The originality of Descartes's conception of analysis as discovery

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According to Descartes, his *Meditations* employ the method of analysis. This method of proof, says Descartes, "...shows the true way by means of which the thing in question was discovered methodically and as it were *a priori*." 1 Such a definition of analysis poses a problem that seems to have attracted little attention among commentators until now, namely, why Descartes considers analysis a method of discovery whereas the Scholastic tradition asserts just the contrary, that synthesis allows the discovery of new things whereas analysis is a method for judging findings or verifying them *a posteriori*. What is more, the definition that Descartes gives of his analysis contains another divergence, this time brought up by the commentators 2, for he curiously considers analysis an *a priori* movement, not an *a posteriori* movement as dictated by tradition.

These issues are all the more important as the analytical method acquired particular prestige in the 17th century. This prestige concerned not only philosophy (Descartes, Leibniz, Locke, etc.) but science as well. Thus, algebra lost some of its rough edges and gradually took on the name of "analysis," 3 Descartes created a type of geometry that was soon dubbed "analytic," and Leibniz developed his *Analysis infinitorum* or differential and integral calculus. If the scientific revolution and the new philosophies that were associated with it thus attached so much importance to a method that, after all, had already been known in ancient Greece,

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3 From François Viète, *In Artem Analyticem Isagoge* (Turonis, 1591).
was it not due to a change of the meaning of analysis or at least a rearranging of some of its functions? This is the question that we propose to tackle while trying to determine to what extent Cartesian analysis breaks with the forms of analysis that preceded it.

To do this, we shall first (1) examine the teachings of Scholastic philosophy concerning the connections between analysis and discovery in general, then (2) study the logical possibilities of discovery through analysis that draw their inspiration from Aristotle's Analytics and were developed extensively by the School of Padua in the 16th century, and finally (3) try to determine whether the mathematical definition of analysis derived from the works of Euclid and Pappus gives grounds for the correlation with discovery.

1) The link between analysis and discovery according to the Scholastics

One might well think that Descartes was completely unaware of the contradiction between his definition of analysis and the teachings of the Scholastics. Had not he always exhibited a true disdain for the latter? However, in September 1640, that is, four months before writing his text on analysis, Descartes began girding for the Jesuit Fathers' objections to his Meditations. This involved looking for an "abstract of the whole of scholastic philosophy; this would save [him] the time it would take to read their huge tomes."\(^4\) He finally found what he was looking for in the shops of Leiden, namely, Eustachius a Sancto Paulo's Summa Philosophiae. There is reason to believe that Descartes read this opus with interest, for two months later he referred to it as "the best book of its kind ever made".\(^5\) Later he said that he would definitely have chosen the Summa Philosophiae as the basis for refuting the School's

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\(^4\)This question leads to another one, namely, how the Cartesian idea of analysis marked modern science and philosophy until Kant. For more on this, see B. Timmermans, *La Résolution des Problèmes de Descartes à Kant. L’Analyse à l’Age de la Révolution Scientifique* (Paris, 1995).

\(^5\)Letter to Mersenne, September 30, 1640, AT III, 185.

\(^6\)Letter to Mersenne, November 11, 1640, AT III, 232.
doctrine if he had not given up such a project. Now, Eustachius a Sancto Paulo's *Summa Philosophiae* is categorical on the subject of analysis versus synthesis:

"[Analysis of a definition] is perfectly compatible with the doctrines for teaching and learning; for the other disciplines of discovery, the other method, that is the composite or synthetic method, must be used."  

According to the Scholastic doctrine as stated by Eustachius a Sancto Paulo, analysis thus plays a part in the teaching of well-known things and in all processes of appraisal, assessment, or judgment. This is easy to explain if one considers that, etymologically, analysis breaks links (*ana-luein*), goes backward, or climbs back up from the consequences to the principles. Starting from an observation, proposition, or question, it wends its way back to the simplest, surest items of knowledge that shed light on and verify said starting point. Synthesis, on the contrary, comes into play in discovery, for it descends from the principle to its consequences and thus discovers, starting from simple, known principles, their previously unknown, complex consequences. Thomas Aquinas said nothing else when he wrote:

"Human reasoning in the order of inquiry and discovery starts from certain truths quite simply understood, namely first principles, and then in the order of judgment by analysis returns to first principles, in the light of which it studies what has been found."

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7A. Baillet, *La Vie de Monsieur Descartes* (reed. Hildesheim-New York, 1972), II, 122. We know that Descartes cherished a project at least from September 1640 to write a philosophy course that would contrast and compare the Scholastics' principles and those of his own philosophy (*Letter to Mersenne*, September 30, 1640, AT III, 185), but later gave up the idea (*Letter to Mersenne*, December 22, 1641, AT III, 470).


