}{y} = \frac{\pi \lambda - \alpha (1 - \mu) + a \pi^\lambda t}{\kappa_0 \pi \lambda - \alpha (1 - \mu) + a \pi^\lambda t}, \]

(15)

where \( \kappa_0 \) denotes the capital-output ratio in the base year.

It is important to note that although \( \pi, \alpha, \mu \) and \( \lambda \) are constants, the equilibrium rate of growth does change gradually over time. This is a consequence of the fact that the different variables have to adapt themselves to the particular values of these constants in the period considered. As is easily seen, however, \( \frac{\Delta y}{y} \) approaches an asymptotic value when \( t \) becomes very large:

\[ \lim_{t \to \infty} \frac{\Delta y}{y} = \frac{\pi \lambda}{1 - \mu}. \]

(16)

Thus the equilibrium rate of growth, that guarantees full employment for labour as well as capital, appears to equal \( \pi \lambda \) divided by \( 1 - \mu \). Because these parameters are considered as constants,
the expression (16) can be regarded as the «long-term-equilibrium rate of growth» of the economic system.

A few numerical examples are tabulated in my paper (3). The first column shows different values for the coefficients of the production function, whereas alternative possibilities with regard to \( \pi \) and the efficiency trend \( (\nu) \) are indicated in the heading of the table. If we disregard \( \nu \) and we set \( \lambda \) and \( \mu \) at their traditional values, i.e. .75 for \( \lambda \) and .25 for \( \mu \), a population increase with 1 % appears to result in an increase of national income with, likewise, 1 %.

On the other hand, if apart from a population increase of 1 %, the production function shows annually a shift of also 1 %, the \[ \frac{\Delta y}{y} \]

growth rate of \( - \) will be 2 \( \frac{1}{3} \) %.

The most interesting point with regard to the asymptotic growth rate is, no doubt, the fact that the propensity to save, \( \alpha \), does not enter the picture at all, although its numerical value can be chosen arbitrarily. At first sight it seems strange indeed that the long-term rate of growth should be independent from the prevailing rate of saving. It should be remembered, however, that so long as the economy is moving towards its long-term-equilibrium development, its rate of growth is described by Eq. (15), and so long its growth rate is by no means independent of \( \alpha \). At the same time, however, the economy's capital-output ratio has had ample opportunity to adapt itself from its original value \( \kappa_0 \) to the asymptotic value that is compatible with the given values of \( \alpha, \pi, \lambda \) and \( \mu \):

\[
\lim_{t \to \infty} \kappa = \frac{\alpha (1 - \mu)}{\pi \lambda}.
\] (17)

Remembering that the equilibrium-rate of growth of national product is characterized by:

\[
\frac{\Delta y}{y} = \frac{\pi \lambda}{1 - \mu},
\] (18)

(3) Page 62, table 1.
it follows that Eq. (17) can also be written as:

\[ \frac{\Delta y}{y} = \frac{\alpha}{\kappa} \]  \hspace{1cm} (19)

Algebraically we thus find the same relationship as that implied by the Harrod-Domar theorem. But in the latter case, \( \kappa \) is a technological constant, whereas in the present one the constancy of \( \kappa \) is the result of adaptation to long-term equilibrium. This fact also explains why development, at least equilibrium development, is neutral with respect to the rate of capital formation. For if we consider two countries, both in an equilibrium situation, the production function and the size of working population being the same, the country with the higher value of \( \alpha \) will, according to this formula, also show the higher capital-output ratio. Consequently, in the country with the higher value of \( \alpha \), also total output will be higher since the available capital stock is one of the variables that enters the production function. Thus, in equilibrium development, the propensity to save affects the level of income rather than the rate of growth.

Nevertheless, this picture of neutrality with regard to the rate of saving, discouraging as it might seem to less developed economies, is altered drastically if the traditional rate of saving happens to change for one reason or another. Let us assume that the economy finds itself in a situation of equilibrium development with a growth rate \( \gamma_{lim} \):

\[ \gamma_{lim} = \lim_{t \to \infty} \frac{\Delta y}{y} = \frac{\pi \lambda}{1 - \mu} \]  \hspace{1cm} (20)

whereas the prevailing rate of saving equals \( \alpha_0 \). Now, suppose that the latter changes at \( t = 0 \) from \( \alpha_0 \) to \( \alpha_1 \). In this case the rate of growth will change also and we find:

\[ \frac{\Delta y}{y} = \frac{\pi \lambda}{1 - \mu} = \left[ 1 + \mu \frac{\alpha_1 - \alpha_0}{\alpha_1 \left( e^{\pi \lambda t} - 1 \right) + \alpha_0} \right] \]  \hspace{1cm} (21)

The formula shows that in the case of a changing saving ratio, the difference between the old rate of saving, \( \alpha_0 \), and the new rate of saving, \( \alpha_1 \), is of crucial importance for the rate of growth. Nevertheless, even if this change of \( \alpha \) means a sustained change, i.e. if \( \alpha \) will be maintained at the same level, the influence is shown
to taper off gradually for increasing values of \( t \). Consequently, the income raising effect of a change in the rate of saving is the greatest in the initial year \(( t = 0)\). For this year, the rate of growth is given by:

\[
\gamma_0 = \frac{\pi \lambda}{1 - \mu} \left( 1 + \frac{\alpha_1 - \alpha_0}{\alpha_0} \right).
\]  

(22)

The percentage difference between this «take-off rate» \((\gamma_0)\) and the equilibrium rate \((\gamma_{lim})\) appears to be proportional to the relative difference between the two rates of saving:

\[
\frac{\gamma_0 - \gamma_{lim}}{\gamma_{lim}} = \mu \frac{\alpha_1 - \alpha_0}{\alpha_0}.
\]  

(23)

Although this difference with the equilibrium-rate of growth must ultimately vanish altogether, a permanent shift in the level of national income is to be expected as the result of a sustained change of the rate of saving. The magnitude of this shift from the old level \((y'_t)\) to the new one \((y_t)\) is given by:

\[
\lim_{t \to \infty} \frac{y_t}{y'_t} = \frac{1 - \mu}{\alpha_0} \left( \frac{\alpha_1}{\alpha_0} \right).
\]  

(24)

A tabulation of the expected shifts for different values of \(\frac{\alpha_1}{\alpha_0}\) and \(\mu\) is given in the Appendix (Table 2). Since \(\mu\), as a rule, can be taken to be less than .50, the ultimate percentage shift will be smaller than the corresponding ratio \(\frac{\alpha_1}{\alpha_0}\). In the case of the traditional values of .75 and .25 for resp. \(\lambda\) and \(\mu\), an increase of the saving ratio by 25 % will lead to a relative shift of national income by only \((1.25)^{1/3} - 1\) or 7.7 %.

For this reason it becomes increasingly difficult to obtain an appreciable shift in the equilibrium level of national income by pushing up the rate of saving once savings take already a substan-
tial part of income. Moreover, it is easy to verify that the level of income available for consumption cannot be increased any more by increasing $\alpha$, as soon as the latter surpasses the numerical value of $\mu$. There are, of course, other possibilities to increase national product apart from saving. The greater part of these possibilities is connected with technical and institutional developments. But Mr Chairman, you will allow me to refer to my paper for this particular point.