"Reason reaches conclusions from principles by way of discovery, and by way of judgment examines the conclusions which have been found, analyzing them back to the principles."\textsuperscript{10}

Other authors, such as Abelard, Albertus Magnus, and John Scot Erigena, also related analysis to judgment and synthesis to discovery.\textsuperscript{11} This conception goes back to Boethius, who himself claimed Aristotle and Cicero as his roots.\textsuperscript{12} Indeed, Boethius translated the title of Aristotle's \textit{Analytics} by \textit{Resolutaria} and asserted that this work developed the method that served to judge the form and value of a given reasoning whereas the \textit{Topics} showed how to find new reasons or arguments.\textsuperscript{13} There thus exists a tradition from Boethius to Eustachius a


\textsuperscript{12}He backs up his position with a famous passage in Cicero's \textit{Topics} that breaks logic (\textit{ars disserendi}) down into two parts, \textit{inventio} and \textit{judicio}: "Every systematic treatment of argumentation [\textit{ars disserendi}] has two branches, one concerned with invention of arguments and the other with judgement of their validity; Aristotle was the founder of both in my opinion. The Stoics have worked in only one of the two fields. That is to say, they have followed diligently the ways of judgement by means of the science which they call dialectic, but they have totally neglected the art which is called topics, an art which is both more useful and certainly prior in the order of nature"(\textit{Topics}, II, 6, tr. H.M. Hubbell (Harvard, 1949)). Compare also this text with Aristotle's \textit{Rhetoric}, 1355 a 9, 1356 a 25-27, 1359 b 9-11.

\textsuperscript{13}There is no doubt that Aristotle already gave this treatise the name \textit{Analytics}. See P. Moreau, \textit{Les Listes Anciennes des Ouvrages d'Aristote} (Louvain, 1951), 54, 87.

\textsuperscript{14}Cf. Boethius, \textit{In Topica Ciceronis Commentaria}, I, tr. 64, c. 1044-1047; \textit{De Topicis Differentiis}, I, tr. 64, c. 1173-1174; \textit{In Porphyrium Commentarium}, I, tr. 64, c. 71.
Sancto Paulo, via John Scot Erigena, Thomas Aquinas, and Abelard, that connects analysis to judgment and synthesis to invention. Descartes opposed this tradition, and his correspondence with Father Mersenne intimates that he did so knowingly.

However, perhaps Descartes was not unaware either of the existence of certain derogations allowing analysis to be connected to discovery? Of what did these derogations consist? In discovery the well-known principle that is taken as the starting point to discover its unknown consequences can sometimes be not only first in a certain order but last in another order. This is what happens, Aristotle explains, in moral deliberation, for moral deliberation starts from the end to achieve, which is both first in the order of reasons or intentions, since it is well known to man, and last in the order of being or execution. Consequently, the search for the means to achieve this end is related, from the standpoint of being, to analysis, climbing back from the consequences to find the principles. At the same time, this analysis is a discovery when seen from the point of view of reasons or intention, since the end is the simple, better-known principle that leads us to discover the appropriate unknown means to achieve it.

"We deliberate not about ends but about what contributes to ends. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall convince, nor a statesman whether he shall produce law and order, nor does any one else deliberate about his end. Having set the end they consider how and by what means it is so attained; and if it seems to be produced by several means they consider by which it is most easily and best produced, while if it is achieved by one only they consider how it will be achieved by this and by what means this will be achieved, till they come to the first cause, which in the order of discovery is last. For the person who deliberates seems to inquire and analyse in the way described as though he were analysing a geometrical construction (not all inquiry appears to be deliberation - for instance,
mathematical inquiries - but all deliberation is inquiry), and what is last in the order of analysis seems to be first in the order of becoming."\(^{15}\)

In other words, analysis, which by definition always goes from the consequences to the principles, can be considered heuristic from a certain point of view when, from this point of view, it is synthetic, that is, when it starts from something that, while a consequence, is also a simple, well-known principle from another point of view that leads us to discover unknown things.\(^{16}\) Aristotle explains that that is what happens not only in moral deliberation, but in geometry as well. Thus, a figure is known at the outset and one tries to discover from that starting point the means for constructing it. These means are first in the order of action but last in the order of discovery. Galen, for his part, compares this way of proceeding to the method used by physicians and architects, who likewise start from an end (for example, the idea of a healthy body or a house) that is both last in the order of execution and first in the order of intentions, for it is known *a priori*, and lead us to discover, starting from this idea, the means to achieve it.

"By analysis or dialysis we mean both that which derives from the idea of the house as that which derives from the knowledge of the human body. Just as,


\(^{16}\)That, in any event, is how Thomas Aquinas interpreted this passage in his *Summa theologiae*, I-II, Q. 14, a. 5, repl. : "Every inquiry has to start from some principle. If this comes first both in our knowledge and in reality, the discourse will not be analytic, but rather synthetic, for to proceed from causes to effects is a putting together, since causes are simpler than effects. If, however, the principle which comes first to our knowledge is later in reality, the discourse will be analytic, as when our judgment deals with known effects which we resolve into their simple causes. Now the principle in deliberative inquiry is the end, which comes first in intention though last in realization. On these grounds the discourse must be analytic, that is to say, starting from that which is intended in the future and continuing until it arrives at decision on what is to be here and now." (tr. T. Gilby O.P. (New York, 1970)).
indeed, God, nature, or the house built from scratch precede their parts, which are engendered by custom, we today learn to build houses.\textsuperscript{17}

Thus, without contradicting the definition of analysis as going from consequences to principles, we can use it as a method of discovery if the consequence from which we start is, when seen from another point of view, a first or \textit{a priori} idea from which we can draw new or unknown information.

This particular idea of analysis was rather favorably received by Renaissance doctors and experimenters\textsuperscript{18} and, in the same vein, by the Masters of the School of Padua, who tried to give it the necessary foundations, that is, to establish the logical conditions of its validity. We shall consider in the next section how successful this attempt was and whether Descartes's writings reflected it, but first let us see to what extent Galen's views may shed light on Descartes's texts.

If the consequence from which analysis starts is at the same time an \textit{a priori principle} of discovery when seen from another point of view, the passage in the \textit{Second Replies} in which Descartes talks about analysis that "shows the true way by means of which the thing in question was discovered methodically and as it were \textit{a priori}" can already be seen in a different way. In his edition of the works of Descartes, Ferdinand Alquié considered the words "\textit{et tanquam a priori}" "completely incomprehensible." Thus, \textit{a priori} reasoning, Alquié explains, is the reasoning that goes from the cause to the effect, from the principle to the

\textsuperscript{17}\textit{De Constitutione Artis Medicæ ad Patrophilum}, ed. C. G. Kühn (Hildesheim, 1964), I, 231. See also \textit{Arte Medica}, \textit{ibid.}, 305.

consequence, and *a posteriori* reasoning is the reasoning that goes from the effects back to the causes, from the consequences to the principles. But, Alquié adds, it so happens that analysis goes back from the consequences to the principles, although it is said here to operate as *a priori* reasoning:

"le raisonnement *a priori*... est celui qui va de la cause à l'effet, du principe à la conséquence, et le raisonnement *a posteriori* est celui qui remonte des effets aux causes, des conséquences aux principes. Mais il se trouve précisément." Alquié adds, "que l'analyse remonte des conséquences aux principes, alors qu'elle est dite ici opérer comme *a priori*."

Now, the contradiction that Alquié pinpointed might disappear if one assumes that Descartes sees analysis above all through the prism of its heuristic power. Thus, since analysis is above all heuristic, our point of view is that of the order of reasons, and the starting point of analysis may thus appear to be a *principle* that is well known *a priori*. This would explain why Descartes, on the one hand, describes synthesis as being "a directly opposite method where the search is, as it were, *a posteriori*" (as if he were still adopting the point of view of the order of reasons), yet adds, on the other hand and between parentheses, that "the proof itself is often more *a priori* than it is in the analytic method." In the latter statement he is talking about the proof rather than the discovery of the proof, which might mean that in this remark between parentheses he is leaving the order of reasons (temporarily) for the order of things or subject matters.

This interpretation, which narrows the gap between Descartes and Aristotle and, above all, Galen, is definitely tempting, but it cannot satisfy us fully, for it supposes that Cartesian

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20 Following in Galen's footsteps, the Discourse on Method already associated medicine (AT VI, 62-63) and architecture (AT VI, 12) with the search for a path to find truth. Much later, Robert Hooke gave a definition of analysis that was amazingly close to Descartes's Second Replies and Galen's *Constitutione Artis Medicæ*, to wit:
analysis is heuristic because it descends the order of reasons as it works its way back up the order of things, starting from well-known principles to discover their unknown consequences in the order of reasons that are also their causes in the order of things. However, the order that Descartes follows is not vertical, but horizontal. Descartes stresses that the proof of each question does not depend solely on everything that precedes it, but also to a great extent on that which follows it, so that "those who do not bother to grasp the proper order of my arguments and the connection between them, but merely try to quarrel with individual passages, will not get much benefit from [my work]." The order is not imposed on us by some natural hierarchy; it is up to us to arrange it. So, there are myriad orders, myriad ways of arranging things. The method that Descartes advocates goes neither from principles to consequences nor from effects to causes, but simply from what is easiest to what is most difficult to consider. Consequently, if Descartes sees discovery as analysis, this cannot be because, as Galen stated or Aristotle laid down in his Nichomachean Ethics, the a priori given from which it starts is a principle in a certain order and a consequence in another one. One cannot content oneself with reversing two predefined orders, that of things and that of reasons, to explain the heuristic nature of Cartesian analysis when Descartes himself claims that the order depends only on the way we consider things.

"[Analysis] begins from the highest, most general and universal principles or causes of things, and branches itself out into the more particular and subordinate... [It] resembles fitly enough by the example of an architect, who hath a full comprehension of what he designs to do, and acts accordingly... From a hypothesis being supposed, on a premeditated design, all the phenomena of the subject will be a priori foretold, and the effects naturally follow, as proceeding from a cause so and so qualified and limited" (R. Hooke's Posthumous Works, 330ff, quoted by Dugald Stewart, Elements of the Philosophy of the Human Mind (Edinburgh, 1821), II, 392-393).

21 Fifth Replies, AT VII, 379.

22 Rules for the Direction of the Mind, title of Rule V.

23 Letter to Mersenne, December 24, 1640.

24 Principles of Philosophy, I, 55.
The analytic method as Descartes sees it thus cannot be heuristic in the Scholastic sense, that is, because it ascends or descends a pre-established order. It discovers the real dependency between causes and effects gradually, as it follows an order arranged in a certain way by a subject. The problem is how to characterize such a method more precisely. Should it be defined in logical terms (point 2), as the Masters of Padua tried to do, or in mathematical terms (point 3), in line with Euclid and Pappus?

2) The logical possibilities of discovery by analysis

Descartes was not the first one to have considered that order is not inherent in things but boils down to the way they are arranged and the method or way (via) that follows this order does not comply with some natural hierarchy but goes simply from what we find easiest to understand to what is least easy to understand. Zabarella - one of the main representatives of the School of Padua in the second half of the 16th century - already defended the same opinion and for which, one may add, he was roundly criticized by his Platonist-influenced colleague, Piccolomini. His *De Regressu* is doubtless the most complete culmination of a long tradition that arises with Aristotle and Galen and continues via Averroes, William of Ockham, Robert Grosseteste, Jacopo da Forli, and Agostino Nifo. All of these thinkers tried to various degrees to apply the heuristic power of analysis, as defined by Aristotle and Galen, to physics, while keeping the logical validity of analysis as intact as possible. To do this, they took from Aristotle's *Analytics* a method of proof that would permit the discovery, starting from what is known to us, of the causes or principles of things, and would do so with a maximum of certitude.

What does Aristotle's *Analytics* teach us on this score? The search for principles may proceed by induction, but we know that this procedure is not certain and is connected more to

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25 *Posterior Analytics*, II, 19.
dialectics than to science." On the other hand, there is another method - called the "demonstration of fact" (demonstratio quia) by Aristotle - that is not always utilizable but is much more certain and was considered from Averroes on to be a certain type of analysis. This method consists in starting from a consequence or effect and establishing a connection (called "convertible") between this effect and the general observation that is the best known and at the same time the closest to this effect. Aristotle gives as an example of an effect the fact that some planets do not twinkle and as an example of a general observation the fact that all that does not twinkle is near. Consequently, the conclusion that necessarily follows is that some planets are near. Aristotle explains that one thereby discovers a new fact (the closeness of some planets). This fact may also be the cause of the effect from which we started, that is to say, that those planets do not twinkle, but one cannot be as sure that this new fact is actually the cause of the effect from which one started.

The demonstratio quia leads to a fact that is only a possible cause but not necessarily a real cause. This is because the starting premises are not absolutely certain and one can perfectly well draw truth from falsehood. That, in fact, is the true problem of logical analysis in general. Since it starts from an effect that is better known to us but uncertain itself, it cannot claim to yield absolute truth. Indeed, Aristotle stressed this flaw explicitly while observing that it disappeared in fields such as mathematics in which the relations of implication are reversible.

"If it were impossible to prove truth from falsehood, it would be easy to make an analysis; for they would convert from necessity... In mathematics things convert more because they assume nothing accidental... but only definitions."28

26 Topics, I, 12.

27 Posterior Analytics, I, 13.

28 Posterior Analytics, I, 12, 78a 7, trad. J. Barnes.
All of the authors who have tackled analysis have had to deal with this problem. The question is whether the logicians of Padua - especially Zabarella - were able to handle it in such a way as to feel authorized to say, with Descartes, that analysis "shows the true way by means of which the thing in question was discovered methodically."

At first glance, the writings of Zabarella and Descartes appear to be amazingly close. Both philosophers make the distinction between order and the way (via) or method of proof, namely, that order is only the arrangement of subject matters, whereas the method of proof always proceeds from the known (or what is easiest to know) to the unknown (or what is least easy to know)\(^2\). This similarity between the ideas of Zabarella and Descartes might strengthen the thesis defended by E. Cassirer\(^3\), J.H. Randall\(^3\), A.C. Crombie\(^3\) and W.F. Edwards\(^3\), that the regressive method of *demonstratio quia* developed most particularly by the School of Padua was a forerunner on many accounts of modern science's interest in observation and experimentation, both of which would be handled by mathematics as well as logic. Did not Galileo, who himself taught mathematics in Padua for eighteen years, write that if analysis was used correctly it was an excellent means of discovery?\(^4\)


\(^{3}\)*Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* (Berlin, 1922), I, 117ff, 136ff.

\(^{4}\)"The Development of Scientific Method in the School of Padua", *Journal of the History of Ideas*, 1 (1940), 177-206; *The School of Padua and the Emergence of Modern Science* (Padua, 1961).


Nevertheless, Descartes's conception of analysis differs radically from that of Zabarella on a number of points that in any event refute the idea of a direct filiation between the two. First and most obviously is the fact that Zabarella's method proceeds by syllogisms\(^3\), whereas Descartes's method relies on the reasoning of geometry\(^4\). This difference is important, for Descartes is known to have held that "with regard to logic, syllogisms and most of its other techniques are of less use for learning things than for explaining to others the things one already knows or even, as in the art of Lully, for speaking without judgment about matters of which one is ignorant."\(^5\) "Mathematicians alone," Descartes adds, "have been able to find any demonstrations."\(^6\) That is why, instead of engaging in logic, it is better to make and follow "those long chains composed of very simple and easy reasonings, which geometers customarily use to arrive at their most difficult demonstrations."\(^7\)

Secondly, the *demonstratio quia* defended by Zabarella proceeds *a posteriori*, that is, backwards, or from effects to their causes, whereas the Cartesian resolution of the analysis is, as we have seen, "as if it were *a priori*.\(^8\) Yet how should one take this last assertion? It should not be misconstrued as meaning that analysis would paradoxically proceed from the causes to the effects or from the principles to the consequences but rather that it starts from the primary idea of the dependency or relation between cause and effect.\(^9\) The analyst uses the (supposed) order or dependency between things to build a foundation for their knowledge, that is, to discover from this their various forms or expressions. He does not start from either a cause or

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\(^3\) *Op. cit.*, cap. 18.

\(^4\) *AT VII*, 155-156.

\(^5\) *AT VI*, 17.

\(^6\) *AT VI*, 19.


\(^8\) *AT VII*, 155.
an effect but from the relation between the two, which he considers a "foundation" and from which he hopes to draw a "solution." Thus, the following passage may explain why Descartes talks about an "a priori" analysis (or solution).

"As for the objections of your analysts, I shall try to solve [soudre, from the Latin resolvere] them without setting them out; that is, I shall lay the foundations, from which those who know the objections to them may derive their solution, without teaching them to those who have never heard of them." 42

We shall come back to the mathematical interpretation that should be given to this "a priori" procedure later in this article.

Thirdly, Descartes differs from Zabarella in that he does not make the success of analysis contingent on that of synthesis. Zabarella and his Paduan predecessors were not unaware of the logical problem of analysis. They thus knew that demonstratio quia, in starting from uncertain premises, will discover a fact that may be the cause thereof but for which there is no a priori proof that it actually is the cause. To prove the latter, one must further show, according to Aristotelian logic, that the fact in question is the middle term of a new syllogism, the starting point of which is the real, well-known cause and the conclusion the effect (demonstratio propter quid). This means, moreover, that one must think through

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41 Although Descartes frequently used the expression "a priori" to mean "from the causes" and "a posteriori" to mean "from the effects" (The World, AT IX, 47; Letter to the 'Hyperaspistes', August 1641, AT III, 421-2; Letter to F. Vatier, February 22, 1638, AT I, 563), he did so above all, it seems, to provide an explanation using the School's terms, as he put it: "...to express myself in scholastics terms" (The World, op. cit.). However, he also used "a priori" in a narrower sense to characterize "the key and the foundation" of knowledge (Letter to Mersenne, May 10, 1632, AT I, 250), that is, "knowledge of the order" that binds things to each other (ibid.).

42 Letter to Mersenne, December 25, 1639, AT II, 629-630, our emphasis. Compare with Letter to Mersenne, May 10, 1632, AT I, 250. See also Geometry, AT VI, 372 : "we must unravel the difficulty in any way that shows most naturally the relations [la dépendance] between these lines".
(negotiato) the definition or essence of the fact in question most clearly. So, while the type of solution used in demonstratio quia admittedly permits discovery, it does not procure any science; it is "a subordinate procedure, the servant of the demonstrative...The end of the demonstrative method is a perfect science, which is knowledge of things through their causes; but the end of the resolutive method is discovery rather than science". So, according to Zabarella, analysis demands synthesis (demonstratio propter quid) as well as reflection (negotiatio) if it is to be validated, whereas according to Descartes, it suffices to "engage the minds of those who are eager to learn...," so that "...if the reader is willing to follow it and give sufficient attention to all points, he will make the thing his own and understand it just as perfectly as if he had discovered it for himself." 

One must not conclude from the preceding that Descartes was unaware of the logical problem of analysis. On the contrary, he took a very different path from that of Zabarella to get around it by using mathematics, which played a crucial role in his analytical method, instead of logic. This last point rules out any idea of a direct derivation from Zabarella's thinking once and for all, pointing to Euclid and Pappus instead.

3) The mathematical possibilities of discovery through analysis

43 De Methodis Libri Quatuor, Liber de Regressu, III, cap. 18; tr. J.H. Randall, op. cit., (1940), 198.

44 Not only is discovery not science; discovery occasionally falls short of its goal, in which case it is reduced to a search. The inventio medi (discovery of the middle term of a syllogism), which is often identified with analysis, is also sometimes separate from it. In Leiden, for instance, Petrus Bertius (1565-1629) thus differentiates analysis (the search for the principles) - from euporia (discovery of the middle term) and the genesis (the synthetic exposition) of a syllogism (Logicae Peripateticae Libri Sex (Argentorati, 1616), 117, 142, 152, quoted by W. Risse, Die Logik der Neuzeit (Stuttgart-Bad Cannstatt, 1964), I, 466). Accordingly, analysis, which is the victim of the logical problem raised by Aristotle, i.e., that truth can be drawn from falsity, seems to be stuck between the (uncompleted) search for the causes and the discovery of the (uncertain) means for proving a cause.

45 AT VII, 156 and 155.
The 4th-century mathematician Pappus of Alexandria gave us the most important text available to us today to define the meaning of analysis as used by the Greek geometers, namely, his *Synagoge* (Collection). The excerpt below comes from Heath's translation:

"Analysis then takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis; for in analysis we assume that which is sought as if it were (already) done, and we inquire what it is from what it results, and again what is the antecedent cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles, and such a method we call analysis as being solution backwards."  

Analysis thus starts from something that is sought but considered to be admitted. This recalls the process already described by Aristotle and Galen concerning ethics, architecture, and medicine, that is, going backward from the consequence (which is "sought" or less known in itself) to the principle (which is better known in itself) can also be, from another point of view, going forward from a principle (which is "admitted" or known by us) toward its consequences (less known by us). However, Pappus does differ from Galen and Aristotle in that the analyst, says Pappus, examines "from what it results, and again what is the antecedent of the things...," that is, he searches for both what is derived from his starting point and that from which his starting point is derived. One way to understand this way of thinking is to follow Hintikka's example and interpret the word *akolouthon* as meaning "concomitance" or "that which goes with" instead of "consequence."  

In other words, before worrying about what is the cause and what is the effect, what is principle and what is consequence, the analyst will focus only on the relations or operations that create a correspondence between two different terms (one "unknown" but considered admitted, the other known). According to

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Hintikka, these operations are the auxiliary geometric constructions that Euclid, Pappus, and Diophantus used to solve mathematical problems. What appears certain, in any case, is that the nascent algebra of the 16th century was wonderfully well suited for an interpretation of this type, since it used arithmetic operations to establish correspondences between unknown and known quantities without even calling geometric figures into play.⁴⁸

Descartes took up this characterization of analysis for his own purposes as the following passage shows:

"We are well aware that the geometers of antiquity employed a sort of analysis which they went on to apply to the solution of every problem, though they begrudged revealing it to posterity. At the present time a sort of arithmetic called 'algebra' is flourishing, and this is achieving for numbers what the ancients did for figures. The two disciplines are simply the spontaneous fruits which have sprung from the innate principles of this method."⁴⁹

There is no doubt that his Second Replies refers to analysis of this type, for in it he uses almost the same words: "...the ancient geometers...were not utterly ignorant of analysis, but...they had such a high regard for it that they kept it to themselves like a sacred mystery."⁵⁰

⁴⁸François Viète is the first one to have qualified algebra as a form of analysis. He explains that his analysis allows one to find equality by means of the proportion between the magnitude that one is seeking and the magnitude that is given ("one finds an equation by the proportion between a term that is to be found and the given terms... therefore the whole analytic art... may be called the science of correct discovery" (In Artem Analyticem Isagoge (Turonis, 1591), 1).

⁴⁹Rules for the Direction of the Mind, IV, AT X, 373.

⁵⁰AT VII, 156.
Thus, the idea emerged that analysis according to Descartes might not go backward from the consequences to the principles any more than it divided a complex whole into simple parts, but revealed the correspondences, order, or relations between different terms, knowns and unknowns, consequences and principles, wholes and parts. Descartes gives us the following example of this new approach. Thus, in mathematics, there are two ways of solving proportions, the direct method and the indirect method. The former, which is synthetic, presents no difficulties. It consists in following a given order "without ever interrupting the order." For example, given the magnitudes 3 and 6, one can easily find a third magnitude, X, in continued proportion, that is, 12, as expressed by the following equation:

\[ \frac{3}{6} = \frac{6}{X} \]

The second method, which is analytic, presents a "quite different type of problem", for it consists in treating the unknowns as if they were known. In this case, given the two extremes, that is, 3 and 12, one searches for the mean proportional, X, viz.:

\[ \frac{3}{X} = \frac{X}{12} \]

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51 Let us reread the second precept of Descartes's Discourse on Method: "...to divide each of the difficulties I examined into as many parts as possible and as may be required in order to resolve them better" (AT VI, 18, our emphasis). A clear distinction is made between division and resolution: division is one of the means, one of the steps to resolve or analyze the difficulties ("in order to" is rendered by the Latin ad in Etienne de Courcelles' translation, reviewed and corrected by Descartes), but the resolution or analysis requires other operations as well: first listing the givens of the problem, then determining the links between and composition of the simple terms resulting from the division, and finally intuiting the obviousness of the solution. Descartes never reduced analysis to division. Nicolas Poisson was the first one to make this mistake in his Commentaire, ou Remarques sur la Méthode de R. Descartes, (Vendôme, 1670), 78.

52 Rules for the Direction of the Mind, XVII, and the start of Geometry.

Here Descartes states that the order is interrupted or "complicated". To (re-)establish it, the unknown terms (X) must be considered to be of the same nature as the known terms. Otherwise, they cannot be placed in an equation with the knowns.

Another illustration of this method applied to both metaphysics and mathematics is found in the beginning of *Geometry*. Of course, mathematics works with lines or algebraic quantities, whereas metaphysics handles not only lines and quantities, but things in general. Still, the prescription that Descartes gives in *Geometry* seems to be valid in both areas:

"If... we wish to solve any problem..., then, making no distinction between known and unknown lines [or quantities or things], we must unravel the difficulty in any way that shows most naturally the relations between these lines [quantities, things], until we find it possible to express a single quantity [line, thing] in two ways. This will constitute an equation, since the terms of one of these two expressions are together equal to the terms of the other."54

The method of analysis common to both mathematics and metaphysics would thus consist in, notably, setting up an equation or correspondance between the known and unknown. For example, one could say that, while doubting everything, I take everything that surrounds me as being so many unknown or poorly known things. On the other hand, given my doubts, I know myself also to be a thinking thing and can thus take the thinking subject that I am to be a known thing that has been placed in a correspondance with the unknown by the very fact of doubting.

By this means Descartes managed to get round the logical problem of analysis in a different way than the logicians did. It was a matter no longer of drawing an uncertain consequence from a recognized principle or deducing truth from falsehood, but of bringing to

light the relations (operations) that connected the terms that one was taking into account, without supplying any prior hypothesis as to their order or rank, truth or falsity.

If we continue in this vein, we can understand that analysis may be said to be *a priori* because, instead of following a given (*i.e.* already known) order from principles to consequences, it works out *a priori* an unknown order.

"...to work out an order is no mean feat, as our method makes clear throughout, that being virtually its entire message. But there is no difficulty whatever in recognizing an order since we have come upon one."

"... there are innumerable instances of order, each one different from the other, yet all regular... what we shall do is to invent an order...," [knowing well that it is] "...a matter for individual choice..."

In mathematics, for example, one can choose *a priori* several units or common measures, the "bases and foundations", as Descartes calls them, of all the relations that can be established between known and unknown things. Similarly, in philosophy one can take the thinking thing as one's starting point and treat it like an *absolute* as defined by Descartes, that is, not the simplest in itself, but the simplest in that it enables us to gauge or account for other things most easily. The starting point is thus something that may be treated as *if* it were independent and all the other things that we consider (the relatives) depended on its nature.


56 *Ibid.*, X, AT X, 404


We can also understand how analysis can said to be heuristic. From the standpoint of logic, an analysis that starts from an unknown thing "as if it were known" and "goes backward" to a known thing is not a discovery but a verification or justification. On the contrary, from a mathematical standpoint, an analysis that considers the known and unknown through their relations takes quite another shape. In such a case, the vague and obscure unknown becomes a place that is homogeneous with other places and can accordingly be made to correspond to or placed in an equation with that which is known, obvious and clear. In other words, setting an absolute right from the start (or "a priori"), then using the absolute as a unit of measurement to compare known and unknown terms, reveals the space of the real relations between these known and unknown terms. Mathematical analysis is thus heuristic because it discovers places, that is, because it belongs to the Topics as opposed to the

60 The French translation of *Secondae Responsones*, reviewed and corrected by Descartes, replaces the expression "et tanguam a priori" by "et fait voir comment les effets dependence des causes" ("and reveals how the effects depend on the causes"). Notice that Descartes avoids saying that analysis goes backward from effects to their causes, but says rather that it reveals the dependency (or relations) between the two.

61 "The secret of this technique consists entirely in our attentively noting in all things which is absolute... I call 'absolute' whatever ... is viewed as if were independent, a cause, simple, universal, single, equal, similar, straight, and other qualities of that sort... [Now] we have artificially [de industria] listed 'cause' and 'equal' among the absolutes, although their nature really is relative" (Rules for the Direction of the Mind, VI, AT X, 381-383). "So it will be to the reader's advantage to... think of all knowledge whatever -save knowledge obtained through simple and pure intuition of a single, solitary thing -as resulting from a comparison between two or more things. In fact, the business of human reason consists almost entirely in preparing for this operation..." (ibid., XIV, AT X, 440).

62 The notion of *topos* embraces the fields of physics, mathematics, logic, and rhetoric. In its broadest sense, the *topos* is a general principle, a set of elements or "points" that brings together or concentrates the same property, answers the same question/solves the same problem, or characterizes the same type of arguments. Alexander of Aphrodisias relates that Theophrastus defined it as a principle or element permitting the deduction of the principles of each particular thing (In Aristotelis Topicorum, Libros Octo Commentaria, ed. Wallies, 124). The
Analytics, which are justificative.\textsuperscript{63} The junction, officialized by Descartes, between analysis and topical discovery may thus have been greatly facilitated by the application of algebra to geometry. Thus, in proceeding by convertible relations, analysis gained an algebraic dimension and situated these relations within a uniform, isotropic space no longer governed by medieval notions of hierarchy.

The far-reaching originality of Cartesian analysis thus resides in the fact that, far from limiting analysis's power of topical discovery to algebra or geometry, Descartes extends this power to metaphysics and ethics. So, Descartes's Meditations runs through the various battle tactics the analyst employs to win knowledge. These tactics remain analytical to the very end, to the extent that, although one becomes "accustomed... to leading [one's] mind away from the senses,"\textsuperscript{64} ceaselessly pushing back the unknown or uncertainty that separates it from substance, God, or corporeal bodies, the mind pushes back the unknown precisely to be able to find support therein, just as the algebraist relies on unknown quantities and their differences (or ratios) with known quantities, for he sees substance from the vantage point of his own doubt or uncertainty. Man discovers God through his own flaws or imperfection, he grasps the body by imagining both the link between and the difference with his idea of it. Similarly, when it comes to ethics the Second Replies states clearly that analysis is no longer the careful

place or topos is thus a sort of preferred meeting point in a given region; it is a common procedure for solving several problems of the same type that may be revealed by rhetoric (common places), physics (above and below, fore and aft, right and left), and mathematics (planes, solids, and lines) (\textit{Rhetoric}, 1403 a 18; \textit{Physics}, 205 b 32; Proclus, \textit{In Primum Euclidis Librum Commentarii}, ed. G. Friedlein (Leipzig, 1873) 394-5). In this context, places were considered "hidden" (Quintilian, \textit{Institutio Oratoria}, V, 10) "treasures" (Cicero, \textit{De Finibus Honorum et Malorum}, IV, 4) that had to be discovered or finded.

\textsuperscript{63}This was already the case for Pappus, who explained that "the treasury of analysis" (\textit{topos analuomenos}) consisted in "discovering" (\textit{heuriskein}) the solutions to problems. But Pappus added that this method comprises two orders, analysis and synthesis.

\textsuperscript{64}\textit{Meditations}, AT VII, 52.
deliberation or uncertain regression towards eminent causes of yore. Analysis becomes just the opposite, that is, the resolve to confront the unknown or difficulties and chooses to consider them as such to advance. Analysis, Descartes stresses, leads to discovery only if he who performs it "...is willing to follow it" and pays careful attention to the main difficulty of "making our perception of the primary notions clear and distinct," whereas when it comes to synthesis "...there is no difficulty there, except in the proper deduction of the consequences." That is why all things deserve to be considered and all places are of equal worth, not only in mathematics and physics, but in the order of metaphysical reasons as well. One manages to discover new places or situate them with regard to other places because it is possible to reveal in each of these fields relations or correspondences between different places, both known and unknown. Descartes generalizes or extends to (analytical method) the principle of the homogenization of space that emerged in the Renaissance and came to a head with Newton's enunciation of the principle of inertia. In doing so, he also paved the way for the advent of differential and integral calculus, which is based on the idea of space that is not only homogeneous, but continuous as well.

So, Descartes contradicts the Scholastics by considering analysis to be heuristic, contradicts traditional logic by considering analysis to be a priori, and continues the mathematical tradition of analysis, with the difference that he applies its heuristic power to metaphysics and ethics as well.

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66 Ibid., 156-157.